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Mark Scheme (Results)

Summer 2012

GCE Mechanics M4
(6680) Paper 1

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6680 Mechanics M4
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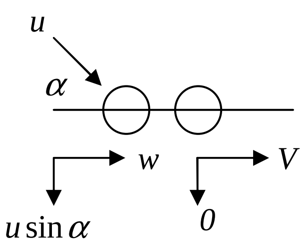
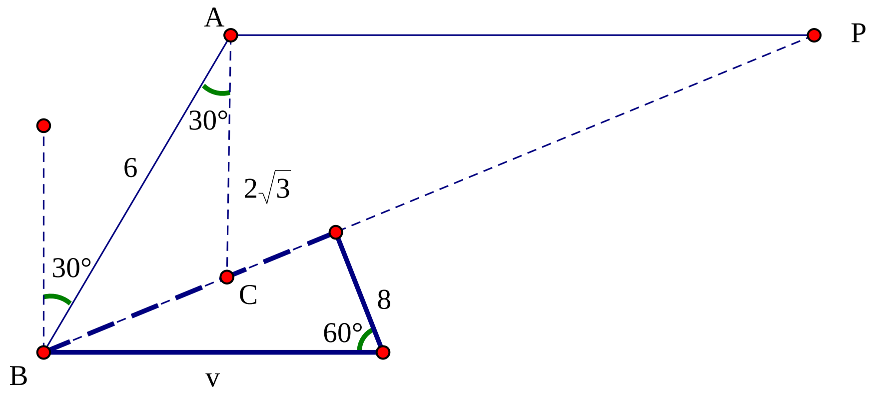
Question Number	Scheme	Marks	Notes
1. (a)			
	$mu\cos\alpha = mw + 2mV$	M1 A1	CLM parallel to the line of centres. $\frac{4}{5}u = w + 2V$. Need all terms but condone sign errors.
	$eu\cos\alpha = -w \ + \ \ V$	M1 A1	Impact law. Must be the right way round. $\frac{3}{4}\times\frac{4}{5}u = V - w$
	$u\cos\alpha(e+1) = 3V \Rightarrow$ (i) $u = \frac{15V}{7}$	M1 A1	Eliminate w and solve for u in terms of V or v.v. 2.14V or better
	$\Rightarrow w = -\frac{2V}{7}$	A1	Solve for w in terms of V. -0.286V or better
	(ii) speed of S $= \sqrt{(\frac{-2V}{7})^2 + (u\sin\alpha)^2} = \frac{V\sqrt{85}}{7}$	M1 A1	Use of Pythagoras with their $u\sin\alpha$ and w. $\sqrt{\left(\frac{-2V}{7}\right)^2 + \left(\frac{15V}{7}\times\frac{3}{5}\right)^2}$ $\sqrt{\frac{85}{49}}V$, accept 1.32V or better
		(9)	

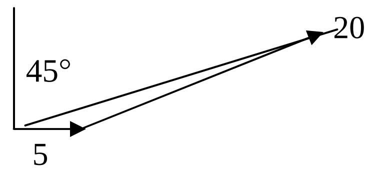
Question Number	Scheme	Marks	Notes
(b)	$\tan\theta = \dfrac{\frac{9V}{7}}{\frac{2V}{7}} = \frac{9}{2}$	M1	Direction of S after the collision. Condone $\dfrac{2}{9}$
		A1	$77.5°$ or $12.5°$ seen or implied
	defln angle = $180° - (\theta + \alpha)$	DM1	Combine their θ and α to find the required angle. e.g. $12.5° + \tan^{-1}\left(\dfrac{4}{3}\right)$
	$= 65.7°$ (3 sf)	A1	Accept $66°$
		(4)	
		13	

