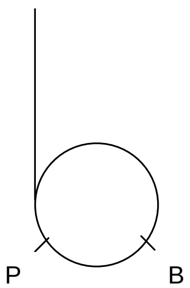
4764 Mechanics 4

1(i)	If δm is change in mass over time δt		
	PCLM $mv = (m+\delta m)(v+\delta v) + \lvert\delta m\rvert(v-u)$ [N.B. $\delta m < 0$]	M1	Change in momentum over time δt
	$(m+\delta m)\frac{\delta v}{\delta t} + u\frac{\delta m}{\delta t} = 0 \Rightarrow m\frac{\mathrm{d}v}{\mathrm{d}t} = -u\frac{\mathrm{d}m}{\mathrm{d}t}$	M1 A1	Rearrange to produce DE Accept sign error
	$\frac{\mathrm{d}m}{\mathrm{d}t} = -k \Rightarrow m = m_0 - kt$	M1	Find m in terms of t
	$\Rightarrow (m_0 - kt)\frac{\mathrm{d}v}{\mathrm{d}t} = uk$	E1	Convincingly shown
			5
(ii)	$v = \int \frac{uk}{m_0 - kt}\mathrm{d}t$	M1	Separate and integrate
	$= -u\ln(m_0 - kt) + c$	A1	cao (allow no constant)
	$t = 0, v = 0 \Rightarrow c = u\ln m_0$	M1	Use initial condition
	$v = u\ln\left(\frac{m_0}{m_0 - kt}\right)$	A1	All correct
			4
(iii)	$m = \frac{1}{3}m_0 \Rightarrow m_0 - kt = \frac{1}{3}m_0$	M1 A1	Find expression for mass or time Or $t = 2m_0/3k$
	$\Rightarrow v = u\ln 3$	A1	
			3

2(i)	$P = Fv$	M1	Used, not just quoted
	$= mv\frac{\mathrm{d}v}{\mathrm{d}x}v$	M1	Use N2L and expression for acceleration
	$\Rightarrow mv^2\frac{\mathrm{d}v}{\mathrm{d}x} = m\left(k^2 - v^2\right)$	A1	Correct DE
	$\Rightarrow \frac{v^2}{k^2 - v^2}\frac{\mathrm{d}v}{\mathrm{d}x} = 1$	M1	Rearrange
	$\Rightarrow \left(\frac{k^2}{k^2 - v^2} - 1\right)\frac{\mathrm{d}v}{\mathrm{d}x} = 1$	E1	Convincingly shown
	$\int\left(\frac{k^2}{k^2 - v^2} - 1\right)\mathrm{d}v = \int \mathrm{d}x$	M1	Separate and integrate
	$\frac{1}{2}k\ln\left(\frac{k+v}{k-v}\right) - v = x + c$	A1	LHS
	$x = 0, v = 0 \Rightarrow c = 0$	M1	Use condition
	$x = \frac{1}{2}k\ln\left(\frac{k+v}{k-v}\right) - v$	A1	cao
			9
(ii)	Terminal velocity when acceleration zero $\Rightarrow v = k$	M1 A1	
	$v = 0.9k \Rightarrow x = \frac{1}{2}k\ln\left(\frac{1.9}{0.1}\right) - 0.9k = \left(\frac{1}{2}\ln 19 - 0.9\right)k \approx$ 0.572k	F1	Follow their solution to (i)
			3

