Paper Reference(s)
6680

Edexcel GCE

Mechanics M4
(New Syllabus)
Advanced/Advanced Subsidiary

Friday 25 January 2002 – Morning
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions. Pages 6, 7 and 8 are blank.

Advice to Candidates

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You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N6989 83019/R6678 3/3/3/3/3/8100

1. A river of width 40 m flows with uniform and constant speed between straight banks. A swimmer crosses as quickly as possible and takes 30 s to reach the other side. She is carried 25 m downstream.

Find

(*a*) the speed of the river, **(2)**

(*b*) the speed of the swimmer relative to the water. **(2)**

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2. A ball of mass m is thrown vertically upwards from the ground with an initial speed u. When the speed of the ball is v, the magnitude of the air resistance is mkv, where k is a positive constant.

By modelling the ball as a particle, find, in terms of u, k and g, the time taken for the ball to reach its greatest height. **(8)**

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3. A smooth uniform sphere P of mass m is falling vertically and strikes a fixed smooth inclined plane with speed u. The plane is inclined at an angle θ, $\theta < 45°$, to the horizontal. The coefficient of restitution between P and the inclined plane is e. Immediately after P strikes the plane, P moves horizontally.

(*a*) Show that $e = \tan^2 \theta$. **(6)**

(*b*) Show that the magnitude of the impulse exerted by P on the plane is $mu \sec \theta$. **(4)**

---

4. A pilot flying an aircraft at a constant speed of 2000 kmh^{-1} detects an enemy aircraft 100 km away on a bearing of 045°. The enemy aircraft is flying at a constant velocity of 1500 kmh^{-1} due west. Find

(i) the course, as a bearing to the nearest degree, that the pilot should set up in order to intercept the enemy aircraft,

(ii) the time, to the nearest s, that the pilot will take to reach the enemy aircraft. **(11)**

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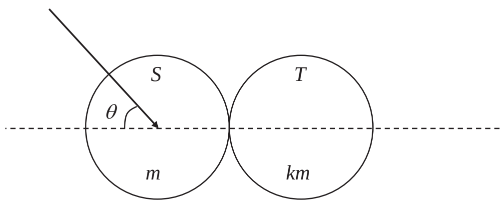
5. **Figure 1**



A smooth uniform sphere S of mass m is moving on a smooth horizontal table. The sphere S collides with another smooth uniform sphere T, of the same radius as S but of mass km, $k > 1$, which is at rest on the table. The coefficient of restitution between the spheres is e. Immediately before the spheres collide the direction of motion of S makes an angle θ with the line joing their centres, as shown in Fig. 1.

Immediately after the collision the directions of motion of S and T are perpendicular.

(*a*) Show that $e = \frac{1}{k}$.

(6)

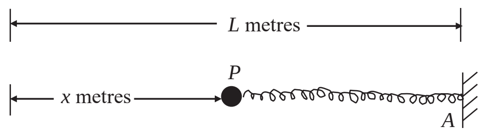
Given that $k = 2$ and that the kinetic energy lost in the collision is one quarter of the initial kinetic energy,

(*b*) find the value of θ.

(6)

TURN OVER FOR QUESTION 6

6. **Figure 2**



In a simple model of a shock absorber, a particle P of mass m kg is attached to one end of a light elastic horizontal spring. The other end of the spring is fixed at A and the motion of P takes place along a fixed horizontal line through A. The spring has natural length L metres and modulus of elasticity $2mL$ newtons. The whole system is immersed in a fluid which exerts a resistance on P of magnitude $3mv$ newtons, where v m s^{-1} is the speed of P at time t seconds. The compression of the spring at time t seconds is x metres, as shown in Fig. 2.

(*a*) Show that

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 0.$$

(4)

Given that when $t = 0$, $x = 2$ and $\frac{dx}{dt} = -4$,

(*b*) find x in terms of t.

(8)

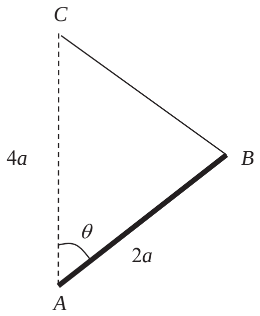
(*c*) Sketch the graph of x against t.

(2)

(*d*) State, with a reason, whether the model is realistic.

(1)

7. **Figure 3**



A uniform rod *AB*, of mass *m* and length $2a$, can rotate freely in a vertical plane about a fixed smooth horizontal axis through *A*. The fixed point *C* is vertically above *A* and $AC = 4a$. A light elastic string, of natural length $2a$ and modulus of elasticity $\frac{1}{2}mg$, joins *B* to *C*. The rod *AB* makes an angle θ with the upward vertical at *A*, as shown in Fig. 3.

