Edexcel Maths M4

Past Paper Pack

2009-2013

June 2009

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 8 0 / 0 1	Signature	

Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Monday 15 June 2009 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Orange or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

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Check that you have the correct question paper.
Answer ALL the questions. You must write your answers to each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 6 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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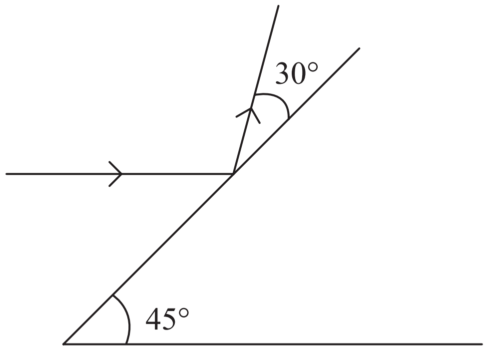

Figure 1

A fixed smooth plane is inclined to the horizontal at an angle of 45°. A particle P is moving horizontally and strikes the plane. Immediately before the impact, P is moving in a vertical plane containing a line of greatest slope of the inclined plane. Immediately after the impact, P is moving in a direction which makes an angle of 30° with the inclined plane, as shown in Figure 1.

Find the fraction of the kinetic energy of P which is lost in the impact.

(6)

Leave blank

2. At time $t = 0$, a particle P of mass m is projected vertically upwards with speed $\sqrt{\dfrac{g}{k}}$, where k is a constant. At time t the speed of P is v. The particle P moves against air resistance whose magnitude is modelled as being mkv^2 when the speed of P is v. Find, in terms of k, the distance travelled by P until its speed first becomes half of its initial speed.

(9)

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Question 2 continued

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3. At noon a motorboat P is 2 km north-west of another motorboat Q. The motorboat P is moving due south at 20 m s^{-1}. The motorboat Q is pursuing motorboat P at a speed of 12 m s^{-1} and sets a course in order to get as close to motorboat P as possible.

(a) Find the course set by Q, giving your answer as a bearing to the nearest degree.
(4)

(b) Find the shortest distance between P and Q.
(3)

(c) Find the distance travelled by Q from its position at noon to the point of closest approach.
(5)

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Question 3 continued

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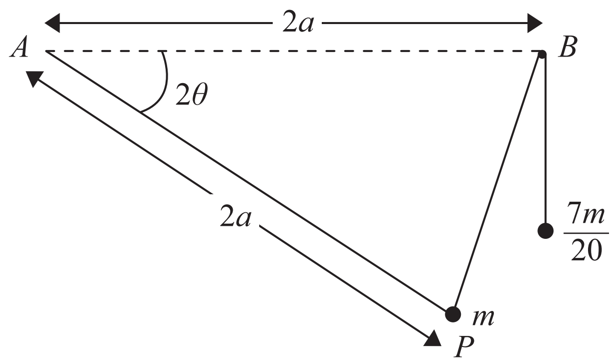

Figure 2

A light inextensible string of length $2a$ has one end attached to a fixed point A. The other end of the string is attached to a particle P of mass m. A second light inextensible string of length L, where $L > \frac{12a}{5}$, has one of its ends attached to P and passes over a small smooth peg fixed at a point B. The line AB is horizontal and $AB = 2a$. The other end of the second string is attached to a particle of mass $\frac{7}{20}m$, which hangs vertically below B, as shown in Figure 2.

(a) Show that the potential energy of the system, when the angle $PAB = 2\theta$, is

$$\tfrac{1}{5}mga(7\sin\theta - 10\sin 2\theta) + \text{constant}.$$

(4)

(b) Show that there is only one value of $\cos\theta$ for which the system is in equilibrium and find this value.

(8)

(c) Determine the stability of the position of equilibrium.

(4)

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Question 4 continued

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5. Two small smooth spheres A and B, of mass 2 kg and 1 kg respectively, are moving on a smooth horizontal plane when they collide. Immediately before the collision the velocity of A is $(\mathbf{i} + 2\mathbf{j})$ m s^{-1} and the velocity of B is $-2\mathbf{i}$ m s^{-1}. Immediately after the collision the velocity of A is $\mathbf{j}$ m s^{-1}.

(a) Show that the velocity of B immediately after the collision is $2\mathbf{j}$ m s^{-1}.

(3)

(b) Find the impulse of B on A in the collision, giving your answer as a vector, and hence show that the line of centres is parallel to $\mathbf{i} + \mathbf{j}$.

(4)

(c) Find the coefficient of restitution between A and B.

(6)

M 3 4 2 7 6 A 0 1 6 2 4

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Question 5 continued

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6. A light elastic spring AB has natural length $2a$ and modulus of elasticity $2mn^2a$, where n is a constant. A particle P of mass m is attached to the end A of the spring. At time $t = 0$, the spring, with P attached, lies at rest and unstretched on a smooth horizontal plane. The other end B of the spring is then pulled along the plane in the direction AB with constant acceleration f. At time t the extension of the spring is x.

