Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Mechanics M4 (6680/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- $\lfloor$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

6. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

  HL Hooke's Law

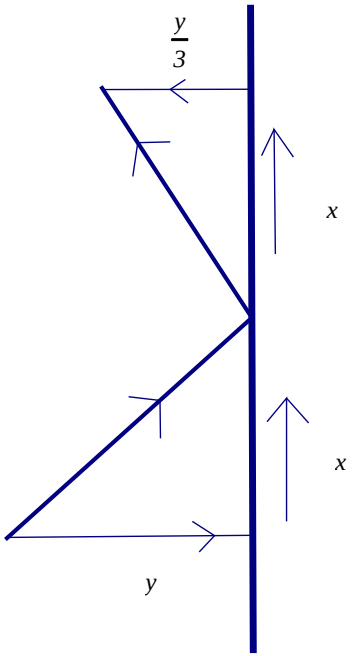
  SHM Simple harmonic motion

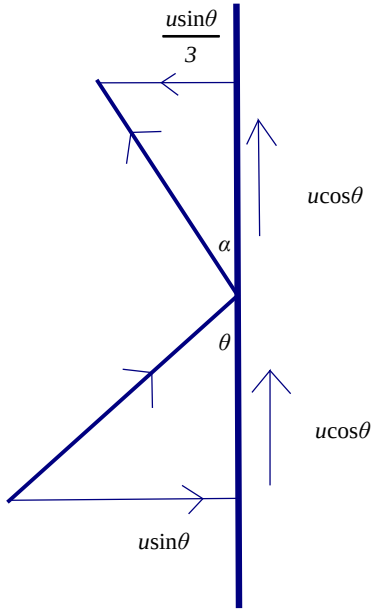
  PCLM Principle of conservation of linear momentum

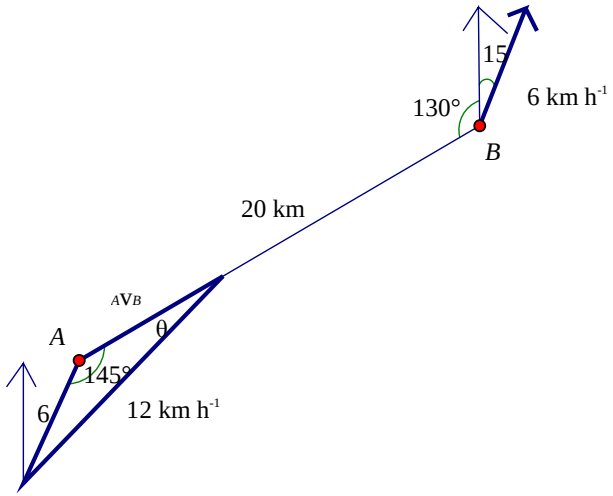
  RHS, LHS Right hand side, left hand side.