(*a*) Show that the potential energy of the system is

$$-mga[\cos\theta + \sqrt{(5 - 4\cos\theta)}] + \text{constant}.$$

(9)

(b) Hence determine the values of θ for which the system is in equilibrium.

(6)

END

Paper Reference(s)
6680

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Monday 24 June 2002 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

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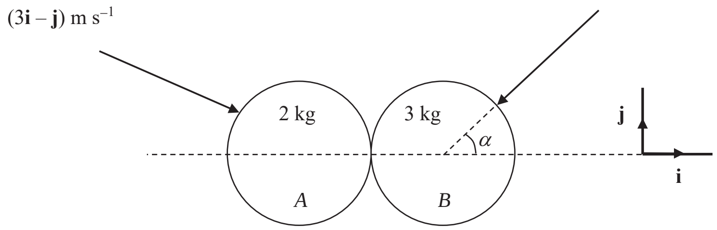
1. **Figure 1**



Two smooth uniform spheres *A* and *B*, of equal radius, are moving on a smooth horizontal plane. Sphere *A* has mass 2 kg and sphere *B* has mass 3 kg. The spheres collide and at the instant of collision the line joining their centres is parallel to **i**. Before the collision *A* has velocity (3**i** – **j**) m s^{-1} and after the collision it has velocity (–2**i** – **j**) m s^{-1}. Before the collision the velocity of *B* makes an angle α with the line of centres, as shown in Fig. 1, where $\tan \alpha = 2$. The coefficient of restitution between the spheres is $\frac{1}{2}$.

Find, in terms of **i** and **j**, the velocity of *B* before the collision.

(9)

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2. Ship *A* is steaming on a bearing of 060° at 30 km h^{-1} and at 9 a.m. it is 20 km due west of a second ship *B*. Ship *B* steams in a straight line.

(*a*) Find the least speed of *B* if it is to intercept *A*.

(3)

Given that the speed of *B* is 24 km h^{-1},

(*b*) find the earliest time at which it can intercept *A*.

(7)

---

3. The engine of a car of mass 800 kg works at a constant rate of 32 kW. The car travels along a straight horizontal road and the resistance to motion of the car is proportional to the speed of the car. The car starts from rest and *t* seconds later it has a speed of *v* m s^{-1}.

(*a*) Show that

$$800v\frac{\mathrm{d}v}{\mathrm{d}t} = 32\,000 - kv^2, \text{ where } k \text{ is a positive constant.}$$

(3)

Given that the limiting speed of the car is 40 m s^{-1}, find

(*b*) the value of *k*,

(2)

(*c*) *v* in terms of *t*.

(7)

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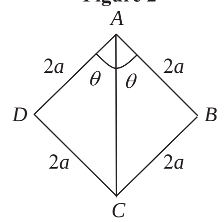
4. **Figure 2**



Four identical uniform rods, each of mass *m* and length 2*a*, are freely jointed to form a rhombus *ABCD*. The rhombus is suspended from *A* and is prevented from collapsing by an elastic string which joins *A* to *C*, with $\angle BAD = 2\theta$, $0 \le \theta \le \frac{1}{3}\pi$, as shown in Fig. 2. The natural length of the elastic string is 2*a* and its modulus of elasticity is 4*mg*.

(*a*) Show that the potential energy, *V*, of the system is given by

$$V = 4mga[(2\cos\theta - 1)^2 - 2\cos\theta] + \text{constant}.$$

(5)

(*b*) Hence find the non-zero value of θ for which the system is in equilibrium.

(4)

(*c*) Determine whether this position of equilibrium is stable or unstable.

(4)

---

5. At time $t = 0$ particles P and Q start simultaneously from points which have position vectors $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ m and $(-\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ m respectively, relative to a fixed origin O. The velocities of P and Q are $(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ m s^{-1} and $(2\mathbf{i} + \mathbf{k})$ m s^{-1} respectively.

(*a*) Show that P and Q collide and find the position vector of the point at which they collide.

(5)

A third particle R moves in such a way that its velocity relative to P is parallel to the vector $(-5\mathbf{i} + 4\mathbf{j} - \mathbf{k})$ and its velocity relative to Q is parallel to the vector $(-2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$.

Given that all three particles collide simultaneously, find

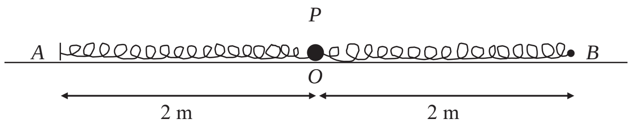
(*b*) (i) the velocity of R,

(ii) the position vector of R at time $t = 0$.

