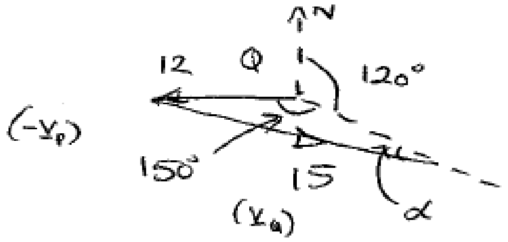
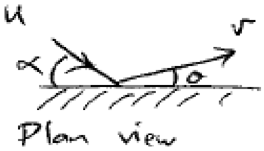
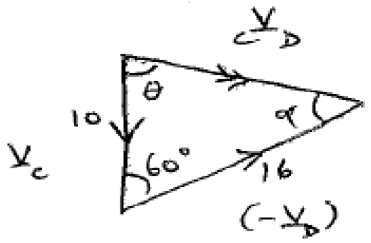
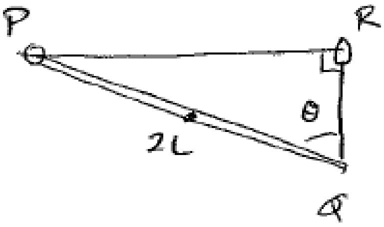
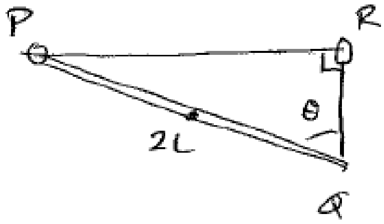
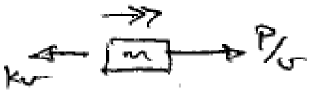
June 2006
6680 Mechanics M4
Mark Scheme

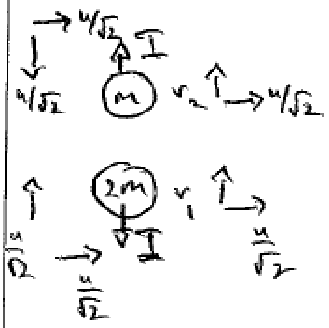
Question Number	Scheme	Marks
1.	 $\frac{\sin\alpha}{12} = \frac{\sin 150°}{15}$ $\Rightarrow \sin\alpha = \frac{6}{15}$ $\Rightarrow \alpha = 23.6°$ $\therefore$ Course is 096 (.4°)	M1 M1 A1 A1 A1 (5)
2.	 $(\rightarrow)$ $u\cos\alpha = v\cos\theta$ $(\uparrow)$ $eu\sin\alpha = v\sin\theta$ $\Rightarrow v^2 = u^2(\cos^2\alpha + e^2\sin^2\alpha)$ $\Rightarrow \underline{KE} = \frac{1}{2}mu^2\underline{(\cos^2\alpha + e^2\sin^2\alpha)}$	M1 A1 M1 A1 M1 A1 (6)

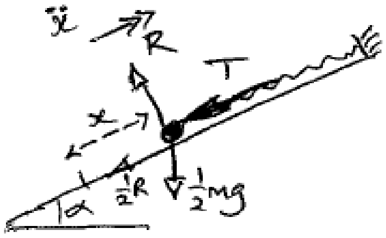
Question Number	Scheme	Marks
3.(a)		
	$\lvert \underset{CD}{v} \rvert^2 = 10^2 + 16^2 - 2\times10\times16\cos 60°$	M1 A1
	$= 196$	
	$\lvert \underset{CD}{v} \rvert = 14 \text{ ms}^{-1}$ *	A1 (3)
(b)	α is acute (opposite shortest side)	
	$\dfrac{\sin\alpha}{10} = \dfrac{\sin 60°}{14}$	M1
	$\Rightarrow \alpha = 38.213°\ldots$	A1
		