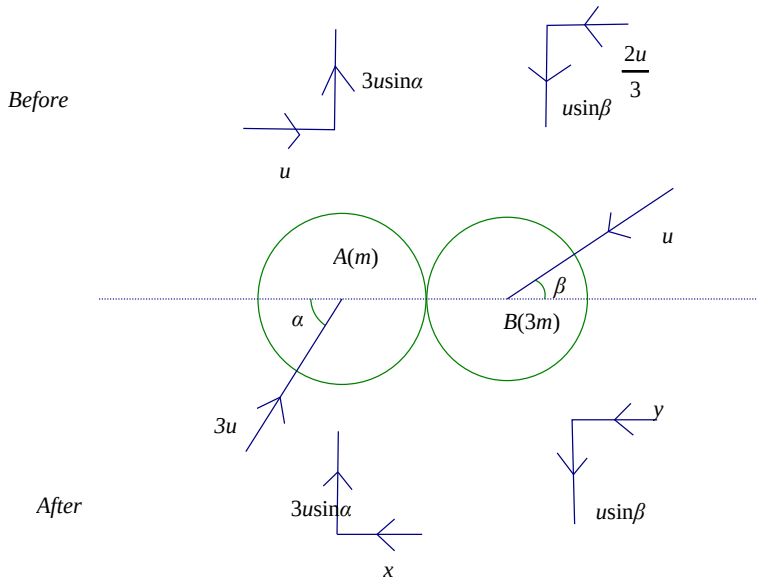
Question Number	Scheme	Marks	Notes
1a	$\mathbf{r}_A = (-6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}) + t(3\mathbf{i} + \mathbf{j}) = ((-6 + 3t)\mathbf{i} + (4 + t)\mathbf{j} + (-3)\mathbf{k})$ $\mathbf{r}_B = (-2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + t(\mathbf{i} - \mathbf{k}) = ((-2 + t)\mathbf{i} + (2)\mathbf{j} + (3 - t)\mathbf{k})$	M1 A1	Position vector for A or B Both position vectors correct (seen or implied)
	${}_B\mathbf{r}_A = (-2 + t + 6 - 3t)\mathbf{i} + (2 - 4 - t)\mathbf{j} + (3 - t + 3)\mathbf{k}$ $= (4 - 2t)\mathbf{i} + (-2 - t)\mathbf{j} + (6 - t)\mathbf{k}$	M1	Position of B relative to A (or A relative to B)
	$\left\lvert {}_B\mathbf{r}_A \right\rvert^2 = (4-2t)^2 + (t+2)^2 + (6-t)^2$	M1	Use of Pythagoras
alt1	$= 6t^2 - 24t + 56 = 6(t-2)^2 + 32$	M1	Complete the square
	Minimum distance = $\sqrt{32} = 4\sqrt{2}$ m **	A1 [6]	Reach **given answer** correctly
	$\left\lvert {}_B\mathbf{r}_A \right\rvert^2 = (4-2t)^2 + (t+2)^2 + (6-t)^2 \left(= 6t^2 - 24t + 56\right)$	M1	Use of Pythagoras
alt2	$12t - 24 = 0 \Rightarrow t = 2$	M1	Differentiate and solve for t
	Minimum distance = $\sqrt{32} = 4\sqrt{2}$ m **	A1	Reach **given answer** correctly
alt3	$\begin{pmatrix} 4-2t \\ -2-t \\ 6-t \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 0 \Rightarrow 8 - 4t - 2 - t + 6 - t = 12 - 6t = 0$	M1	Scalar product of position vector with relative velocity = zero and form equation in t
	Distance $= \sqrt{0^2 + 4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$	M1 A1	Use of Pythagoras Reach **given answer** correctly
1b	When $t = 2$, $\mathbf{r}_A = 6\mathbf{j} - 3\mathbf{k}$	B1 B1 [2]	Seen or implied cso

Question Number	Scheme	Marks	Notes
2a	$\frac{P}{v}-10v=ma\,;\frac{25000}{v}-10v=1000a$	M1	Equation of motion
	$v=20,\ (\text{m s}^{-2})\quad a=\frac{\frac{25000}{20}-10\times 20}{1000}=\frac{\frac{25}{2}-2}{10}$	DM1	Substitute $v=20$
	$=1.05\ \ (\text{m s}^{-2})$ **	A1 [3]	Obtain **given answer** correctly
2b	$v\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{\frac{25000}{v}-10v}{1000}=\frac{25000-10v^2}{1000v}=\frac{2500-v^2}{100v}$	M1	Differential equation in v and x
		A1	Any equivalent form
	$\int\frac{100v^2}{2500-v^2}\mathrm{d}v=\int 1\mathrm{d}x\qquad\left(=100\int -1+\frac{2500}{2500-v^2}\mathrm{d}v\right)$	M1	Separate the variables
alt1	$=100\int -1+\frac{25}{50-v}+\frac{25}{50+v}\,dv$	DM1	Split using partial fractions
		A1	Or equivalent
	$x\left(+C\right)=100\left\{-v+25\ln\left\lvert\frac{50+v}{50-v}\right\rvert\right\}$	A1	Integration correct
		DM1	Correct use of limits
	$x=100\left(-20+25\ln\frac{70}{30}\right)-100\left(-10+25\ln\frac{60}{40}\right)=105\,(\text{m})$	A1 [8]	Or better $\left(2500\ln\left(\frac{14}{9}\right)-1000\right)$
alt2	$=100\left(v-50\operatorname{arctanh}\left(\frac{v}{50}\right)\right)$	DM1	Use of arctanh
		A1	correct
	$x\left(+C\right)=100\left\{-v+25\ln\left\lvert\frac{50+v}{50-v}\right\rvert\right\}$	A1	Convert to log form
		DM1	Correct use of limits
	$x=100\left(-20+25\ln\frac{70}{30}\right)-100\left(-10+25\ln\frac{60}{40}\right)=105\,(\text{m})$	A1	Or better $\left(2500\ln\left(\frac{14}{9}\right)-1000\right)$
	NB A correct numerical answer that does not follow from integration scores no marks.		

