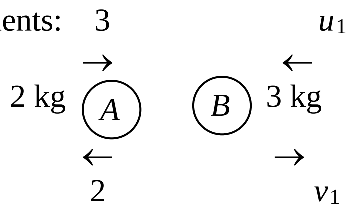
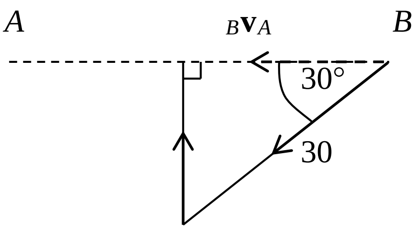
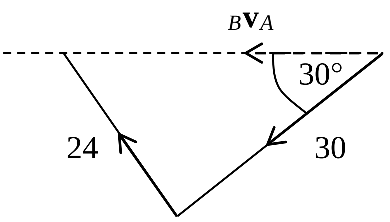
EDEXCEL MECHANICS M4 (6680) - JUNE 2002 PROVISIONAL MARK SCHEME

Question Number	Scheme	Marks
1.	**i** components: 	
	Momentum ⟷: $2 \times 3 - 3u_1 = -2 \times 2 + 3v_1$	M1
	NLI: $10 = 3u_1 + 3v_1$	A1
	$\frac{1}{2}(3 + u_1) = 2 + v_1$	M1
	$1 = u_1 - 2v_1$	A1
	Solve for u_1, $23 = 9u_1$ $\Rightarrow u_1 = \frac{23}{9}$	M1 A1
	j component $= -u_1 \tan\alpha = -\frac{46}{9}$	M1 A1 ft
	Hence $\mathbf{u}_B = -\frac{23}{9}\mathbf{i} - \frac{46}{9}\mathbf{j}$	A1
		(9 marks)
2.	 $_B\mathbf{v}_A$ as ←	M1
	Correct △ for $\mathbf{v}_B$ minimum	M1
	$v = 30 \sin 30° = 15$ km h^{-1}	A1 (3)
	 Correct △	M1
	$\frac{30}{\sin\alpha} = \frac{24}{\sin 30°}$	M1
	$\Rightarrow \sin\alpha = \frac{5}{8}$	A1
	$\lvert {}_B\mathbf{v}_A \rvert = 30\cos 30° + 24\cos\alpha$	M1 A1
	(≈ 44.716)	
	$T = \frac{20}{44.716} \approx 0.4473$	M1
	$\Rightarrow$ 0927 hrs (awrt)	A1 (7)
		(10 marks)

(ft = follow through mark; awrt = anything which rounds to)

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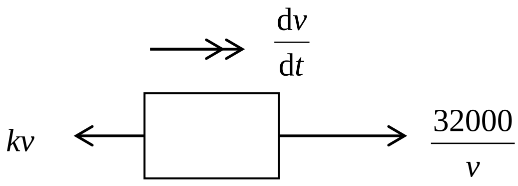
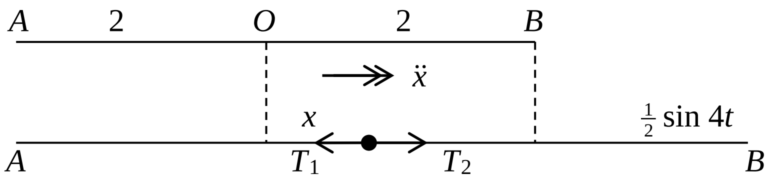
3. (*a*)		
	$800\frac{dv}{dt} = \frac{32000}{v} - kv$	M1 A1
	$\Rightarrow \quad 800v\frac{dv}{dt} = 32000 - kv^2 \quad (*)$	A1 (3)
(*b*)	$v = 40, \quad \frac{dv}{dt} = 0 \quad \Rightarrow \quad 32000 = k \times 40^2$	M1
	$\Rightarrow \quad k = 20$	A1 (2)
(*c*)	$\int dt = 800\int \frac{v\,dv}{32000 - 20v^2} = \int \frac{40v\,dv}{1600 - v^2}$	M1
	$t = -20 \ln (1600 - v^2) \, (+ C)$	M1 A1 ft
	$t = 0, \ v = 0 \ \Rightarrow \ C = 20 \ln 1600$ (or use of limits)	
	$t = 20 \ln 1600 - 20 \ln (1600 - v^2)$	M1 A1 ft
	$\Rightarrow \quad t = 20 \ln \left(\frac{1600}{1600 - v^2}\right)$	
	$\frac{1600}{1600 - v^2} = e^{\frac{t}{20}}$	M1
	$1600 e^{-\frac{t}{20}} = 1600 - v^2$	
	$v = 40\sqrt{\left(1 - e^{-\frac{t}{20}}\right)}$	A1 (7)
		(12 marks)

((*) indicates answer is given on the examination paper)

