OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4764

Mechanics 4

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Thursday 11 June 2009
Morning

Duration: 1 hour 30 minutes

4764

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\ \mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (24 marks)

1 A raindrop increases in mass as it falls vertically from rest through a stationary cloud. At time t s the velocity of the raindrop is $v\,\mathrm{m\,s^{-1}}$ and its mass is m kg. The rate at which the mass increases is modelled as $\dfrac{mg}{2(v+1)}\,\mathrm{kg\,s^{-1}}$. Resistances to motion are neglected.

(i) Write down the equation of motion of the raindrop. Hence show that

$$\left(1-\frac{1}{v+2}\right)\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{1}{2}g.$$ **[5]**

(ii) Solve this differential equation to find an expression for t in terms of v. Calculate the time it takes for the velocity of the raindrop to reach $10\,\mathrm{m\,s^{-1}}$. **[5]**

(iii) Describe, with reasons, what happens to the acceleration of the raindrop for large values of t. **[2]**

2 A uniform rigid rod AB of mass m and length $4a$ is freely hinged at the end A to a horizontal rail. The end B is attached to a light elastic string BC of modulus $\frac{1}{2}mg$ and natural length a. The end C of the string is attached to a ring which is small, light and smooth. The ring can slide along the rail and is always vertically above B. The angle that AB makes below the rail is θ. The system is shown in Fig. 2.

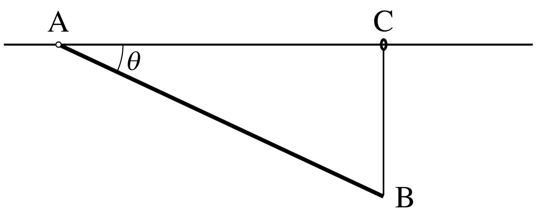

Fig. 2

(i) Find the potential energy, V, of the system when the string is stretched and show that

$$\frac{\mathrm{d}V}{\mathrm{d}\theta}=4mga\cos\theta(2\sin\theta-1).$$ **[5]**

(ii) Hence find any positions of equilibrium of the system and investigate their stability. **[7]**

Section B (48 marks)

3 A uniform circular disc has mass M and radius a. The centre of the disc is at point C.

(i) Show by integration that the moment of inertia of the disc about an axis through C and perpendicular to the disc is $\frac{1}{2}Ma^2$. **[6]**

The point A on the disc is at a distance $\frac{1}{10}a$ from its centre.

(ii) Show that the moment of inertia of the disc about an axis through A and perpendicular to the disc is $0.51Ma^2$. **[2]**

The disc can rotate freely in a vertical plane about an axis through A that is horizontal and perpendicular to the disc. The disc is held slightly displaced from its stable equilibrium position and is released from rest. In the motion that follows, the angle that AC makes with the downward vertical is θ.

(iii) Write down the equation of motion for the disc. Assuming θ remains sufficiently small throughout the motion, show that the disc performs approximate simple harmonic motion and determine the period of the motion. **[6]**

A particle of mass m is attached at a point P on the circumference of the disc, so that the centre of mass of the system is now at A.

(iv) Sketch the position of P in relation to A and C. Find m in terms of M and show that the moment of inertia of the system about the axis through A and perpendicular to the disc is $0.6Ma^2$. **[5]**

The system now rotates at a constant angular speed ω about the axis through A.

(v) Find the kinetic energy of the system. Hence find the magnitude of the constant resistive couple needed to bring the system to rest in n revolutions. **[5]**

4 A parachutist of mass 90 kg falls vertically from rest. The forces acting on her are her weight and resistance to motion R N. At time t s the velocity of the parachutist is $v\,\mathrm{m\,s^{-1}}$ and the distance she has fallen is x m.

While the parachutist is in free-fall (i.e. before the parachute is opened), the resistance is modelled as $R = kv^2$, where k is a constant. The terminal velocity of the parachutist in free-fall is $60\,\mathrm{m\,s^{-1}}$.

(i) Show that $k = \frac{g}{40}$. **[2]**

(ii) Show that $v^2 = 3600\left(1 - e^{-\frac{gx}{1800}}\right)$. **[7]**

When she has fallen 1800 m, she opens her parachute.

(iii) Calculate, by integration, the work done against the resistance before she opens her parachute. Verify that this is equal to the loss in mechanical energy of the parachutist. **[7]**

As the parachute opens, the resistance instantly changes and is now modelled as $R = 90v$.

(iv) Calculate her velocity just before opening the parachute, correct to four decimal places. **[1]**

(v) Formulate and solve a differential equation to calculate the time it takes after opening the parachute to reduce her velocity to $10\,\mathrm{m\,s^{-1}}$. **[7]**