Question Number	Scheme	Marks	Notes
3 alt1			
	Speed perpendicular to wall after collision = $\frac{y}{3}$	B1	
	Speed parallel to the wall is unchanged	B1	
	$\frac{1}{2}\left(x^2+y^2\right)=x^2+\frac{1}{9}y^2$	M1 A1	Use the speeds to form an equation in x & y (or equivalent) Correct unsimplified
	$9\left(x^2+y^2\right)=2\left(9x^2+y^2\right),\ \ 9x^2=7y^2,\ \ x=\frac{\sqrt{7}}{3}y$	A1	Correct ratio for x & y (any equivalent form)
	direction deflected by $\tan^{-1}\frac{y}{x}+\tan^{-1}\frac{y}{3x}$	M1 A1	To find the correct angle Correct in x & y
	$=\tan^{-1}\sqrt{\frac{27}{5}}+\tan^{-1}\sqrt{\frac{3}{5}}=104.5°$ (104)	A1 [8]	

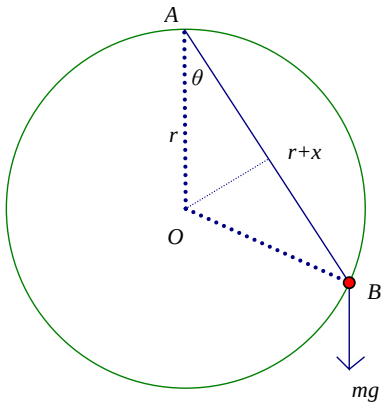
Question Number	Scheme	Marks	Notes
alt2			
	Speed perpendicular to wall after collision = $\frac{u\sin\theta}{3}$	B1	
	Speed parallel to the wall is unchanged	B1	
	$\frac{u^2}{4}=\frac{u^2}{9}\sin^2\theta+u^2\cos^2\theta$	M1 A1	Use the speeds to form an equation in u & θ (or equivalent) Correct unsimplified
	$27\cos^2\theta=5\sin^2\theta$, $\tan^2\theta=\frac{27}{5}$	A1	Correct trig ratio for θ (or equivalent)
	deflected by $\theta+\alpha$, $\tan(\theta+\alpha)=\frac{\tan\theta+\frac{1}{3}\tan\theta}{1-\frac{1}{3}\tan^2\theta}$ $\left(=-\sqrt{15}\right)$	M1 A1	To find the correct angle Correct in θ (or equivalent)
	$\theta+\alpha=104.5°$ (104)	A1 [8]	

Question Number	Scheme	Marks	Notes
4a			
	Relative velocity triangle	M1	Seen or implied
	$\frac{\sin 145}{12} = \frac{\sin\theta}{6}$, $\theta = 16.7°$	M1	Use of trig to find a relevant angle
	Bearing = $15 + (180 - 145 - 16.7) = 33.3°$	M1	To find the required angle
	Bearing 033°	A1 [4]	They were asked for an answer "to the nearest degree". Accept N 33º E
4b	$\frac{{}_A v_B}{\sin 18.3} = \frac{12}{\sin 145}$	M1	Correct method to find the relative velocity
	${}_A v_B = 6.58 \,(\text{km h}^{-1})$	A1	
	Time taken = $\frac{20}{6.58}$ (hrs)	M1	For their 6.58
	Time is 3:02 pm (1502)	A1 [4]	

Question Number	Scheme	Marks	Notes
5a			
	CLM: $mx+3my=3m\times u\cos\beta-m\times 3u\cos\alpha=mu \quad (x+3y=u)$	M1 A1	Terms of correct structure but condone sign errors
	NEL: $x-y=\frac{1}{5}(3u\cos\alpha+u\cos\beta)\left(=\frac{1}{5}\left(u+\frac{2}{3}u\right)=\frac{1}{3}u\right)$	M1 A1	equation of correct structure but condone sign errors
	$x=\frac{u}{2}$, or $y=\frac{u}{6}$	DM1 A1	Dependent on the two previous M marks. Solve for x or y
	Magnitude of the impulse on $A=mu-\left(m\times-\frac{u}{2}\right)=\frac{3mu}{2}$	M1 A1 [8]	Correct for their x or y Must be positive

