OCR
Oxford Cambridge and RSA

Monday 22 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4764/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4764/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

* 4 4 2 8 5 5 8 3 4 5 *

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\ \text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (24 Marks)

1 A rocket is launched vertically upwards from rest. The initial mass of the rocket, including fuel and payload, is m_0 and the propulsion system ejects mass at a constant mass rate k with constant speed u relative to the rocket. The only other force acting on the rocket is its weight. The acceleration due to gravity is constant throughout the motion.

At time t after launch the speed of the rocket is v.

(i) Show that while mass is being ejected from the rocket $v = u\ln\left(\dfrac{m_0}{m_0 - kt}\right) - gt$. **[9]**

The rocket initially has 2400 kg of fuel which is ejected at a constant rate of $100\,\text{kg}\,\text{s}^{-1}$ with constant speed $3000\,\text{m}\,\text{s}^{-1}$ relative to the rocket.

(ii) Given that the rocket must reach a speed of $7910\,\text{m}\,\text{s}^{-1}$ before releasing its payload, find the maximum possible value of m_0. **[3]**

2 Fig. 2 shows a system in a vertical plane. A uniform rod AB of length $2a$ and mass m is freely hinged at A. The angle that AB makes with the horizontal is θ, where $-\frac{2}{3}\pi < \theta < \frac{2}{3}\pi$. Attached at B is a light spring BC of natural length a and stiffness $\dfrac{mg}{a}$. The other end of the spring is attached to a small light smooth ring C which can slide freely along a vertical rail. The rail is at a distance of a from A and the spring is always horizontal.

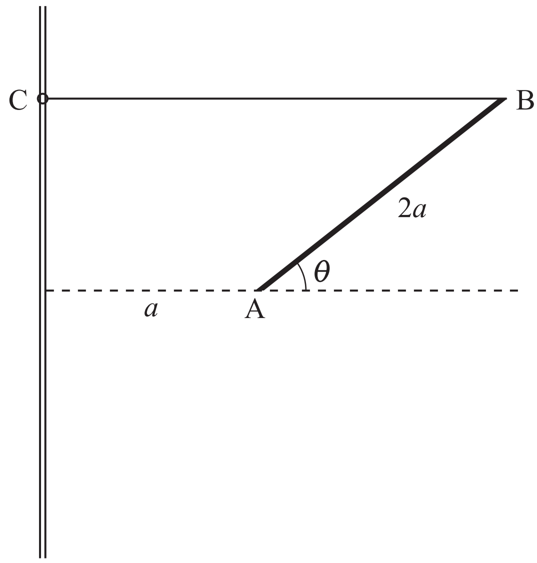

Fig. 2

(i) Find the potential energy, V, of the system and hence show that $\dfrac{\mathrm{d}V}{\mathrm{d}\theta} = mga\cos\theta(1 - 4\sin\theta)$. **[5]**

(ii) Hence find the positions of equilibrium of the system and investigate their stability. **[7]**

Section B (48 Marks)

3 A particle of mass 4 kg moves along the x-axis. At time t seconds the particle is x m from the origin O and has velocity $v\,\mathrm{m\,s^{-1}}$. A driving force of magnitude $20te^{-t}$ N is applied in the positive x direction. Initially $v = 2$ and the particle is at O.

(i) Find, in either order, the impulse of the force over the first 3 seconds and the velocity of the particle after 3 seconds. **[8]**

From time $t = 3$ a resistive force of magnitude $\frac{1}{2}t$ N and the driving force are applied until the particle comes to rest.

(ii) Show that, after the resistive force is applied, the only time at which the resultant force on the particle is zero is when $t = \ln 40$. Hence find the maximum velocity of the particle during the motion. **[11]**

(iii) Given that the time T seconds at which the particle comes to rest is given by the equation $T = \sqrt{121 - 80e^{-T}(1+T)}$, without solving the equation deduce that $T \approx 11$. **[2]**

(iv) Use a numerical method to find T correct to 4 decimal places. **[3]**

4 A solid cylinder of radius a m and length $3a$ m has density ρ kg m^{-3} given by $\rho = k\left(2+\frac{x}{a}\right)$ where x m is the distance from one end and k is a positive constant. The mass of the cylinder is M kg where $M = \frac{21}{2}\pi a^3 k$. Let A and B denote the circular faces of the cylinder where $x = 0$ and $x = 3a$, respectively.

(i) Show by integration that the moment of inertia, I_A kg m^2, of the cylinder about a diameter of the face A is given by $I_A = \frac{109}{28}Ma^2$. **[9]**

(ii) Show that the centre of mass of the cylinder is $\frac{12}{7}a$ m from A. **[4]**

(iii) Using the parallel axes theorem, or otherwise, show that the moment of inertia, I_B kg m^2, of the cylinder about a diameter of the face B is given by $I_B = \frac{73}{28}Ma^2$. **[4]**

You are now given that $M = 4$ and $a = 0.7$. The cylinder is at rest and can rotate freely about a horizontal axis which is a diameter of the face B as shown in Fig. 4. It is struck at the bottom of the curved surface by a small object of mass 0.2 kg which is travelling horizontally at speed 20 m s^{-1} in the vertical plane which is both perpendicular to the axis of rotation and contains the axis of symmetry of the cylinder. The object sticks to the cylinder at the point of impact.

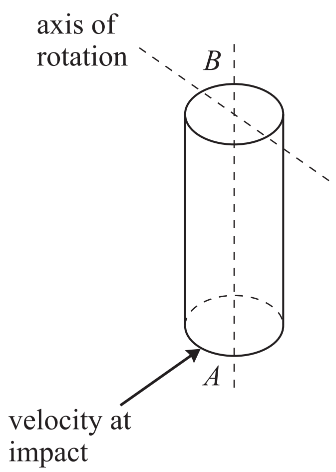

Fig. 4

(iv) Find the initial angular speed of the combined object after the collision. **[7]**

END OF QUESTION PAPER