	(i) $DN = 4000 \sin 8.213$	M1
	$\simeq$ 571 m $\left(\frac{4000}{7}\right)$	A1
	(ii) $t = \dfrac{4000\cos 8.213°}{14}$ secs	M1 A1
	$\simeq 282.78\ldots$ secs	
	Time is 2.05pm (nearest minute)	A1 (7)
		(10)

Question number	Scheme	Marks
4.(a)		
	PE of rod $= -mgl\cos\theta$	B1
	EPE of string $= \frac{kmg}{2l}(2l\cos\theta - l)^2$	M1 A1
	Total PE of system, $V = -mgl\cos\theta + \frac{kmgl}{2}(2\cos\theta-1)^2 + c$	M1
	$= -mgl\cos\theta + \frac{kmgl}{2}(4\cos^2\theta - 4\cos\theta + 1) + c$	M1 A1
	$= mgl(-\cos\theta + 2k\cos^2\theta - 2k\cos\theta) + c'$	
	$= \underline{mgl[2k\cos^2\theta - (2k+1)\cos\theta] + c'}$ *	A1 (7)
(b)	$\frac{dV}{d\theta} = mgl(-4k\cos\theta\sin\theta + (2k+1)\sin\theta)$	M1 A1
	At equil^m, $mgl\sin\theta(-4k\cos\theta + (2k+1)) = 0$	M1
	$\Rightarrow \sin\theta = 0$ or $\cos\theta = \frac{2k+1}{4k}$	
	$\Rightarrow \theta = 0$ $(\theta>0)$ $\frac{2k+1}{4k} < 1$	M1
	$2k+1 < 4k$	
	$1 < 2k$	
	$\underline{\frac{1}{2} < k}$ *	A1 (5)
		(12)

Question number	Scheme	Marks
4. (a)	 PE of rod $= -mgL\cos\theta$	B1
	EPE of string $= \frac{kmg}{2L}(2L\cos\theta - L)^2$	M1 A1
	Total PE of system, $V = -mgL\cos\theta + \frac{kmgL}{2}(2\cos\theta-1)^2 + c$	M1
	$= -mgL\cos\theta + \frac{kmgL}{2}(4\cos^2\theta - 4\cos\theta + 1) + c$	M1 A1
	$= mgL(-\cos\theta + 2k\cos^2\theta - 2k\cos\theta) + c'$	
	$= mgL[2k\cos^2\theta - (2k+1)\cos\theta] + c'$ *	A1 (7)
(b)	$\frac{dV}{d\theta} = mgL(-4k\cos\theta\sin\theta + (2k+1)\sin\theta)$	M1 A1
	At equil^{m}, $mgL\sin\theta(-4k\cos\theta + (2k+1)) = 0$	M1
	$\Rightarrow \sin\theta = 0$ or $\cos\theta = \frac{2k+1}{4k}$	
	$\Rightarrow \theta = 0$ $(\theta > 0)$ $\frac{2k+1}{4k} < 1$	M1
	$2k+1 < 4k$	
	$1 < 2k$	
	$\frac{1}{2} < k$ *	A1 (5)
		(12)

Question number	Scheme	Marks
5.(a)	 $\frac{P}{v}$	B1
	$(\rightarrow):\quad \frac{P}{v} - kv = m\frac{dv}{dt}$	M1
	$\Rightarrow \underline{P = mv\frac{dv}{dt} + kv^2}$ *	A1 (3)
(b)	$\int_0^T dt = \int_U^{2U} \frac{mv\,dv}{P-kv^2} \qquad \left(U = \frac{1}{3}\sqrt{\frac{P}{k}}\right)$	M1 A1
	$\Rightarrow \quad T = \frac{-m}{2k}\left[\ln(P-kv^2)\right]_U^{2U}$	A2
	$= \frac{m}{2k}\left\{\ln\left(P - \frac{\cancel{k}}{9}\cdot\frac{P}{\cancel{k}}\right) - \ln\left(P - \frac{4\cancel{k}}{9}\cdot\frac{P}{\cancel{k}}\right)\right\}$	M1 A1
	$= \frac{m}{2k}\left\{\ln\frac{8P}{9} - \ln\frac{5P}{9}\right\}$	
	$= \frac{m}{2k}\ln\left(\frac{8P}{9} \times \frac{9}{5P}\right)$	M1
	$= \underline{\frac{m}{2k}\ln\frac{8}{5}}$	A1 (8)
		(11)