Question Number	Scheme	Marks	Notes
2.	With B as origin,		
	$\mathbf{r}_A = (6\sin 30\mathbf{i} + 6\cos 30\mathbf{j})$	M1	Express the original relative positions in component (vector) form – one term correct.
	$= (3)\mathbf{i} + (3\sqrt{3})\mathbf{j}$	A1	Both terms correct (substitution of trig values not required).
	$\mathbf{r}_B = vt\mathbf{i}$ or $\mathbf{v}_B = v\mathbf{i}$	B1	Position of B at time t (seen or implied)
	$(v-4)\mathbf{i} + (4\sqrt{3})\mathbf{j}$	M1	Express the relative velocity in component form – one term correct.
	or $\left(v - 8\sin 30\right)\mathrm{i} + \left(8\cos 30\right)\mathrm{j}$	A1	Both terms correct
	When B is $2\sqrt{3}$ km south of A,		
	$-3\sqrt{3} + 4\sqrt{3}t = -2\sqrt{3} \Rightarrow t = \frac{1}{4}$	M1 A1	Compare $\mathbf{j}$ displacement with $\pm 2\sqrt{3}$ and solve for t cao
	$vt - 3 - 4t = 0 \quad \Rightarrow v = 16$	M1 A1	Equate $\mathbf{i}$ displacement to zero and substitute their value of t. cao
	When B is due east of A,		
	$-3\sqrt{3} + 4\sqrt{3}t = 0 \Rightarrow t = \frac{3}{4}$ i.e. at 12.45 pm	M1 A1	Equate $\mathbf{j}$ displacement to zero and solve for t. Any equivalent form for the time.
	then distance AB = $16\text{x}\frac{3}{4} - 3 - 4\text{x}\frac{3}{4} = 6$ km.	M1 A1	Substitute their v & t in the $\mathbf{i}$ displacement and evaluate cao. Must be a scalar.
		13	
			See over page for geometrical alternative

or	Triangle *ABC*: cosine rule gives $BC^2 = 36 + 12 - 2 \times 6 \times 2\sqrt{3}\cos 30$	M1A1	The given information provides us with two triangles - velocities in bold. Fix *A* and *B* follows the path *BP*. *C* is the point when *B* is due South of *A*, and *P* when it is due East.
	Solve for *BC* and $\angle ABC$ $BC = 2\sqrt{3}$, $\longrightarrow$ triangle is isosceles	M1A1	
	$\angle B$ in velocity triangle is 30°	B1	
	Trig in rt$\angle$ triangle gives relative velocity $=8 \times \tan 60 = 8\sqrt{3}$	M1A1	
	$\angle$APB = 30° (angles of a triangle) so triangle is isosceles and distance *AP* = 6km	M1A1	
	Using cosine rule or symmetry of isosceles triangle, distance $BP = 6\sqrt{3}$	M1A1	
	Time taken $= \frac{6\sqrt{3}}{8\sqrt{3}} = \frac{3}{4}$ hr, time is now 12.45	M1A1	