(a) Show that $$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + n^2x = f.$$
(6)

(b) Find x in terms of n, f and t.
(8)

Hence find

(c) the maximum extension of the spring,
(3)

(d) the speed of P when the spring first reaches its maximum extension.
(2)

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Question 6 continued

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Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 8 0 / 0 1	Signature	

Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Tuesday 22 June 2010 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
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1. [*In this question* **i** *and* **j** *are unit vectors due east and due north respectively*]

A man cycles at a constant speed u m s^{-1} on level ground and finds that when his velocity is $u\mathbf{j}$ m s^{-1} the velocity of the wind appears to be $v(3\mathbf{i} - 4\mathbf{j})$ m s^{-1}, where v is a positive constant.

When the man cycles with velocity $\frac{1}{5}u(-3\mathbf{i}+4\mathbf{j})$ m s^{-1}, the velocity of the wind appears to be $w\mathbf{i}$ m s^{-1}, where w is a positive constant.

Find, in terms of u, the true velocity of the wind.

(7)

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2. Two smooth uniform spheres S and T have equal radii. The mass of S is 0.3 kg and the mass of T is 0.6 kg. The spheres are moving on a smooth horizontal plane and collide obliquely. Immediately before the collision the velocity of S is $\mathbf{u}_1$ m s^{-1} and the velocity of T is $\mathbf{u}_2$ m s^{-1}. The coefficient of restitution between the spheres is 0.5. Immediately after the collision the velocity of S is $(-\mathbf{i}+2\mathbf{j})$ m s^{-1} and the velocity of T is $(\mathbf{i}+\mathbf{j})$ m s^{-1}. Given that when the spheres collide the line joining their centres is parallel to $\mathbf{i}$,

(a) find

(i) $\mathbf{u}_1$,

(ii) $\mathbf{u}_2$.

(6)

After the collision, T goes on to collide with a smooth vertical wall which is parallel to $\mathbf{j}$. Given that the coefficient of restitution between T and the wall is also 0.5, find

(b) the angle through which the direction of motion of T is deflected as a result of the collision with the wall,

(5)

(c) the loss in kinetic energy of T caused by the collision with the wall.

(3)

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Question 2 continued

N 3 5 3 9 3 A 0 5 2 4

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3. At 12 noon, ship A is 8 km due west of ship B. Ship A is moving due north at a constant speed of 10 km h^{-1}. Ship B is moving at a constant speed of 6 km h^{-1} on a bearing so that it passes as close to A as possible.

(a) Find the bearing on which ship B moves.

(4)

(b) Find the shortest distance between the two ships.

(3)

(c) Find the time when the two ships are closest.

(3)

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Question 3 continued

N 3 5 3 9 3 A 0 9 2 4

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4. A particle of mass m is projected vertically upwards, at time $t = 0$, with speed U. The particle is subject to air resistance of magnitude $\frac{mgv^2}{k^2}$, where v is the speed of the particle at time t and k is a positive constant.

(a) Show that the particle reaches its greatest height above the point of projection at time $\frac{k}{g}\tan^{-1}\left(\frac{U}{k}\right)$.
(6)

(b) Find the greatest height above the point of projection attained by the particle.
(6)

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Question 4 continued

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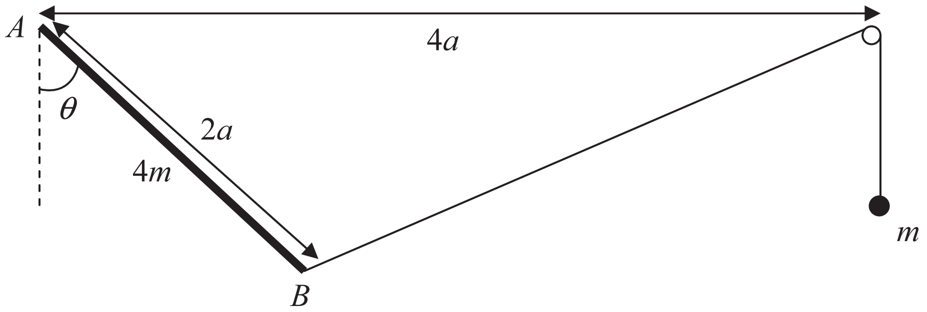

Figure 1

The end A of a uniform rod AB, of length $2a$ and mass $4m$, is smoothly hinged to a fixed point. The end B is attached to one end of a light inextensible string which passes over a small smooth pulley, fixed at the same level as A. The distance from A to the pulley is $4a$. The other end of the string carries a particle of mass m which hangs freely, vertically below the pulley, with the string taut. The angle between the rod and the downward vertical is θ, where $0 < \theta < \frac{\pi}{2}$, as shown in Figure 1.

(a) Show that the potential energy of the system is

$$2mga(\sqrt{(5 - 4\sin\theta)} - 2\cos\theta) + \text{constant}\,.$$

(5)

(b) Hence, or otherwise, show that any value of θ which corresponds to a position of equilibrium of the system satisfies the equation

$$4\sin^3\theta - 6\sin^2\theta + 1 = 0\,.$$

(5)

(c) Given that $\theta = \frac{\pi}{6}$ corresponds to a position of equilibrium, determine its stability.