(9)

6.

Figure 3



A particle P of mass 2 kg is attached to the mid-point of a light elastic spring of natural length 2 m and modulus of elasticity 4 N. One end A of the elastic spring is attached to a fixed point on a smooth horizontal table. The spring is then stretched until its length is 4 m and its other end B is held at a point on the table where $AB = 4$ m. At time $t = 0$, P is at rest on the table at the point O where $AO = 2$ m, as shown in Fig. 3. The end B is now moved on the table in such a way that AOB remains a straight line. At time t seconds, $AB = (4 + \frac{1}{2}\sin 4t)$ m and $AP = (2 + x)$ m.

(*a*) Show that

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 4x = \sin 4t.$$

(5)

(*b*) Hence find the time when P first comes to instantaneous rest.

(12)

Paper Reference(s)

6680

Edexcel GCE

Mechanics M4
Advanced/Advanced Subsidiary

Wednesday 22 January 2003 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

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N10808A

1. A boy enters a large horizontal field and sees a friend 100 m due north. The friend is walking in an easterly direction at a constant speed of 0.75 m s^{-1}. The boy can walk at a maximum speed of 1 m s^{-1}.

Find the shortest time for the boy to intercept his friend and the bearing on which he must travel to achieve this.

(6)

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2. Boat *A* is sailing due east at a constant speed of 10 km h^{-1}. To an observer on *A*, the wind appears to be blowing from due south. A second boat *B* is sailing due north at a constant speed of 14 km h^{-1}. To an observer on *B*, the wind appears to be blowing from the south west. The velocity of the wind relative to the earth is constant and is the same for both boats.

Find the velocity of the wind relative to the earth, stating its magnitude and direction.

(7)

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3. A small pebble of mass *m* is placed in a viscous liquid and sinks vertically from rest through the liquid. When the speed of the pebble is *v* the magnitude of the resistance due to the liquid is modelled as mkv^2, where *k* is a positive constant.

Find the speed of the pebble after it has fallen a distance *D* through the liquid.

(11)

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4. **Figure 1**

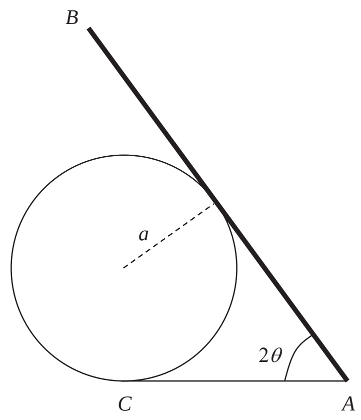

Figure 1 shows a uniform rod *AB*, of mass *m* and length 4*a*, resting on a smooth fixed sphere of radius *a*. A light elastic string, of natural length *a* and modulus of elasticity $\frac{3}{4}mg$, has one end attached to the lowest point *C* of the sphere and the other end attached to *A*. The points *A*, *B* and *C* lie in a vertical plane with $\angle BAC = 2\theta$, where $\theta < \frac{\pi}{4}$.

Given that *AC* is always horizontal,

(*a*) show that the potential energy of the system is

$$\frac{mga}{8}(16\sin 2\theta + 3\cot^2\theta - 6\cot\theta) + \text{constant},$$

(7)

(b) show that there is a value of θ for which the system is in equilibrium such that $0.535 < \theta < 0.545$.

(6)

(c) Determine whether this position of equilibrium is stable or unstable.

(3)

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5. A particle P moves in a straight line. At time t seconds its displacement from a fixed point O on the line is x metres. The motion of P is modelled by the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + 2x = 12\cos 2t - 6\sin 2t.$$

When $t = 0$, P is at rest at O.

(*a*) Find, in terms of t, the displacement of P from O.

(11)

(*b*) Show that P comes to instantaneous rest when $t = \frac{\pi}{4}$.

(2)

(*c*) Find, in metres to 3 significant figures, the displacement of P from O when $t = \frac{\pi}{4}$.

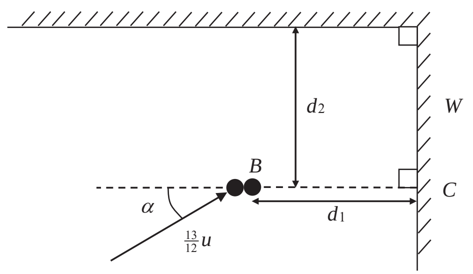
(2)

(*d*) Find the approximate period of the motion for large values of t.

(2)

6.

