OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4764/01**

MATHEMATICS (MEI)

Mechanics 4

WEDNESDAY 18 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (24 marks)

1 A rocket in deep space starts from rest and moves in a straight line. The initial mass of the rocket is m_0 and the propulsion system ejects matter at a constant mass rate k with constant speed u relative to the rocket. At time t the speed of the rocket is v.

(i) Show that while mass is being ejected from the rocket, $(m_0 - kt)\dfrac{dv}{dt} = uk$. [5]

(ii) Hence find an expression for v in terms of t. [4]

(iii) Find the speed of the rocket when its mass is $\frac{1}{3}m_0$. [3]

2 A car of mass m kg starts from rest at a point O and moves along a straight horizontal road. The resultant force in the direction of motion has power P watts, given by $P = m(k^2 - v^2)$, where $v\,\mathrm{m\,s^{-1}}$ is the velocity of the car and k is a positive constant. The displacement from O in the direction of motion is x m.

(i) Show that $\left(\dfrac{k^2}{k^2 - v^2} - 1\right)\dfrac{dv}{dx} = 1$, and hence find x in terms of v and k. [9]

(ii) How far does the car travel before reaching 90% of its terminal velocity? [3]

Section B (48 marks)

3 A circular disc of radius a m has mass per unit area $\rho\,\mathrm{kg\,m^{-2}}$ given by $\rho = k(a + r)$, where r m is the distance from the centre and k is a positive constant. The disc can rotate freely about an axis perpendicular to it and through its centre.

(i) Show that the mass, M kg, of the disc is given by $M = \frac{5}{3}k\pi a^3$, and show that the moment of inertia, $I\,\mathrm{kg\,m^2}$, about this axis is given by $I = \frac{27}{50}Ma^2$. [9]

For the rest of this question, take $M = 64$ and $a = 0.625$.

The disc is at rest when it is given a tangential impulsive blow of 50 N s at a point on its circumference.

(ii) Find the angular speed of the disc. [4]

The disc is then accelerated by a constant couple reaching an angular speed of $30\,\mathrm{rad\,s^{-1}}$ in 20 seconds.

(iii) Calculate the magnitude of this couple. [3]

When the angular speed is $30\,\mathrm{rad\,s^{-1}}$, the couple is removed and brakes are applied to bring the disc to rest. The effect of the brakes is modelled by a resistive couple of $3\dot{\theta}$ N m, where $\dot{\theta}$ is the angular speed of the disc in $\mathrm{rad\,s^{-1}}$.

(iv) Formulate a differential equation for $\dot{\theta}$ and hence find $\dot{\theta}$ in terms of t, the time in seconds from when the brakes are first applied. [7]

(v) By reference to your expression for $\dot{\theta}$, give a brief criticism of this model for the effect of the brakes. [1]

4 A uniform smooth pulley can rotate freely about its axis, which is fixed and horizontal. A light elastic string AB is attached to the pulley at the end B. The end A is attached to a fixed point such that the string is vertical and is initially at its natural length with B at the same horizontal level as the axis. In this position a particle P is attached to the highest point of the pulley. This initial position is shown in Fig. 4.1.

The radius of the pulley is a, the mass of P is m and the stiffness of the string AB is $\frac{mg}{10a}$.

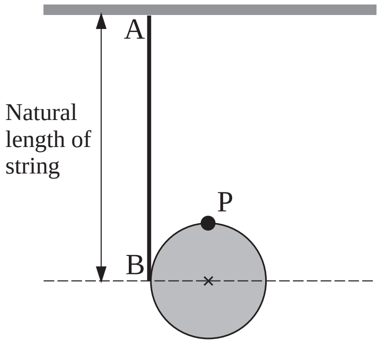

Fig. 4.1

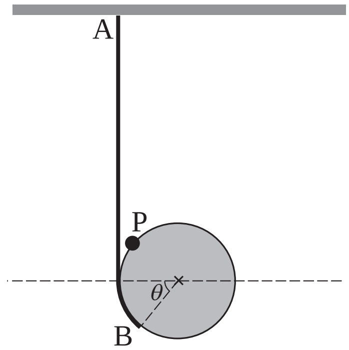

Fig. 4.2

(i) Fig. 4.2 shows the system with the pulley rotated through an angle θ and the string stretched. Write down the extension of the string and hence find the potential energy, V, of the system in this position. Show that $\frac{dV}{d\theta} = mga\left(\frac{1}{10}\theta - \sin\theta\right)$. [6]

(ii) Hence deduce that the system has a position of unstable equilibrium at $\theta = 0$. [6]

(iii) Explain how your expression for V relies on smooth contact between the string and the pulley. [2]

Fig. 4.3 shows the graph of the function $f(\theta) = \frac{1}{10}\theta - \sin\theta$.

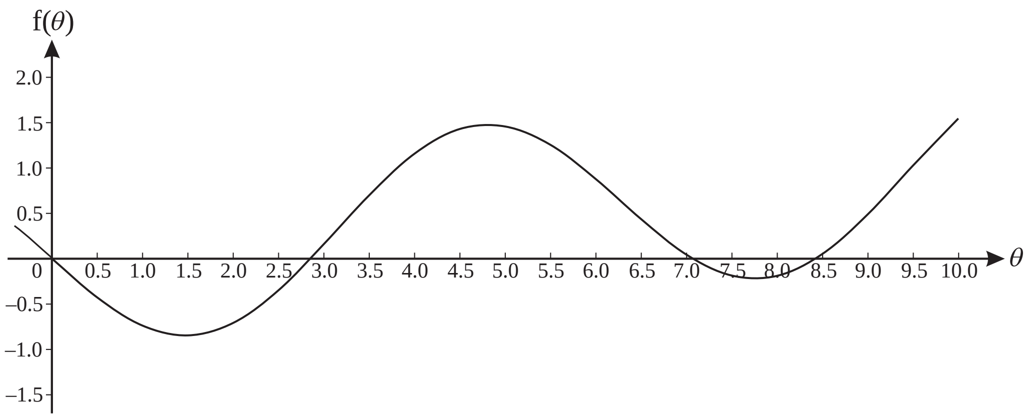

Fig. 4.3

(iv) Use the graph to give rough estimates of three further values of θ (other than $\theta = 0$) which give positions of equilibrium. In each case, state with reasons whether the equilibrium is stable or unstable. [6]

(v) Show on a sketch the physical situation corresponding to the least value of θ you identified in part **(iv)**. On your sketch, mark clearly the positions of P and B. [2]

(vi) The equation $f(\theta) = 0$ has another root at $\theta \approx -2.9$. Explain, with justification, whether this necessarily gives a position of equilibrium. [2]