(5)

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Question 5 continued

N 3 5 3 9 3 A 0 1 7 2 4

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6. Two points A and B lie on a smooth horizontal table with $AB = 4a$. One end of a light elastic spring, of natural length a and modulus of elasticity $2mg$, is attached to A. The other end of the spring is attached to a particle P of mass m. Another light elastic spring, of natural length a and modulus of elasticity mg, has one end attached to B and the other end attached to P. The particle P is on the table at rest and in equilibrium.

(a) Show that $AP = \dfrac{5a}{3}$.

(4)

The particle P is now moved along the table from its equilibrium position through a distance $0.5a$ towards B and released from rest at time $t = 0$. At time t, P is moving with speed v and has displacement x from its equilibrium position. There is a resistance to motion of magnitude $4m\omega v$ where $\omega = \sqrt{\left(\dfrac{g}{a}\right)}$.

(b) Show that $\dfrac{d^2x}{dt^2} + 4\omega\dfrac{dx}{dt} + 3\omega^2 x = 0$.

(5)

(c) Find the velocity, $\dfrac{dx}{dt}$, of P in terms of a, ω and t.

(8)

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Question 6 continued

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June 2011

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 8 0 / 0 1	Signature	

Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Wednesday 22 June 2011 – Morning

Time: 1 hour 30 minutes

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Team Leader's use only

Question Number	Leave Blank
1	
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Total	

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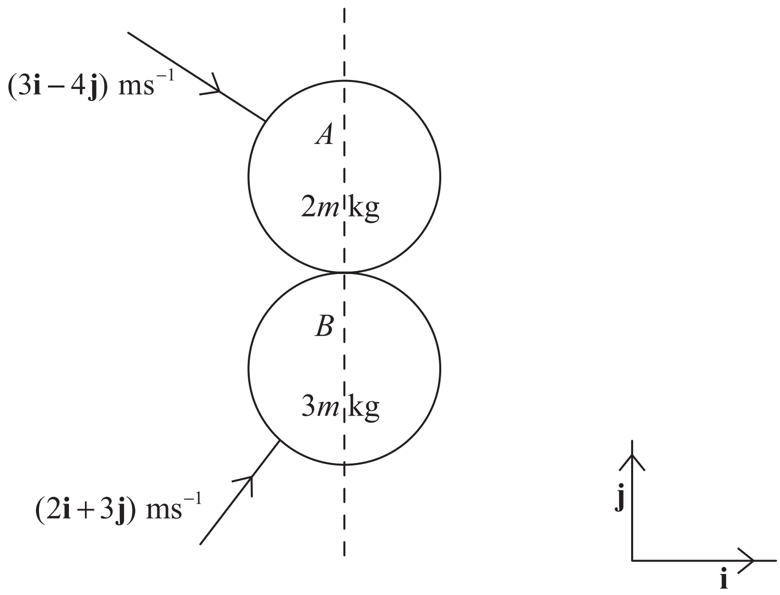

Figure 1

Two smooth uniform spheres A and B have masses $2m$ kg and $3m$ kg respectively and equal radii. The spheres are moving on a smooth horizontal surface. Initially, sphere A has velocity $(3\mathbf{i} - 4\mathbf{j})$ m s^{-1} and sphere B has velocity $(2\mathbf{i} - 3\mathbf{j})$ m s^{-1}. When the spheres collide, the line joining their centres is parallel to $\mathbf{j}$, as shown in Figure 1. The coefficient of restitution between the spheres is $\frac{3}{7}$. Find, in terms of m, the total kinetic energy lost in the collision.

(10)

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Question 1 continued

Q1

(Total 10 marks)

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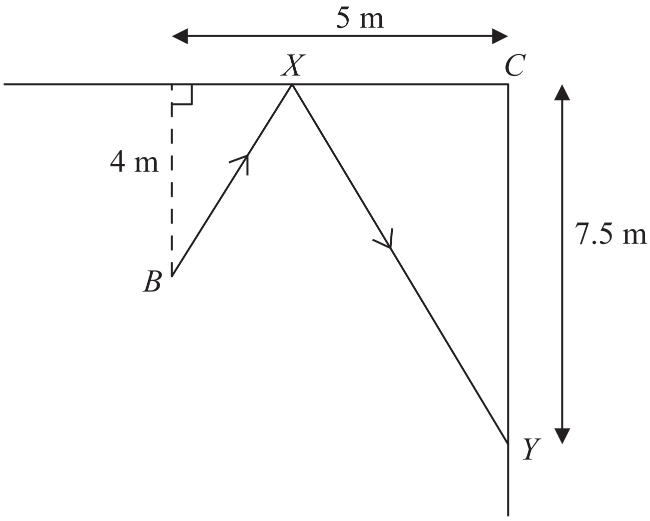

Figure 2

Figure 2 represents part of the smooth rectangular floor of a sports hall. A ball is at B, 4 m from one wall of the hall and 5 m from an adjacent wall. These two walls are smooth and meet at the corner C. The ball is kicked so that it travels along the floor, bounces off the first wall at the point X and hits the second wall at the point Y. The point Y is 7.5 m from the corner C.
The coefficient of restitution between the ball and the first wall is $\frac{3}{4}$.

Modelling the ball as a particle, find the distance CX.