Question Number	Scheme	Marks	Notes
5b	Component of velocity perpendicular to the line of centres before = component after = $3u\sin\alpha = 3u\times\frac{\sqrt{8}}{3} = \sqrt{8}u$	B1	
	KE lost = $\frac{m}{2}\left(9u^2 - \left(8u^2 + \frac{1}{4}u^2\right)\right)\left[= \frac{3}{8}mu^2\right]$	M1	Change in KE. Does not need to be a fraction at this stage. Does not need to include the (cancelling) component perpendicular to the line of centre.
		A1	Correct unsimplified
	Fraction lost = $\frac{3/8}{9/2} = \frac{3}{8}\times\frac{2}{9} = \frac{1}{12}$	A1 [4]	

Question Number	Scheme	Marks	Notes
6a	$m\ddot{x} = 4mv - \dfrac{5ma \times x}{a}$ $\quad v = -\dot{x}$	M1	Equation of motion as far as $m\ddot{x} = \pm 4mv - T$
		M1	Use of $v = -\dot{x}$
	$\ddot{x} + 4\dot{x} + 5x = 0$ **	A1 [3]	Reach **given answer** correctly.
6b	AE $m^2 + 4m + 5 = 0$, $m = \dfrac{-4 \pm \sqrt{4^2 - 4 \times 5}}{2} = -2 \pm \mathrm{i}$	M1	Solve AE to find GS
	$x = \mathrm{e}^{-2t}(A\cos t + B\sin t)$	A1	
	$t = 0, x = a = A$	M1	Use $t = 0, x = a$ to find A
		A1	
	$\dot{x} = -2\mathrm{e}^{-2t}(a\cos t + B\sin t) + \mathrm{e}^{-2t}(-a\sin t + B\cos t)$	M1	Differentiate and use boundary conditions to find B
	$t = 0, \quad \dot{x} = 0 = -2a + B$ $x = \mathrm{e}^{-2t}(a\cos t + 2a\sin t)$	A1 [6]	
6c	String goes slack when $x = \mathrm{e}^{-2t}(a\cos t + 2a\sin t) = 0$		
	$\cos t = -2\sin t$, $\tan t = -\dfrac{1}{2}$	M1	Set $x = 0$ and solve for t or $\tan t$
		A1	
	$\dot{x} = -2\mathrm{e}^{-2t}(a\cos t + 2a\sin t) + \mathrm{e}^{-2t}(-a\sin t + 2a\cos t)$	M1	Substitute a positive value of t to find the speed. An answer of 0.88... indicates a negative t.
	$= \mathrm{e}^{-2t}(-5a\sin t) = -0.01\ldots a$ Speed $= 0.011a\ (\mathrm{ms}^{-1})$	A1 [4]	The question specifies 2 sf

Question Number	Scheme	Marks	Notes
7a			
	Measuring GPE from A, GPE $= -mg\cos\theta(r+x)$	B1	Or $-2mgr\cos^2\theta$, or $-mgr(1+\cos 2\theta)$ or equivalent
	EPE $= \dfrac{kmgx^2}{2r}$	B1	
	From the isosceles triangle, $\cos\theta = \dfrac{x+r}{2r}$	B1	
	$V = -mg\cos\theta(r+x) + \dfrac{kmgx^2}{2r}$	M1	Correct unsimplified total
	$= -mg\cos\theta\times 2r\cos\theta + \dfrac{kmgr^2(2\cos\theta-1)^2}{2r}$	A1	In terms of r & θ
	$= mgr\left\{-2\cos^2\theta + 2k\cos^2\theta - 2k\cos\theta + \dfrac{k}{2}\right\}$		
	$= 2mgr\left\{(k-1)\cos^2\theta - k\cos\theta\right\} + \text{constant}$ **	A1 (6)	Reach **given answer** correctly

Question Number	Scheme	Marks	Notes
7b	$V = 2mgr\left(2\cos^2\theta - 3\cos\theta\right)$ +constant		
	$V' = 2mgr\left(-4\cos\theta\sin\theta + 3\sin\theta\right)$	M1 A1	Differentiate V
	$V' = 0 \Rightarrow \sin\theta = 0$ or $\cos\theta = \frac{3}{4}$	M1	Derivative = 0 and solve for θ
	$\theta = 0$ or $\theta = \pm 0.72$ rads	A3	-1 for each missing solution
	$V'' = 2mgr\left(-4\cos 2\theta + 3\cos\theta\right)$	M1	Second derivative of V
	$\theta = 0, V'' = -2mgr < 0$, unstable equilibrium	A1	Need to see $-2mgr$ or equivalent
	$\cos\theta = \frac{3}{4}$, $V'' = \frac{7mgr}{2} > 0$, stable equilibrium	A1 (9)	Do not need to consider the symmetrical position as well