Question	Scheme	Marks	Notes
3. (a)	$2mg - T - kv^2 = 2ma$	M1 A1	Equation of motion for particle of mass $2m$ aef
	$T - mg - kv^2 = ma$	M1 A1	Equation of motion for particle of mass m aef
	Adding, $mg - 2kv^2 = 3ma$		
	$\frac{2g}{3} - \frac{4kv^2}{3m} = 2v\frac{\mathrm{d}v}{\mathrm{d}x}$	DM1	Eliminate T, substitute for a and rearrange. Dependent on both previous M marks.
	$\frac{\mathrm{d}(v^2)}{\mathrm{d}x} + \frac{4kv^2}{3m} = \frac{2g}{3}$ *	A1 (6)	Reach **given answer** correctly
(b)	$IF = \mathrm{e}^{\int \frac{4\mathrm{k}}{3\mathrm{m}} dx} = \mathrm{e}^{\frac{4kx}{3m}}$	B1	
	$v^2 \mathrm{e}^{\frac{4kx}{3m}} = \frac{2g}{3}\int \mathrm{e}^{\frac{4kx}{3m}}\mathrm{d}x = \frac{mg}{2k}\mathrm{e}^{\frac{4kx}{3m}} \left(+C\right)$	M1 A1	Use integrating factor to obtain $\frac{d}{dx}\left(v^2 e^{\frac{4kx}{3m}}\right) = \frac{2g}{3}e^{\frac{4kx}{3m}}$ and integrate
	$v^2 = \frac{mg}{2k} + C\mathrm{e}^{\frac{-4kx}{3m}}$		
	$x = 0, v = 0 \Rightarrow C = -\frac{mg}{2k}$	M1	Use initial values to evaluate C or as limits in a definite integral and find an expression for v^2.
	$v^2 = \frac{mg}{2k}(1 - \mathrm{e}^{\frac{-4kx}{3m}})$	A1 (5)	aef.
OR	Separate variables: $\int \frac{3m}{2mg - 4kv^2}\mathrm{d}v^2 = \int 1\mathrm{d}x$	B1	CF $v^2 = Ae^{-\frac{4k}{3m}x}$
	$x = -\frac{3m}{4k}\ln\left\|2mg - 4kv^2\right\|\left(+C\right)$	M1A1	PI $v^2 = b \Rightarrow 0 + \frac{4k}{3m}b = \frac{2g}{3}$; GS $v^2 = Ae^{-\frac{4k}{3m}x} + \frac{mg}{2k}$
	$x = -\frac{3m}{4k}\ln\left\|\frac{2mg}{2mg - 4kv^2}\right\|$	M1	$x = 0,\ v = 0\ \Rightarrow A = -\frac{mg}{2k}$
	$v^2 = \frac{mg}{2k}(1 - \mathrm{e}^{\frac{-4kx}{3m}})$	A1	$v^2 = \frac{mg}{2k}(1 - \mathrm{e}^{\frac{-4kx}{3m}})$

(c)	When $x = 0, T = \dfrac{4mg}{3}$	M1 A1	Substitute v = 0 in the initial equations and solve for T
	As $x \to \infty, T \to \dfrac{9mg}{6} = \dfrac{3mg}{2}$	M1 A1	For large x, $v^2 \to \dfrac{mg}{2k}$. Substitute in the initial equations and solve for T
	Hence, $\dfrac{4mg}{3} \le T < \dfrac{3mg}{2}$. *	A1 (5)**16**	cwo – **answer is given**.

4.(a)			
	$\dfrac{\sin\theta}{5}=\dfrac{\sin 45}{20}$	M1	Use a vector triangle to find θ. Condone the 5 ms^{-1} in the wrong direction.
		A1	Correct equation for θ
	$\theta = 10.182...$		
	Bearing is $45^{\circ} - \theta = 34.8 = 35^{o}$ (nearest degree)	M1	Use their angle correctly in their triangle to find the bearing.
		A1 (4)	Accept alternative forms e.g. N 35 E
OR	SW $\rightarrow (20\sin\theta)T = (5+20\cos\theta)T$	M1	45° rt angle triangle
	$3t^2+8t-5=0$, $t=\dfrac{-8+\sqrt{124}}{6}=0.5225...$	A1	t substitution leading to correct equation in t, use of $R\cos(\theta+\alpha)$ o.e.
	$\theta = 55.18...$ Bearing is $90-\theta = 34.8°$	M1A1 (4)	
(b)	$v^2 = 5^2 + 20^2 - 2\text{x}5\text{x}20\cos 124.818...$ OR $v=\dfrac{20}{\sin 45}\times\sin 124.8$ OR $v = 5\cos 45 + 20\cos\theta$	M1	Complete method to find v
	$v = 23.22$	A1	Or better $\left(\dfrac{5\sqrt{2}+5\sqrt{62}}{2}\right)$
	$t=\dfrac{15}{23.22}=0.646$ h= 39 min (nearest min)	M1	$\dfrac{15}{\text{their } v}$
		A1 (4)	The Q specifies "nearest minute"
(c)	Due N, (since current affects both equally)	B1 (1)	cao
(d)	$t=\dfrac{4}{20}=0.2$ h = 12 min	B1 (1) **10**	cso