(9)

P 3 5 4 1 5 A 0 4 2 4

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Question 2 continued

(Total 9 marks)

Q2

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3. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A coastguard patrol boat C is moving with constant velocity $(8\mathbf{i} + u\mathbf{j})$ km h^{-1}. Another ship S is moving with constant velocity $(12\mathbf{i} + 16\mathbf{j})$ km h^{-1}.

(a) Find, in terms of u, the velocity of C relative to S.

(2)

At noon, S is 10 km due west of C.
If C is to intercept S,

(b) (i) find the value of u.

(ii) Using this value of u, find the time at which C would intercept S.

(4)

If instead, at noon, C is moving with velocity $(8\mathbf{i} + 8\mathbf{j})$ km h^{-1} and continues at this constant velocity,

(c) find the distance of closest approach of C to S.

(5)

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Question 3 continued

P 3 5 4 1 5 A 0 7 2 4

Turn over

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4. A hiker walking due east at a steady speed of 5 km h^{-1} notices that the wind appears to come from a direction with bearing 050. At the same time, another hiker moving on a bearing of 320, and also walking at 5 km h^{-1}, notices that the wind appears to come from due north.

Find

(a) the direction from which the wind is blowing,

(3)

(b) the wind speed.

(4)

Leave blank

5. A particle Q of mass 6 kg is moving along the x-axis. At time t seconds the displacement of Q from the origin O is x metres and the speed of Q is v m s^{-1}. The particle moves under the action of a retarding force of magnitude $(a + bv^2)$ N, where a and b are positive constants. At time $t = 0$, Q is at O and moving with speed U m s^{-1} in the positive x-direction. The particle Q comes to instantaneous rest at the point X.

(a) Show that the distance OX is

$$\frac{3}{b}\ln\left(1+\frac{bU^2}{a}\right) \text{ m}$$

(6)

Given that $a = 12$ and $b = 3$,

(b) find, in terms of U, the time taken to move from O to X.

(5)

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Question 5 continued

P 3 5 4 1 5 A 0 1 3 2 4

Turn over

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6. A particle P of mass 4 kg moves along a horizontal straight line under the action of a force directed towards a fixed point O on the line. At time t seconds, P is x metres from O and the force towards O has magnitude $9x$ newtons. The particle P is also subject to air resistance, which has magnitude $12v$ newtons when P is moving with speed v m s^{-1}.

(a) Show that the equation of motion of P is

$$4\frac{d^2x}{dt^2}+12\frac{dx}{dt}+9x=0$$

(4)

It is given that the solution of this differential equation is of the form

$$x = e^{-\lambda t}(At + B)$$

When $t = 0$ the particle is released from rest at the point R, where $OR = 4$ m.

Find,

(b) the values of the constants λ, A and B,

(4)

(c) the greatest speed of P in the subsequent motion.

(5)

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Question 6 continued

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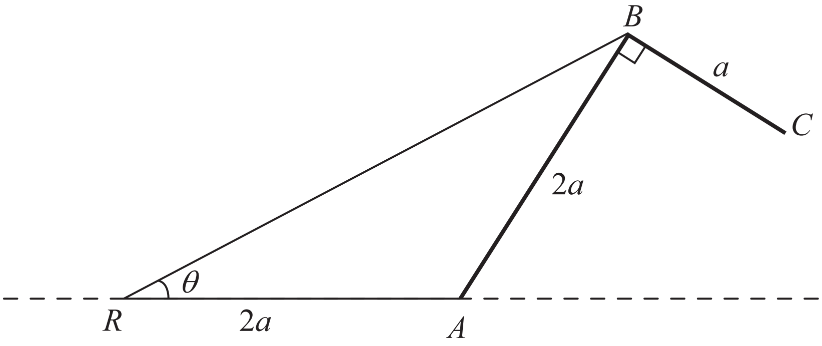

Figure 3

Figure 3 shows a framework ABC, consisting of two uniform rods rigidly joined together at B so that $\angle ABC = 90°$. The rod AB has length $2a$ and mass $4m$, and the rod BC has length a and mass $2m$. The framework is smoothly hinged at A to a fixed point, so that the framework can rotate in a fixed vertical plane. One end of a light elastic string, of natural length $2a$ and modulus of elasticity $3mg$, is attached to A. The string passes through a small smooth ring R fixed at a distance $2a$ from A, on the same horizontal level as A and in the same vertical plane as the framework. The other end of the string is attached to B. The angle ARB is θ, where $0 < \theta < \frac{\pi}{2}$.

(a) Show that the potential energy V of the system is given by

$$V = 8amg \sin 2\theta + 5amg \cos 2\theta + \text{constant}$$

(7)

(b) Find the value of θ for which the system is in equilibrium.

(4)

(c) Determine the stability of this position of equilibrium.

(3)

P 3 5 4 1 5 A 0 2 0 2 4

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Question 7 continued

June 2012

Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	8	0	/	0	1	Signature	

Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Friday 1 June 2012 – Morning

Time: 1 hour 30 minutes

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Team Leader's use only

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1. A smooth uniform sphere S, of mass m, is moving on a smooth horizontal plane when it collides obliquely with another smooth uniform sphere T, of the same radius as S but of mass $2m$, which is at rest on the plane. Immediately before the collision the velocity of S makes an angle α, where $\tan\alpha = \frac{3}{4}$, with the line joining the centres of the spheres. Immediately after the collision the speed of T is V. The coefficient of restitution between the spheres is $\frac{3}{4}$.