Question Number	Scheme	Marks
4. (*a*)	$AC = 4a\cos\theta \Rightarrow$ extension in spring $= 4a\cos\theta - 2a$	B1
	$V = -2mga\cos\theta - 2mg \times 3a\cos\theta + \frac{4mg}{4a}(4a\cos\theta - 2a)\ (+C)$	M1 A1 A1
	$= -8mga\cos\theta + 4mg(2\cos\theta - 1)^2\ (+C)$	
	$= 4mga[(2\cos\theta - 1)^2 - 2\cos\theta]\ (+C)$	A1 (5)
(*b*)	$\frac{dV}{d\theta} = 4mga[2(2\cos\theta - 1)(-2\sin\theta) + 2\sin\theta)]$	M1 A1
	$= 8mga\sin\theta(3 - 4\cos\theta)$	
	$= 0 \Rightarrow \cos\theta = \frac{3}{4}\ (\theta \neq 0, \pi)$	M1 A1
	$\Rightarrow \theta = 0.723$	A1 (4)
(*c*)	$\frac{d^2V}{d\theta^2} = 8mga\cos\theta(3 - 4\cos\theta) + 32mga\sin^2\theta$	M1 A1
	as $\theta = \frac{3}{4} \Rightarrow \frac{d^2V}{d\theta^2} = 0 + 32mga \times \frac{7}{16}$	M1
	$= 14mga$	
	$> 0 \Rightarrow$ stable	A1 (4)
		(13 marks)

Question Number	Scheme	Marks
5. (*a*)	$\mathbf{r}_P = \begin{pmatrix}1\\-2\\3\end{pmatrix} + t\begin{pmatrix}1\\2\\-1\end{pmatrix}$ $\quad \mathbf{r}_Q = \begin{pmatrix}-1\\2\\-1\end{pmatrix} + t\begin{pmatrix}2\\0\\1\end{pmatrix}$	B1 (either)
	$\begin{pmatrix}1\\-2\\3\end{pmatrix} + t\begin{pmatrix}1\\2\\-1\end{pmatrix} = \begin{pmatrix}-1\\2\\-1\end{pmatrix} + t\begin{pmatrix}2\\0\\1\end{pmatrix}$	M1
	$\Rightarrow t = 2$ (one component)	A1
	showing true for all components $\Rightarrow$ collide	M1
	$\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$	A1 (5)
(*b*)	$\mathbf{v}_R - \mathbf{v}_P = \lambda\begin{pmatrix}-5\\4\\-1\end{pmatrix}$, $\mathbf{v}_R - \mathbf{v}_Q = \mu\begin{pmatrix}-2\\2\\-1\end{pmatrix}$	B1
	$\mathbf{v}_Q - \mathbf{v}_P = \begin{pmatrix}2\\0\\1\end{pmatrix} - \begin{pmatrix}1\\2\\-1\end{pmatrix} = \lambda\begin{pmatrix}-5\\4\\-1\end{pmatrix} - \mu\begin{pmatrix}-2\\2\\-1\end{pmatrix}$	M1
	$\begin{pmatrix}1\\-2\\2\end{pmatrix} = \lambda\begin{pmatrix}-5\\4\\-1\end{pmatrix} - \mu\begin{pmatrix}-2\\2\\-1\end{pmatrix}$	
	$\left.\begin{array}{l}-5\lambda + 2\mu = 1\\ 4\lambda - 2\mu = -2\\ -\lambda + \mu = 2\end{array}\right\}$	A1
	Solve for either $\lambda = 1$ or $\mu = 3$	M1 A1
	Hence $\mathbf{v}_R = -4\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$	A1
	$\begin{pmatrix}3\\2\\1\end{pmatrix} = \begin{pmatrix}a\\b\\c\end{pmatrix} + 2\begin{pmatrix}-4\\6\\-2\end{pmatrix}$	M1 A1ft
	$\Rightarrow a = 11, b = -10, c = 5$	
	$t = 0$, R is at $11\mathbf{i} - 10\mathbf{j} + 5\mathbf{k}$	A1 (9)
		(14 marks)

(ft = follow through mark)

Question Number	Scheme	Marks
6. (*a*)	$t = 0$ 	
	$T_1 = 4(1 + x)$; $T_2 = 4(1 + \frac{1}{2}\sin 4t - x)$	B1; B1
	$T_2 - T_1 = 2\ddot{x}$	M1
	$2\ddot{x} = 4(1 + \frac{1}{2}\sin 4t - x) - 4(1 + x)$	A1
	$\Rightarrow \quad \ddot{x} + 4x = \sin 4t \quad$ (✱)	A1 (5)
(*b*)	CF: $x = A\sin 2t + B\cos 2t$	B1
	PI: $x = P\sin 4t$	M1
	$-16P\sin 4t + 4P\sin 4t = \sin 4t$	M1
	$\Rightarrow P = -\frac{1}{12}$	A1
	$x = A\sin 2t + B\cos 2t - \frac{1}{12}\sin 4t$	M1
	$t = 0, x = 0 \Rightarrow B = 0$	A1
	$\dot{x} = 2A\cos 2t - \frac{1}{3}\cos 4t$	M1
	$t = 0, \dot{x} = 0 \Rightarrow A = \frac{1}{6}$	A1 ft (8)
	$\dot{x} = 0 \Rightarrow \frac{1}{3}\cos 2t - \frac{1}{3}\cos 4t = 0$	M1
	$\Rightarrow \cos 4t = \cos 2t$	
	$\Rightarrow 4t = 2t + 2\pi$ *or* $2\pi - 2t$	M1
	$\Rightarrow t = \pi$ or $\frac{\pi}{3}$	A1
	$\Rightarrow$ First at rest after $t = 0$ when $t = \frac{\pi}{3}$	A1 cso (4)
		(17 marks)

(ft = follow through mark; (✱) denotes answer is given on paper; cso = correct solution only)