Question			Answer	Marks	Guidance	
1	(i)		$\frac{dm}{dt} = k \Rightarrow m = kt + c$	B1		
			conditions $\Rightarrow m = m_0 + kt$	B1		
			$\left(\frac{d}{dt}(mv) = 0 \Rightarrow\right) \quad mv = m_0 v_0$	M1 A1	Momentum equation	Or derive a differential equation in only two variables
			$v = \frac{m_0 v_0}{m} = \frac{m_0 v_0}{m_0 + kt}$	A1		
			$x = \int \frac{m_0 v_0}{m_0 + kt} dt$	M1	Integrate their expression for v	
			$= \frac{m_0 v_0}{k} \ln(m_0 + kt) \quad (+ c_2)$	A1		
			conditions $\Rightarrow 0 = \frac{m_0 v_0}{k} \ln m_0 + c_2$	M1	Use initial conditions	
			so $x = \frac{m_0 v_0}{k}(\ln(m_0 + kt) - \ln m_0)$			
			$= \frac{m_0 v_0}{k} \ln\left(\frac{m_0 + kt}{m_0}\right) = \frac{m_0 v_0}{k} \ln\left(1 + \frac{kt}{m_0}\right)$	E1		
				[9]		
1	(ii)		$kt = 2m_0 \Rightarrow t = \frac{2m_0}{k} \Rightarrow v = \frac{1}{3} v_0$	B1	Follow through their $v = f(t)$	**SC** If $kt = m_0$ Award B1 either correct on follow through
			$x = \frac{m_0 v_0}{k} \ln 3$	B1	Ft	
				[2]		
2	(i)		$BC = 2 \times 0.5 \sin \frac{1}{2}\theta$	E1		
				[1]		

Question			Answer	Marks	Guidance	
2	(ii)		$V = -0.5g \cdot 0.25\sin\theta +$ $\frac{1}{2}\cdot 2(\text{BC}-0.5)^2 + \frac{1}{2}\cdot 2(\text{BD}-0.5)^2$ $\text{BD}^2 = 1^2 + 0.5^2 - 2\times 1\times 0.5\cos\theta = 1.25 - \cos\theta$ $V = -1.225\sin\theta + (\sin\frac{1}{2}\theta - 0.5)^2 +$ $(\sqrt{1.25-\cos\theta} - 0.5)^2$ $\frac{\text{d}V}{\text{d}\theta} = -1.225\cos\theta + 2(\sin\frac{1}{2}\theta - 0.5)(\frac{1}{2}\cos\frac{1}{2}\theta) +$ $2\left(\sqrt{1.25-\cos\theta} - 0.5\right)\left(\frac{\sin\theta}{2\sqrt{1.25-\cos\theta}}\right)$ $= -1.225\cos\theta + \sin\frac{1}{2}\theta\cos\frac{1}{2}\theta - 0.5\cos\frac{1}{2}\theta +$ $\sin\theta - \frac{0.5\sin\theta}{\sqrt{1.25-\cos\theta}}$ $= 1.5\sin\theta - 1.225\cos\theta - \frac{0.5\sin\theta}{\sqrt{1.25-\cos\theta}} - 0.5\cos\frac{1}{2}\theta$	M1 M1 B1 A1 M1 M1 A1 E1 **[8]**	GPE At least one EPE term oe Differentiate Use of chain rule one EPE term correct Complete argument	
2	(iii)		$\theta \approx 1.2$ and 4.1 Stable and unstable respectively at $\theta \approx 1.2$, $\frac{\text{d}V}{\text{d}\theta}$ increasing because the graph shows that $\text{f}'(\theta)$ is positive so V minimum hence stable at $\theta \approx 4.1$ $\frac{\text{d}V}{\text{d}\theta}$ decreasing because the graph shows that $\text{f}'(\theta)$ is negative, so max. so unstable	B1 B1 M1 A1 **[4]**	Both Consider gradient, relating f to $\frac{\text{d}V}{\text{d}\theta}$ Clear evidence from the graph	Allow B1M1A1 from 1.1 and/or 4.05

Question			Answer	Marks	Guidance	
3	(i)		$2\frac{dv}{dt} = \frac{2v^3+4v}{v} - 6v$	M1 A1	N2L	
			$\frac{dv}{dt} = v^2 - 3v + 2 = (1-v)(2-v)$	E1		
			$\int \frac{1}{(1-v)(2-v)} dv = \int dt$	M1	Separate	
			$\int \left(\frac{1}{1-v} - \frac{1}{2-v}\right) dv = \int dt$	M1	Partial fractions	
			$-\ln\lvert 1-v\rvert + \ln\lvert 2-v\rvert = t + c$	A1 A1	LHS RHS	
			$t = 0, v = 0 \Rightarrow \ln 2 = c$	M1	Use condition	
			$t = \ln(2-v) - \ln(1-v) - \ln 2$	M1	Rearrange	
			$= \ln\frac{2-v}{2(1-v)}$	E1		
				[10]		
3	(ii)		$\Rightarrow \frac{2-v}{2(1-v)} = e^t \Rightarrow 2 - v = 2e^t - 2e^t v$	M1	Rearrange	
			$v = \frac{2(e^t - 1)}{2e^t - 1}$	A1		
				[2]		
3	(iii)		$v = 0.8 \Rightarrow P = 2\times 0.8^3 + 4\times 0.8 = 4.224$	E1		
			$t = \ln\frac{2-0.8}{2(1-0.8)} = \ln 3 \approx 1.10$	B1		
				[2]		