(a) Find, in terms of V, the **speed** of S

(i) immediately before the collision,

(ii) immediately after the collision.

(9)

(b) Find the angle through which the direction of motion of S is deflected as a result of the collision.

(4)

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Question 1 continued

P 4 0 1 1 0 A 0 3 2 8

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2. A ship A is moving at a constant speed of 8 km h^{-1} on a bearing of 150°. At noon a second ship B is 6 km from A, on a bearing of 210°. Ship B is moving due east at a constant speed. At a later time, B is $2\sqrt{3}$ km due south of A.

Find

(i) the time at which B will be due east of A,

(ii) the distance between the ships at that time.

(13)

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Question 2 continued

P 4 0 1 1 0 A 0 7 2 8

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3. Two particles, of masses m and $2m$, are connected to the ends of a long light inextensible string. The string passes over a small smooth fixed pulley and hangs vertically on either side. The particles are released from rest with the string taut. Each particle is subject to air resistance of magnitude kv^2, where v is the speed of each particle after it has moved a distance x from rest and k is a positive constant.

(a) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(v^2) + \frac{4k}{3m}v^2 = \frac{2g}{3}$

(6)

(b) Find v^2 in terms of x.

(5)

(c) Deduce that the tension in the string, T, satisfies

$$\frac{4mg}{3} \leqslant T < \frac{3mg}{2}$$

(5)

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Question 3 continued

P 4 0 1 1 0 A 0 1 1 2 8

Turn over

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4. A rescue boat, whose maximum speed is 20 km h^{-1}, receives a signal which indicates that a yacht is in distress near a fixed point P. The rescue boat is 15 km south-west of P. There is a constant current of 5 km h^{-1} flowing uniformly from west to east. The rescue boat sets the course needed to get to P as quickly as possible. Find

(a) the course the rescue boat sets,

(4)

(b) the time, to the nearest minute, to get to P.

(4)

When the rescue boat arrives at P, the yacht is just visible 4 km due north of P and is drifting with the current. Find

(c) the course that the rescue boat should set to get to the yacht as quickly as possible,

(1)

(d) the time taken by the rescue boat to reach the yacht from P.

(1)

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Question 4 continued

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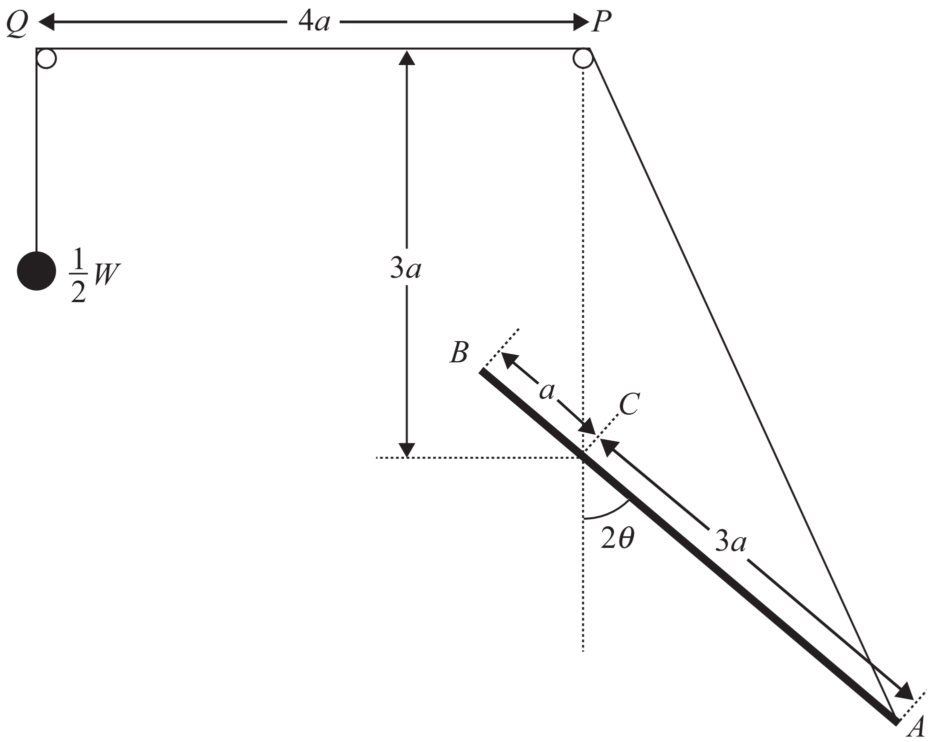

Figure 1

A uniform rod AB, of length $4a$ and weight W, is free to rotate in a vertical plane about a fixed smooth horizontal axis which passes through the point C of the rod, where $AC = 3a$. One end of a light inextensible string of length L, where $L > 10a$, is attached to the end A of the rod and passes over a small smooth fixed peg at P and another small smooth fixed peg at Q. The point Q lies in the same vertical plane as P, A and B. The point P is at a distance $3a$ vertically above C and PQ is horizontal with $PQ = 4a$. A particle of weight $\frac{1}{2}W$ is attached to the other end of the string and hangs vertically below Q. The rod is inclined at an angle 2θ to the vertical, where $-\pi < 2\theta < \pi$, as shown in Figure 1.