Question number	Scheme	Marks
6. (a)	 Form: $I = m\left(v_2 + \frac{u}{\sqrt{2}}\right)$	M1 A1
	CLM(↑): $\frac{2mu}{\sqrt{2}} - \frac{mu}{\sqrt{2}} = 2mv_1 + mv_2$	
	$\frac{u}{\sqrt{2}} = 2v_1 + v_2$ (1)	M1 A1
	NIL: $e\frac{2u}{\sqrt{2}} = \frac{u}{\sqrt{2}} = -v_1 + v_2$ (2)	M1 A1
	$\Rightarrow \frac{\cancel{3}u}{\sqrt{2}} = \cancel{3}v_2$	M1 A1
	$\Rightarrow I = m\left(\frac{u}{\sqrt{2}} + \frac{u}{\sqrt{2}}\right)$	
	$= mu\sqrt{2}$	A1 (9)
(b)	$v_2 - v_1 = \frac{u}{\sqrt{2}}$ (Separation speed)	M1
	time to wall $= \frac{d}{u/\sqrt{2}} = \frac{d\sqrt{2}}{u}$	M1 A1
	$\therefore$ Separation $= \frac{d\sqrt{2}}{u} \times \frac{u}{\sqrt{2}} = d$	M1 A1
		(5)
		14

Question number	Scheme	Marks
7.(a)		
	$F = \frac{1}{2}R$	M1
	$R = mg\cos\alpha$	B1
	$T = \frac{4mgx}{L}$	B1
	$(\nearrow): \quad -F - mg\sin\alpha - T = m\ddot{x}$	M1 A1
	$-\frac{1}{2}\cdot\frac{4}{5}mg - \frac{3}{5}mg - \frac{4mgx}{L} = m\ddot{x}$	
	$\Rightarrow \quad \frac{d^2x}{dt^2} + 4\omega^2 x = -g \quad *$	A1
	$(\omega = \sqrt{g/L})$	(6)
(b)	$m^2 + 4\omega^2 = 0 \Rightarrow m = \pm 2\omega i$	
	C.F. is $x = A\sin 2\omega t + B\cos 2\omega t$	M1
	P.I. is $x = \frac{-g}{4\omega^2} = -L/4$	B1
	G.S. is $x = A\sin 2\omega t + B\cos 2\omega t - \frac{L}{4}$	B1
	$t=0, x=0: \quad B = L/4$	M1 A1
	$\dot{x} = 2\omega A\cos 2\omega t - 2\omega B\sin 2\omega t$	
	$t=0, \dot{x} = \frac{1}{2}\sqrt{gL}: \quad \frac{\sqrt{gL}}{2} = 2\sqrt{\frac{g}{L}}A \Rightarrow A = L/4$	M1
	$\Rightarrow \underline{x = \frac{L}{4}(\sin 2\omega t + \cos 2\omega t - 1)}$	A1 (7)
(c)	$\dot{x} = 0 \Rightarrow \cancel{2\omega}A\cos 2\omega t - \cancel{2\omega}B\sin 2\omega t = 0$	M1
	$\Rightarrow \tan 2\omega t = \frac{A}{B} = 1$	
	$\Rightarrow 2\omega t = \pi/4$ (first value)	A1
	$\Rightarrow x = \frac{L}{4}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1\right)$	M1
	$= \frac{L}{4}(\sqrt{2} - 1)$	A1 (4)
		(17)