Question	Scheme	Marks		Notes
5. **(a)**		B1		GPE of rod e.g. $-Wa\cos 2\theta$
	$V = -Wa\cos 2\theta + \frac{1}{2}W\{3a-(L-6a\cos\theta-4a)\}$	M1		GPE of the particle e.g. $\frac{1}{2}W\{3a-(L-6a\cos\theta-4a)\}$ Condone 3a term missing.
		A1		Correct expression including the 3a (unless in the GPE for the rod) Accept aef e.g. $\sqrt{18a^2(1+\cos 2\theta)}$ for $6a\cos\theta$
	$= -Wa\cos 2\theta + 3Wa\cos\theta + (\frac{7Wa}{2} - \frac{WL}{2})$			
	$= Wa(3\cos\theta - \cos 2\theta) + \text{constant}$ *	A1	(4)	Obtain the **given answer** correctly
(b)	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = Wa(-3\sin\theta + 2\sin 2\theta)$	M1		Differentiate the given V wrt θ
		A1		correct
	For equilibrium, $Wa(-3\sin\theta + 2\sin 2\theta) = 0$			
	$\sin\theta(4\cos\theta - 3) = 0$	DM1		Set their derivative = 0
		A1		First answer
	$\Rightarrow \theta = 0$ or $\theta = \cos^{-1}\left(\frac{3}{4}\right)$	A1		Second answer - ignore $\theta = -\cos^{-1}\left(\frac{3}{4}\right)$. 0.72 rads or better
	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = Wa(-3\cos\theta + 4\cos 2\theta)$	M1		Obtain the second derivative of V and substitute one of their values for θ
	$\theta = 0: \frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = Wa > 0 \Rightarrow$ *stable*	A1		Correct working and conclusion for one value
	$\theta = \cos^{-1}\frac{3}{4}: \frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -\frac{7Wa}{4} < 0 \Rightarrow$ *unstable*	A1		Correct working and reasoning for the second. ISW for work on $-\cos^{-1}\left(\frac{3}{4}\right)$
			(8)	
			12	

6.(a)	$T_1 = mg + T_2$	M1		No resultant force and use of Hooke's law
	$\dfrac{3mge}{a} = mg + \dfrac{mg(2a-e)}{a}$	A1		Correct equation in one unknown $\dfrac{3mg(AP-a)}{a} = mg + \dfrac{mg(3a-AP)}{a}$, $3AP-3a = a+3a-AP$
	$e = \dfrac{3a}{4} \Rightarrow AP = \dfrac{7a}{4}$ *	A1		Derive **given result** correctly.
				Condone verification for 3/3
			(3)	
(b)	$mg + T_2 - T_1 - mkv = m\ddot{x}$	M1		Equation of motion – requires all terms but condone sign errors.
		A1		o.e. Correct equation in T_1 & T_2.
	$mg + \dfrac{mg(\frac{5}{4}a - x)}{a} - \dfrac{3mg(\frac{3}{4}a + x)}{a} - mkv = m\ddot{x}$	DM1		Use Hooke's law with extensions of the form $ka \pm x$
		A1		o.e. Correct unsimplified
	$\ddot{x} + k\dot{x} + \dfrac{4g}{a}x = 0$ *	A1		**Given answer** derived correctly
			(5)	
(c)	For a damped oscillation, $k^2 < \dfrac{16g}{a}$	M1		AE will have complex roots
		A1		Correctly substituted inequality
	i.e. $k < 4\sqrt{\dfrac{g}{a}}$	A1		Only (Q gives k>0) $-4\sqrt{\dfrac{g}{a}} < k < 4\sqrt{\dfrac{g}{a}}$ is A0.
			(3)	
			11	

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