(a) Show that the potential energy of the system is

$$Wa(3\cos\theta - \cos 2\theta) + \text{constant}$$

(4)

(b) Find the positions of equilibrium and determine their stability.

(8)

P 4 0 1 1 0 A 0 2 0 2 8

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Question 5 continued

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6. Two points A and B are in a vertical line, with A above B and $AB = 4a$. One end of a light elastic spring, of natural length a and modulus of elasticity $3mg$, is attached to A. The other end of the spring is attached to a particle P of mass m. Another light elastic spring, of natural length a and modulus of elasticity mg, has one end attached to B and the other end attached to P. The particle P hangs at rest in equilibrium.

(a) Show that $AP = \frac{7a}{4}$

(3)

The particle P is now pulled down vertically from its equilibrium position towards B and at time $t = 0$ it is released from rest. At time t, the particle P is moving with speed v and has displacement x from its equilibrium position. The particle P is subject to air resistance of magnitude mkv, where k is a positive constant.

(b) Show that

$$\frac{d^2x}{dt^2} + k\frac{dx}{dt} + \frac{4g}{a}x = 0$$

(5)

(c) Find the range of values of k which would result in the motion of P being a damped oscillation.

(3)

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Question 6 continued

P 4 0 1 1 0 A 0 2 5 2 8

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Candidate No.						6 6 8 0 / 0 1 R	Signature	

Paper Reference(s)

6680/01R

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Tuesday 18 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

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Information for Candidates

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There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

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PEARSON

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1. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*.]

Boat A is moving with velocity $(3\mathbf{i} + 4\mathbf{j})\ \text{km h}^{-1}$ and boat B is moving with velocity $(6\mathbf{i} - 5\mathbf{j})\ \text{km h}^{-1}$. Find

(a) the magnitude of the velocity of A relative to B,

(3)

(b) the direction of the velocity of A relative to B, giving your answer as a bearing.

(2)

Leave blank

2.

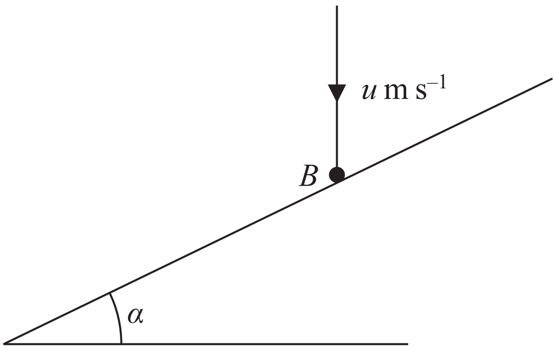

Figure 1

A smooth fixed plane is inclined at an angle α to the horizontal. A smooth ball B falls vertically and hits the plane. Immediately before the impact the speed of B is u m s^{-1}, as shown in Figure 1. Immediately after the impact the direction of motion of B is horizontal. The coefficient of restitution between B and the plane is $\frac{1}{3}$.

Find the size of angle α.

(6)

P 4 2 9 5 9 A 0 4 2 8

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Question 2 continued

(Total 6 marks)

Q2

P 4 2 9 5 9 A 0 5 2 8

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3. A smooth uniform sphere A, of mass $5m$ and radius r, is at rest on a smooth horizontal plane. A second smooth uniform sphere B, of mass $3m$ and radius r, is moving in a straight line on the plane with speed $u \text{ m s}^{-1}$ and strikes A. Immediately before the impact the direction of motion of B makes an angle of 60° with the line of centres of the spheres. The direction of motion of B is turned through an angle of 30° by the impact.

Find

(a) the speed of B immediately after the impact,

(3)

(b) the coefficient of restitution between the spheres.

(6)

Leave blank

Question 3 continued

Turn over

Leave blank

4. At 10 a.m. two walkers *A* and *B* are 4 km apart with *A* due north of *B*. Walker *A* is moving due east at a constant speed of 6 km h^{-1}. Walker *B* is moving with constant speed 5 km h^{-1} and walks in the straight line which allows him to pass as close as possible to *A*.

Find

(a) the direction of motion of *B*, giving your answer as a bearing,

(4)

(b) the least distance between *A* and *B*,

(2)

(c) the time when the distance between *A* and *B* is least.

(4)

Leave blank

Question 4 continued

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5. A van of mass 1200 kg travels along a straight horizontal road against a resistance to motion which is proportional to the speed of the van. The engine of the van is working at a constant rate of 40 kW. The van starts from rest at time $t = 0$. At time t seconds, the speed of the van is v m s^{-1}. When the speed of the van is 40 m s^{-1}, the acceleration of the van is 0.3 m s^{-2}.

(a) Show that

$$75v\frac{\mathrm{d}v}{\mathrm{d}t} = 2500 - v^2$$

(6)

(b) Find v in terms of t.

