

5.

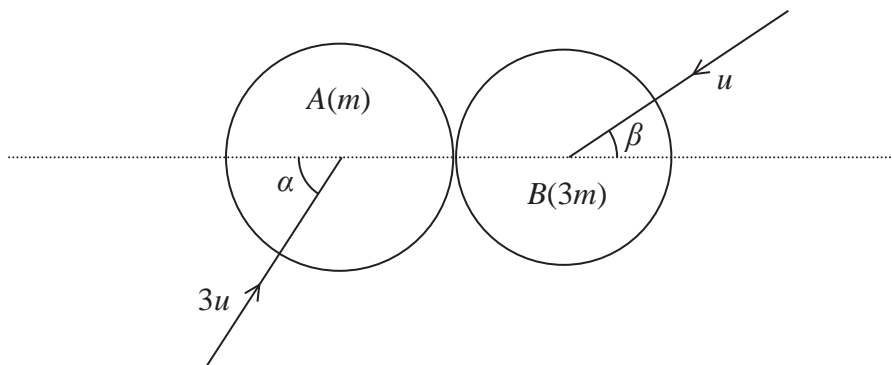


Figure 1

Two smooth uniform spheres A and B have equal radii. The mass of A is m and the mass of B is $3m$. The spheres are moving on a smooth horizontal plane when they collide obliquely. Immediately before the collision, A is moving with speed $3u$ at angle α to the line of centres and B is moving with speed u at angle β to the line of centres, as shown in Figure 1. The coefficient of restitution between the two spheres is $\frac{1}{5}$. It is given that $\cos \alpha = \frac{1}{3}$ and $\cos \beta = \frac{2}{3}$ and that α and β are both acute angles.

(a) Find the magnitude of the impulse on A due to the collision in terms of m and u . (8)

(b) Express the kinetic energy lost by A in the collision as a fraction of its initial kinetic energy. (4)

6. A particle of mass m kg is attached to one end of a light elastic string of natural length a metres and modulus of elasticity $5ma$ newtons. The other end of the string is attached to a fixed point O on a smooth horizontal plane. The particle is held at rest on the plane with the string stretched to a length $2a$ metres and then released at time $t = 0$. During the subsequent motion, when the particle is moving with speed v m s $^{-1}$, the particle experiences a resistance of magnitude $4mv$ newtons. At time t seconds after the particle is released, the length of the string is $(a + x)$ metres, where $0 \leq x \leq a$.

(a) Show that, from $t = 0$ until the string becomes slack,

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = 0$$

(3)

(b) Hence express x in terms of a and t .

(6)

(c) Find the speed of the particle at the instant when the string first becomes slack, giving your answer in the form ka , where k is a constant to be found correct to 2 significant figures.

(4)



