OCR
Oxford Cambridge and RSA

Monday 27 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4764/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4764/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (24 marks)

1 A car of mass m moves horizontally in a straight line. At time t the car is a distance x from a point O and is moving away from O with speed v. There is a force of magnitude kv^2, where k is a constant, resisting the motion of the car. The car's engine has a constant power P. The terminal speed of the car is U.

(i) Show that

$$mv^2\frac{\mathrm{d}v}{\mathrm{d}x} = P\left(1-\frac{v^3}{U^3}\right).$$

[3]

(ii) Show that the distance moved while the car accelerates from a speed of $\frac{1}{4}U$ to a speed of $\frac{1}{2}U$ is

$$\frac{mU^3}{3P}\ln A,$$

stating the exact value of the constant A. **[6]**

Once the car attains a speed of $\frac{1}{2}U$, no further power is supplied by the car's engine.

(iii) Find, in terms of m, P and U, the time taken for the speed of the car to reduce from $\frac{1}{2}U$ to $\frac{1}{4}U$. **[3]**

2 A thin rigid rod PQ has length $2a$. Its mass per unit length, ρ, is given by $\rho = k\left(1+\frac{x}{2a}\right)$ where x is the distance from P and k is a positive constant. The mass of the rod is M and the moment of inertia of the rod about an axis through P perpendicular to PQ is I.

(i) Show that $I = \frac{14}{9}Ma^2$. **[5]**

The rod is initially at rest with Q vertically below P. It is free to rotate in a vertical plane about a smooth fixed horizontal axis passing through P. The rod is struck a horizontal blow perpendicular to the fixed axis at the point where $x = \frac{3}{2}a$. The magnitude of the impulse of this blow is J.

(ii) Find, in terms of a, J and M, the initial angular speed of the rod. **[2]**

(iii) Find, in terms of a, g and M, the set of values of J for which the rod makes complete revolutions. **[5]**

Section B (48 marks)

3 Fig. 3 shows a uniform rigid rod AB of length $2a$ and mass $2m$. The rod is freely hinged at A so that it can rotate in a vertical plane. One end of a light inextensible string of length l is attached to B. The string passes over a small smooth fixed pulley at C, where C is vertically above A and AC = $6a$. A particle of mass λm, where λ is a positive constant, is attached to the other end of the string and hangs freely, vertically below C. The rod makes an angle θ with the upward vertical, where $0 \leqslant \theta \leqslant \pi$. You may assume that the particle does not interfere with the rod AB or the section of the string BC.

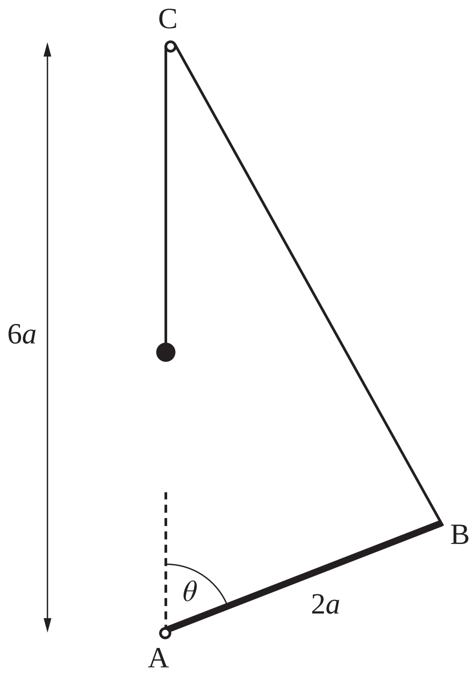

Fig. 3

(i) Find the potential energy, V, of the system relative to a situation in which the rod AB is horizontal, and hence show that

$$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 2mga\sin\theta\left(\frac{3\lambda}{\sqrt{10-6\cos\theta}} - 1\right).$$ **[6]**

(ii) Show that $\theta = 0$ and $\theta = \pi$ are the only values of θ for which the system is in equilibrium whatever the value of λ. **[2]**

(iii) Show that, if there is a third value of θ for which the system is in equilibrium, then $\frac{2}{3} < \lambda < \frac{4}{3}$. **[4]**

(iv) Given that there are three positions of equilibrium, establish whether each of these positions is stable or unstable. **[10]**

It is given that, for small values of θ,

$$\frac{\mathrm{d}V}{\mathrm{d}\theta} \approx 2mga\left[\left(\tfrac{3}{2}\lambda - 1\right)\theta - \left(\tfrac{13}{16}\lambda - \tfrac{1}{6}\right)\theta^3\right].$$

(v) Investigate the stability of the equilibrium position given by $\theta = 0$ in the case when $\lambda = \frac{2}{3}$. **[2]**

4 A raindrop falls from rest through a stationary cloud. The raindrop has mass m and speed v when it has fallen a distance x. You may assume that resistances to motion are negligible.

(i) Derive the equation of motion

$$mv\frac{\mathrm{d}v}{\mathrm{d}x} + v^2\frac{\mathrm{d}m}{\mathrm{d}x} = mg.$$

[4]

Initially the mass of the raindrop is m_0. Two different models for the mass of the raindrop are suggested.

In the first model $m = m_0 e^{k_1 x}$, where k_1 is a positive constant.

(ii) Show that

$$v^2 = \frac{g}{k_1}\left(1 - e^{-2k_1 x}\right),$$

and hence state, in terms of g and k_1, the terminal velocity of the raindrop according to this first model. **[7]**

In the second model $m = m_0(1 + k_2 x)$, where k_2 is a positive constant.

(iii) By considering the expression obtained from differentiating $v^2(1 + k_2 x)^2$ with respect to x, show that, for the second model, the equation of motion in part **(i)** may be written as

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[v^2(1 + k_2 x)^2\right] = 2g(1 + k_2 x)^2.$$

Hence find an expression for v^2 in terms of g, k_2 and x. **[9]**

(iv) Suppose that the models give the same value for the speed of the raindrop at the instant when it has doubled its initial mass. Find the exact value of $\frac{k_1}{k_2}$, giving your answer in the form $\frac{a}{b}$ where a and b are integers. **[4]**

END OF QUESTION PAPER