(6)

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Question 5 continued

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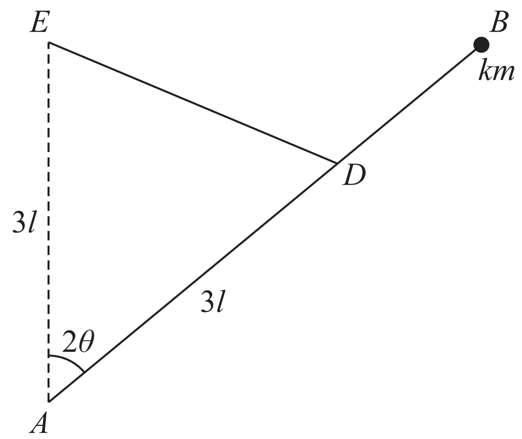

Figure 2

A uniform rod AB has mass $4m$ and length $4l$. The rod can turn freely in a vertical plane about a fixed smooth horizontal axis through A. A particle of mass km, where $k < 7$, is attached to the rod at B. One end of a light elastic string, of natural length l and modulus of elasticity $4mg$, is attached to the point D of the rod, where $AD = 3l$. The other end of the string is attached to a fixed point E which is vertically above A, where $AE = 3l$, as shown in Figure 2. The angle between the rod and the upward vertical is 2θ, where $\arcsin\left(\frac{1}{6}\right) < \theta \leqslant \frac{\pi}{2}$.

(a) Show that, while the string is stretched, the potential energy of the system is

$$8mgl\{(7-k)\sin^2\theta - 3\sin\theta\} + \text{constant}$$

(6)

There is a position of equilibrium with $\theta \leqslant \frac{\pi}{6}$.

(b) Show that $k \leqslant 4$

(5)

Given that $k = 4$,

(c) show that this position of equilibrium is stable.

(5)

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Question 6 continued

P 4 2 9 5 9 A 0 1 9 2 8

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7. A particle P of mass 0.5 kg is attached to the end A of a light elastic spring AB, of natural length 0.6 m and modulus of elasticity 2.7 N. At time $t = 0$ the end B of the spring is held at rest and P hangs at rest at the point C which is vertically below B. The end B is then moved along the line of the spring so that, at time t seconds, the downwards displacement of B from its initial position is $4\sin 2t$ metres. At time t seconds, the extension of the spring is x metres and the displacement of P below C is y metres.

(a) Show that

$$y + \frac{49}{45} = x + 4\sin 2t$$

(3)

(b) Hence show that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 9y = 36\sin 2t$$

(5)

Given that $y = \dfrac{36}{5}\sin 2t$ is a particular integral of this differential equation,

(c) find y in terms of t,

(5)

(d) find the speed of P when $t = \dfrac{1}{3}\pi$.

(4)

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Question 7 continued

P 4 2 9 5 9 A 0 2 5 2 8

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Candidate No.					

Paper Reference						
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Surname	Initial(s)
Signature	

Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Tuesday 18 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
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PEARSON

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1. A particle P of mass 0.5 kg falls vertically from rest. After t seconds it has speed v m s^{-1}. A resisting force of magnitude $1.5v$ newtons acts on P as it falls.

(a) Show that $3v = 9.8(1 - e^{-3t})$.

(8)

(b) Find the distance that P falls in the first two seconds of its motion.

(5)

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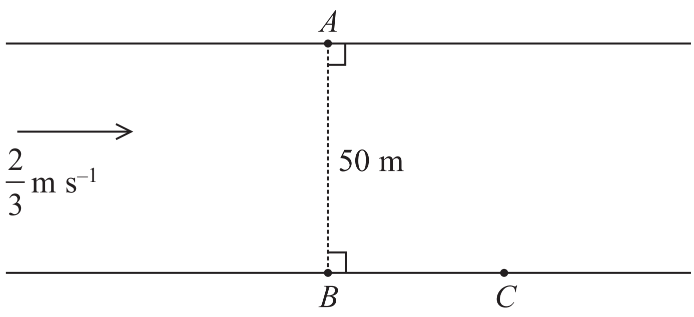

Figure 1

A river is 50 m wide and flows between two straight parallel banks. The river flows with a uniform speed of $\frac{2}{3}$ m s^{-1} parallel to the banks. The points A and B are on opposite banks of the river and AB is perpendicular to both banks of the river, as shown in Figure 1.

Keith and Ian decide to swim across the river. The speed relative to the water of both swimmers is $\frac{10}{9}$ m s^{-1}.

Keith sets out from A and crosses the river in the least possible time, reaching the opposite bank at the point C. Find

(a) the time taken by Keith to reach C,

(2)

(b) the distance BC.

(2)

Ian sets out from A and swims in a straight line so as to land on the opposite bank at B.

(c) Find the time taken by Ian to reach B.

