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1. A smooth uniform sphere  $S$ , of mass  $m$ , is moving on a smooth horizontal plane when it collides obliquely with another smooth uniform sphere  $T$ , of the same radius as  $S$  but of mass  $2m$ , which is at rest on the plane. Immediately before the collision the velocity of  $S$  makes an angle  $\alpha$ , where  $\tan \alpha = \frac{3}{4}$ , with the line joining the centres of the spheres. Immediately after the collision the speed of  $T$  is  $V$ . The coefficient of restitution between the spheres is  $\frac{3}{4}$ .

(a) Find, in terms of  $V$ , the **speed** of  $S$

(i) immediately before the collision,

(ii) immediately after the collision.

(9)

(b) Find the angle through which the direction of motion of  $S$  is deflected as a result of the collision.

(4)

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3. Two particles, of masses  $m$  and  $2m$ , are connected to the ends of a long light inextensible string. The string passes over a small smooth fixed pulley and hangs vertically on either side. The particles are released from rest with the string taut. Each particle is subject to air resistance of magnitude  $kv^2$ , where  $v$  is the speed of each particle after it has moved a distance  $x$  from rest and  $k$  is a positive constant.

(a) Show that  $\frac{d}{dx}(v^2) + \frac{4k}{3m}v^2 = \frac{2g}{3}$  (6)

(b) Find  $v^2$  in terms of  $x$ . (5)

(c) Deduce that the tension in the string,  $T$ , satisfies

$$\frac{4mg}{3} \leq T < \frac{3mg}{2} \quad (5)$$

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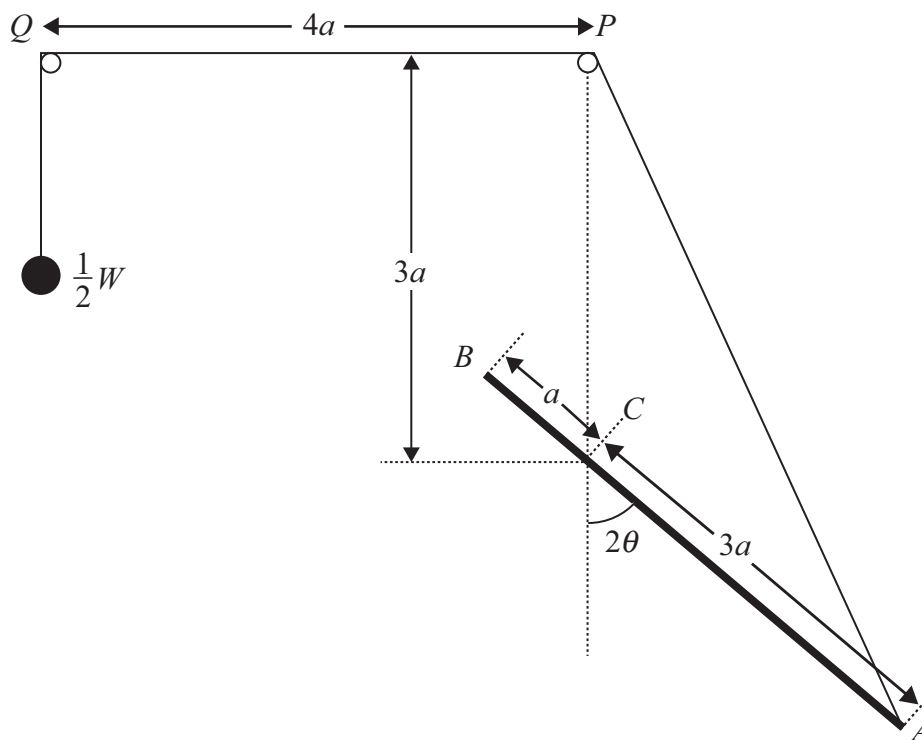


Figure 1

A uniform rod  $AB$ , of length  $4a$  and weight  $W$ , is free to rotate in a vertical plane about a fixed smooth horizontal axis which passes through the point  $C$  of the rod, where  $AC = 3a$ . One end of a light inextensible string of length  $L$ , where  $L > 10a$ , is attached to the end  $A$  of the rod and passes over a small smooth fixed peg at  $P$  and another small smooth fixed peg at  $Q$ . The point  $Q$  lies in the same vertical plane as  $P$ ,  $A$  and  $B$ . The point  $P$  is at a distance  $3a$  vertically above  $C$  and  $PQ$  is horizontal with  $PQ = 4a$ . A particle of weight  $\frac{1}{2}W$  is attached to the other end of the string and hangs vertically below  $Q$ . The rod is inclined at an angle  $2\theta$  to the vertical, where  $-\pi < 2\theta < \pi$ , as shown in Figure 1.

(a) Show that the potential energy of the system is

$$Wa(3\cos\theta - \cos 2\theta) + \text{constant} \tag{4}$$

(b) Find the positions of equilibrium and determine their stability. (8)

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6. Two points  $A$  and  $B$  are in a vertical line, with  $A$  above  $B$  and  $AB = 4a$ . One end of a light elastic spring, of natural length  $a$  and modulus of elasticity  $3mg$ , is attached to  $A$ . The other end of the spring is attached to a particle  $P$  of mass  $m$ . Another light elastic spring, of natural length  $a$  and modulus of elasticity  $mg$ , has one end attached to  $B$  and the other end attached to  $P$ . The particle  $P$  hangs at rest in equilibrium.

(a) Show that  $AP = \frac{7a}{4}$  (3)

The particle  $P$  is now pulled down vertically from its equilibrium position towards  $B$  and at time  $t = 0$  it is released from rest. At time  $t$ , the particle  $P$  is moving with speed  $v$  and has displacement  $x$  from its equilibrium position. The particle  $P$  is subject to air resistance of magnitude  $mkv$ , where  $k$  is a positive constant.

(b) Show that

$$\frac{d^2x}{dt^2} + k \frac{dx}{dt} + \frac{4g}{a}x = 0$$
(5)

(c) Find the range of values of  $k$  which would result in the motion of  $P$  being a damped oscillation. (3)

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**Question 6 continued**

A series of horizontal lines for writing the answer to Question 6 continued.

