Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced Level

Thursday 12 June 2008 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

H30094RA

1. [*In this question* **i** *and* **j** *are unit vectors due east and due north respectively.*]

A ship P is moving with velocity $(5\mathbf{i} - 4\mathbf{j})$ km h^{-1} and a ship Q is moving with velocity $(3\mathbf{i} + 7\mathbf{j})$ km h^{-1}.

Find the direction that ship Q appears to be moving in, to an observer on ship P, giving your answer as a bearing.

(5)

---

2. Two small smooth spheres A and B have equal radii. The mass of A is $2m$ kg and the mass of B is m kg. The spheres are moving on a smooth horizontal plane and they collide. Immediately before the collision the velocity of A is $(2\mathbf{i} - 2\mathbf{j})$ m s^{-1} and the velocity of B is $(-3\mathbf{i} - \mathbf{j})$ m s^{-1}. Immediately after the collision the velocity of A is $(\mathbf{i} - 3\mathbf{j})$ m s^{-1}.

Find the speed of B immediately after the collision.

(5)

---

3. At time $t = 0$, a particle of mass m is projected vertically downwards with speed U from a point above the ground. At time t the speed of the particle is v and the magnitude of the air resistance is modelled as being mkv, where k is a constant.

Given that $U < \dfrac{g}{2k}$, find, in terms of k, U and g, the time taken for the particle to double its speed.

(8)

4.

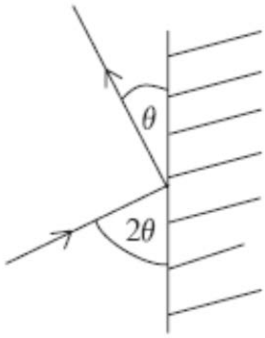

Figure 1

A small smooth ball B, moving on a horizontal plane, collides with a fixed vertical wall. Immediately before the collision the angle between the direction of motion of B and the wall is 2θ, where $0° < \theta < 45°$. Immediately after the collision the angle between the direction of motion of B and the wall is θ, as shown in Figure 1.

Given that the coefficient of restitution between B and the wall is $\frac{3}{8}$, find the value of $\tan\theta$.

(8)

---

5. A light elastic spring has natural length l and modulus of elasticity mg. One end of the spring is fixed to a point O on a rough horizontal table. The other end is attached to a particle P of mass m which is at rest on the table with $OP = l$. At time $t = 0$ the particle is projected with speed $\sqrt{(gl)}$ along the table in the direction OP. At time t the displacement of P from its initial position is x and its speed is v. The motion of P is subject to air resistance of magnitude $2mv\omega$, where $\omega = \sqrt{\frac{g}{l}}$. The coefficient of friction between P and the table is 0.5.

(*a*) Show that, until P first comes to rest,

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 2\omega\frac{\mathrm{d}x}{\mathrm{d}t} + \omega^2 x = -0.5g.$$

(6)

(*b*) Find x in terms of t, l and ω.

(6)

(*c*) Hence find, in terms of ω, the time taken for P to first come to instantaneous rest.

(3)

6.

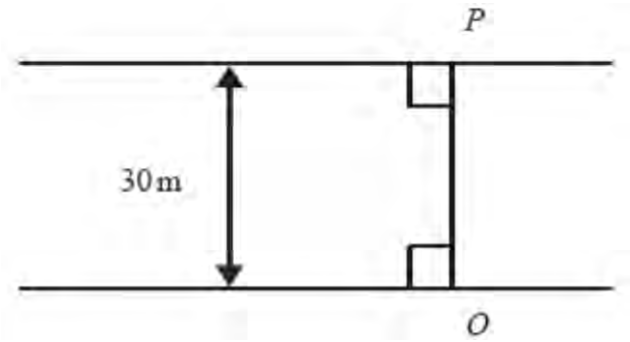

Figure 2

A river is 30 m wide and flows between two straight parallel banks. At each point of the river, the direction of flow is parallel to the banks. At time $t = 0$, a boat leaves a point O on one bank and moves in a straight line across the river to a point P on the opposite bank. Its path OP is perpendicular to both banks and $OP = 30$ m, as shown in Figure 2. The speed of flow of the river, r m s^{-1}, at a point on OP which is at a distance x m from O, is modelled as

$$r = \frac{1}{10}x, \quad 0 \le x \le 30.$$

The speed of the boat relative to the water is constant at 5 m s^{-1}. At time t seconds the boat is at a distance x m from O and is moving with speed v m s^{-1} in the direction OP.

(*a*) Show that

$$100v^2 = 2500 - x^2.$$

(3)

(*b*) Hence show that

$$\frac{d^2x}{dt^2} + \frac{x}{100} = 0.$$

(4)

(*c*) Find the total time taken for the boat to cross the river from O to P.

(9)

7.

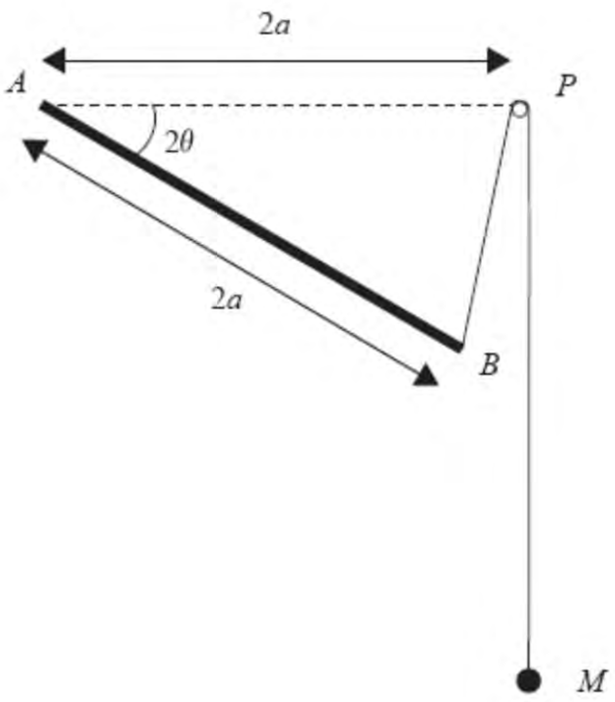

Figure 3

A uniform rod AB, of length $2a$ and mass kM where k is a constant, is free to rotate in a vertical plane about the fixed point A. One end of a light inextensible string of length $6a$ is attached to the end B of the rod and passes over a small smooth pulley which is fixed at the point P. The line AP is horizontal and of length $2a$. The other end of the string is attached to a particle of mass M which hangs vertically below the point P, as shown in Figure 3. The angle PAB is 2θ, where $0° \leq \theta \leq 180°$.

(*a*) Show that the potential energy of the system is

$$Mga(4 \sin \theta - k \sin 2\theta) + \text{constant}.$$

(5)

The system has a position of equilibrium when $\cos \theta = \frac{3}{4}$.

(*b*) Find the value of k.

(5)

(*c*) Hence find the value of $\cos \theta$ at the other position of equilibrium.

(3)

(*d*) Determine the stability of each of the two positions of equilibrium.

(5)

TOTAL FOR PAPER: 75 MARKS

END