(4)

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Question 2 continued

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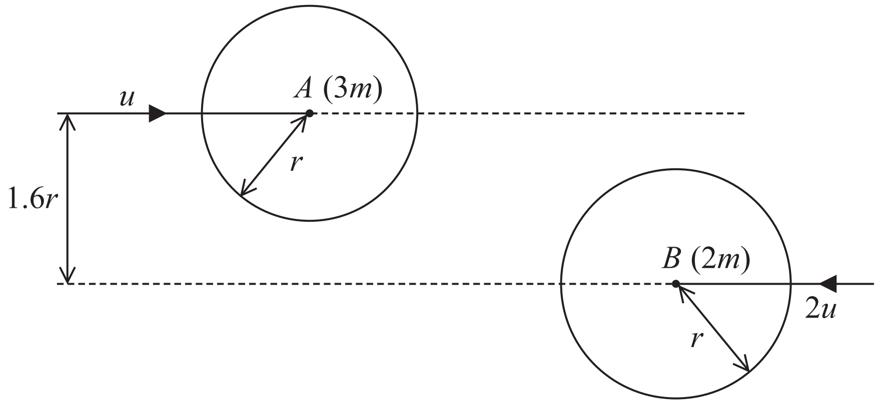

Figure 2

Two smooth uniform spheres A and B, of equal radius r, have masses $3m$ and $2m$ respectively. The spheres are moving on a smooth horizontal plane when they collide. Immediately before the collision they are moving with speeds u and $2u$ respectively. The centres of the spheres are moving towards each other along parallel paths at a distance $1.6r$ apart, as shown in Figure 2.

The coefficient of restitution between the two spheres is $\frac{1}{6}$.

Find, in terms of m and u, the magnitude of the impulse received by B in the collision.

(10)

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Question 3 continued

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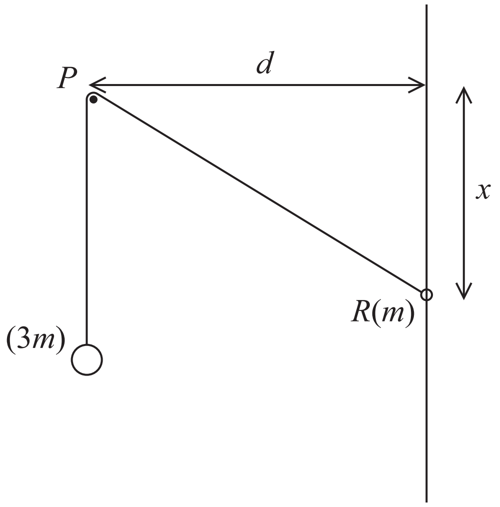

Figure 3

A small smooth peg P is fixed at a distance d from a fixed smooth vertical wire. A particle of mass $3m$ is attached to one end of a light inextensible string which passes over P. The particle hangs vertically below P. The other end of the string is attached to a small ring R of mass m, which is threaded on the wire, as shown in Figure 3.

(a) Show that when R is at a distance x below the level of P the potential energy of the system is

$$3mg\sqrt{(x^2 + d^2)} - mgx + \text{constant}$$

(4)

(b) Hence find x, in terms of d, when the system is in equilibrium.

(3)

(c) Determine the stability of the position of equilibrium.

(3)

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Question 4 continued

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5. A coastguard ship C is due south of a ship S. Ship S is moving at a constant speed of 12 km h^{-1} on a bearing of 140°. Ship C moves in a straight line with constant speed V km h^{-1} in order to intercept S.

(a) Find, giving your answer to 3 significant figures, the minimum possible value for V.

(3)

It is now given that $V = 14$

(b) Find the bearing of the course that C takes to intercept S.

(5)

Leave blank

Question 5 continued

P 4 1 8 1 9 A 0 1 7 2 8

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6. A particle P of mass m kg is attached to the end A of a light elastic string AB, of natural length a metres and modulus of elasticity $9ma$ newtons. Initially the particle and the string lie at rest on a smooth horizontal plane with $AB = a$ metres. At time $t = 0$ the end B of the string is set in motion and moves at a constant speed U m s^{-1} in the direction AB. The air resistance acting on P has magnitude $6mv$ newtons, where v m s^{-1} is the speed of P. At time t seconds, the extension of the string is x metres and the displacement of P from its initial position is y metres.

Show that, while the string is taut,

(a) $x + y = Ut$ **(2)**

(b) $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 6U$ **(5)**

You are given that the general solution of the differential equation in (b) is

$$x = (A + Bt)Ue^{-3t} + \frac{2U}{3}$$

where A and B are arbitrary constants.

(c) Find the value of A and the value of B. **(5)**

(d) Find the speed of P at time t seconds. **(2)**

Leave blank

Question 6 continued

Leave blank

7. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*]

A small smooth ball of mass m kg is moving on a smooth horizontal plane and strikes a fixed smooth vertical wall. The plane and the wall intersect in a straight line which is parallel to the vector $2\mathbf{i} + \mathbf{j}$. The velocity of the ball immediately before the impact is $b\mathbf{i}$ m s^{-1}, where b is positive. The velocity of the ball immediately after the impact is $a(\mathbf{i} + \mathbf{j})$ m s^{-1}, where a is positive.

(a) Show that the impulse received by the ball when it strikes the wall is parallel to $(-\mathbf{i} + 2\mathbf{j})$.

(1)

Find

(b) the coefficient of restitution between the ball and the wall,

(8)

(c) the fraction of the kinetic energy of the ball that is lost due to the impact.

(3)

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Question 7 continued