3(i)	$M=\int_0^a k(a+r)2\pi r\,\mathrm{d}r$	M1 M1	Use circular elements (for *M* or *I*) Integral for mass	
	$=2k\pi\left[\frac{1}{2}ar^2+\frac{1}{3}r^3\right]_0^a$	M1 A1	Integrate (for *M* or *I*) For […]	
	$=\frac{5}{3}k\pi a^3$	E1		
	$I=\int_0^a k(a+r)2\pi r\cdot r^2\,\mathrm{d}r$	M1	Integral for *I*	
	$=2k\pi\left[\frac{1}{4}ar^4+\frac{1}{5}r^5\right]_0^a$	A1	For […]	
	$=\frac{9}{10}k\pi a^5$	A1	cao	
	$=\frac{27}{50}Ma^2$	E1	Complete argument (including mass)	9
(ii)	$I=13.5$ $0.625\times 50=I\omega$ $\Rightarrow \omega\approx 2.31$	B1 M1 M1 A1	Seen or used (here or later) Use angular momentum Use moment of impulse cao	4
(iii)	$\ddot{\theta}=\dfrac{30-2.31}{20}\approx 1.38$	M1	Find angular acceleration	
	Couple $=I\ddot{\theta}$ ≈ 18.7	M1 F1	Use equation of motion Follow their ω and *I*	3
(iv)	$I\ddot{\theta}=-3\dot{\theta}$	B1	Allow sign error and follow their *I* (but not *M*)	
	$I\dfrac{\mathrm{d}\dot{\theta}}{\mathrm{d}t}=-3\dot{\theta}$	M1	Set up DE for $\dot{\theta}$ (first order)	
	$\int\dfrac{\mathrm{d}\dot{\theta}}{\dot{\theta}}=\int-\dfrac{3}{I}\mathrm{d}t$	M1	Separate and integrate	
	$\ln\lvert\dot{\theta}\rvert=-\dfrac{t}{4.5}+c$	B1	$\ln(\text{multiple of }\dot{\theta})$ seen	
	$\dot{\theta}=A\mathrm{e}^{-t/4.5}$	M1	Rearrange, dealing properly with constant	
	$t=0,\dot{\theta}=30\Rightarrow A=30$	M1	Use condition on $\dot{\theta}$	
	$\dot{\theta}=30\,\mathrm{e}^{-t/4.5}$	A1		7
(v)	Model predicts $\dot{\theta}$ never zero in finite time.	B1		1

4(i)	$V = \frac{1}{2}\left(\frac{mg}{10a}\right)(a\theta)^2 + mga\cos\theta$ (relative to centre of pulley)	M1	EPE term	
		B1	Extension $= a\theta$	
		M1	GPE relative to any zero level	
		A1	(± constant)	
	$\frac{dV}{d\theta} = \frac{1}{2}\left(\frac{mg}{10a}\right)\cdot 2a^2\theta - mga\sin\theta$	M1	Differentiate	
	$\frac{dV}{d\theta} = mga\left(\frac{1}{10}\theta - \sin\theta\right)$	E1		
				6
(ii)	$\theta = 0 \Rightarrow \frac{dV}{d\theta} = mga\left(\frac{1}{10}(0) - \sin 0\right) = 0$	M1	Consider value of $\frac{dV}{d\theta}$	
	hence equilibrium	E1		
	$\frac{d^2V}{d\theta^2} = mga\left(\frac{1}{10} - \cos\theta\right)$	M1	Differentiate again	
		A1		
	$V''(0) = -0.9mga < 0$	M1	Consider sign of V''	
	hence unstable	E1	V'' must be correct	
				6
(iii)	If the pulley is smooth, then the tension in the string is constant.	B1		
	Hence the EPE term is valid.	B1		
				2
(iv)	Equilibrium positions at $\theta = 2.8$,	B1	One correct	
	$\theta = 7.1$	B1	All three correct, no extras	
	and $\theta = 8.4$		Accept answers in [2.7,3.0), [7,7.2], [8.3,8.5]	
	From graph, $V''(2.8) = mga\mathrm{f}'(2.8) > 0$	M1	Consider sign of V'' or f'	
	hence stable at $\theta = 2.8$	A1	Accept no reference to V'' for one conclusion but other two must relate to sign of V'', not just f'.	
	$V''(7.1) = mga\mathrm{f}'(7.1) < 0 \Rightarrow$ unstable at $\theta = 7.1$	A1		
	$V''(8.4) = mga\mathrm{f}'(8.4) > 0 \Rightarrow$ stable at $\theta = 8.4$	A1		
				6
(v)		B1	P in approximately correct place	
		B1	B in approximately correct place	2
(vi)	If $\theta < 0$ then expression for EPE not valid	M1		
	hence not necessarily an equilibrium position.	A1		
				2