Question			Answer	Marks	Guidance	
3	(iv)		$2\frac{dv}{dt} = \frac{4.224}{v} - 6v$	B1	N2L	
			$\int \frac{2}{\frac{4.224}{v} - 6v} dv = \int dt$	M1	Separate	
			$\int \frac{v}{2.112 - 3v^2} dv = \int dt$	M1		
			$-\frac{1}{6}\ln\|2.112 - 3v^2\| = t + c_2$	A1	LHS	
				A1	RHS	
			$2.112 - 3v^2 = Ae^{-6t}$			**Alternate**
			$t = \ln 3, v = 0.8 \Rightarrow 2.112 - 3 \times 0.8^2 = Ae^{-6\ln 3}$	M1	Use condition to find constant	
			$A = 139.968$	A1		
				M1	Rearrange to make v the subject	$t \to \infty \Rightarrow 2.112 - 3v^2 \to 0$
			$v = \sqrt{0.704 - 46.656e^{-6t}}$	A1	Correct	
			$t \to \infty \Rightarrow v \to \sqrt{0.704} \approx 0.839$	A1	Ft their expression for v	
				[10]		
4	(i)		$\rho = \frac{m}{\frac{1}{2}a^2 \cdot \frac{\pi}{3}}$	B1	Or let ρ = 1 without lose of generality	
			element with radius x and 'width' δx:			
			$\delta m = \rho x \frac{\pi}{3} \delta x \Rightarrow \delta I = \rho x \frac{\pi}{3} \delta x \cdot x^2$	M1		
			$= \frac{2m}{a^2} x^3 \delta x$	A1	Ft their ρ	
			$I = \int_0^a \frac{2m}{a^2} x^3 \, dx$	M1	Integrate	
			$= \frac{2m}{a^2}\left[\frac{x^4}{4}\right]_0^a$	A1		
			$= \frac{2m}{a^2}\left(\frac{a^4}{4}\right) = \frac{1}{2}ma^2$	E1		
				[6]		

Question			Answer	Marks	Guidance	
4	**(ii)**		$\frac{2a}{\pi}$	B1 **[1]**		
4	**(iii)**		$\frac{1}{2}I\dot{\theta}^2 - mg\left(\frac{2a}{\pi}\right)\cos\theta = -mg\left(\frac{2a}{\pi}\right)\cos\frac{2}{3}\pi$ OE $\frac{1}{2}ma^2\dot{\theta}^2 = 2mg\left(\frac{2a}{\pi}\right)\left(\cos\theta + \frac{1}{2}\right)$ $\dot{\theta}^2 = \frac{4g}{\pi a}(2\cos\theta + 1)$	M1 A1 A1 E1 **[4]**	Energy Two correct terms All correct	RHS: or $mg\left(\frac{2a}{\pi}\right)\cos\frac{1}{3}\pi$
4	**(iv)**		Max $\dot{\theta}$ when $\cos\theta = 1$ $\Rightarrow \dot{\theta}^2 = \frac{12g}{\pi a}$ Speed max. furthest from axis, so max speed $= a\sqrt{\frac{12g}{\pi a}} = \sqrt{\frac{12ag}{\pi}}$	M1 A1 M1 A1 **[4]**	oe	
4	**(v)**		$2\ddot{\theta}\dot{\theta} = \frac{4g}{\pi a}\left(-2\sin\theta\,\dot{\theta}\right)$ $\ddot{\theta} = -\frac{4g}{\pi a}\sin\theta$	M1 A1 **[2]**	Differentiate with respect to time Or use $C = I\ddot{\theta}$	May be seen in (iv) May be seen in (iv)
4	**(vi)**		$J \cdot x = \pm I \cdot \frac{1}{2}\omega \pm I \cdot \omega$ $J \cdot \frac{3}{4}a = \frac{1}{2}ma^2\left(\frac{1}{2}\omega\right) - \frac{1}{2}ma^2(-\omega)$ $\theta = (-)\frac{1}{3}\pi \Rightarrow \omega^2 = \frac{4g}{\pi a}\left(2\left(\frac{1}{2}\right)+1\right) \Rightarrow \omega = \sqrt{\frac{8g}{\pi a}}$ $J = m\sqrt{\frac{8ag}{\pi}}$	M1 A1 B1 A1 **[4]**		

Question			Answer	Marks	Guidance	
4	(vii)		$\frac{1}{2}I\left(\frac{1}{2}\sqrt{\frac{8g}{\pi a}}\right)^2 - mg\left(\frac{2a}{\pi}\right)\cos\frac{1}{3}\pi = -mg\left(\frac{2a}{\pi}\right)\cos\theta$	M1 A1		
			$\Rightarrow \theta = \cos^{-1}\frac{1}{4} \approx 1.32$	A1 **[3]**	CAO	