Figure 2



A small ball Q of mass $2m$ is at rest at the point B on a smooth horizontal plane. A second small ball P of mass m is moving on the plane with speed $\frac{13}{12}u$ and collides with Q. Both the balls are smooth, uniform and of the same radius. The point C is on a smooth vertical wall W which is at a distance d_1 from B, and BC is perpendicular to W. A second smooth vertical wall is perpendicular to W and at a distance d_2 from B. Immediately before the collision occurs, the direction of motion of P makes an angle α with BC, as shown in Fig. 2, where $\tan\alpha = \frac{5}{12}$. The line of centres of P and Q is parallel to BC. After the collision Q moves towards C with speed $\frac{3}{5}u$.

(*a*) Show that, after the collision, the velocity components of P parallel and perpendicular to CB are $\frac{1}{5}u$ and $\frac{5}{12}u$ respectively.

(4)

(*b*) Find the coefficient of restitution between P and Q.

(2)

(*c*) Show that when Q reaches C, P is at a distance $\frac{4}{3}d_1$ from W.

(3)

For each collision between a ball and a wall the coefficient of restitution is $\frac{1}{2}$.

Given that the balls collide with each other again,

(*d*) show that the time between the two collisions of the balls is $\frac{15d_1}{u}$,

(4)

(*e*) find the ratio $d_1 : d_2$.

(5)

Paper Reference(s)

6680

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Wednesday 25 June 2003 – Morning

Time: 1 hour 30 minutes

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Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

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1. A wooden ball of mass 0.01 kg falls vertically into a pond of water. The speed of the ball as it enters the water is 10 m s^{-1}. When the ball is x metres below the surface of the water and moving downwards with speed v m s^{-1}, the water provides a resistance of magnitude $0.02v^2$ N and an upward buoyancy force of magnitude 0.158 N.

(*a*) Show that, while the ball is moving downwards,

$$-2v^2 - 6 = v\frac{dv}{dx}.$$

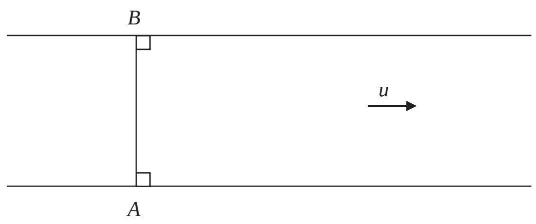
(3)

(*b*) Hence find, to 3 significant figures, the greatest distance below the surface of the water reached by the ball.

(5)

2.

Figure 1



A man, who rows at a speed v through still water, rows across a river which flows at a speed u. The man sets off from the point A on one bank and wishes to land at the point B on the opposite bank, where AB is perpendicular to both banks, as shown in Fig. 1.

(*a*) Show that, for this to be possible, $v > u$.

(3)

Given that $v < u$ and that he rows from A so as to reach a point C, on the opposite bank, which is as close to B as possible,

(*b*) find, in terms of u and v, the ratio of BC to the width of the river.

(5)

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3. A man walks due north at a constant speed u and the wind seems to him to be blowing *from* the direction 30° east of north. On his return journey, when he is walking at the same speed u due south, the wind seems to him to be blowing *from* the direction 30° south of east. Assuming that the velocity, **w**, of the wind relative to the earth is constant, find

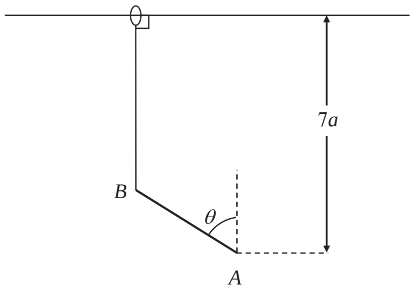
(i) the magnitude of **w**, in terms of u,

(ii) the direction of **w**.

(9)

4.

Figure 2



A uniform rod AB, of length $2a$ and mass $8m$, is free to rotate in a vertical plane about a fixed smooth horizontal axis through A. One end of a light elastic string, of natural length a and modulus of elasticity $\frac{4}{5}mg$, is fixed to B. The other end of the string is attached to a small ring which is free to slide on a smooth straight horizontal wire which is fixed in the same vertical plane as AB at a height $7a$ vertically above A. The rod AB makes an angle θ with the upward vertical at A, as shown in Fig. 2.

(*a*) Show that the potential energy V of the system is given by

$$V = \tfrac{8}{5}mga(\cos^2\theta - \cos\theta) + \text{constant}.$$

(6)

(*b*) Hence find the values of θ, $0 \le \theta \le \pi$, for which the system is in equilibrium.

(5)

(*c*) Determine the nature of these positions of equilibrium.

(4)

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5. A light elastic string, of natural length $2a$ and modulus of elasticity mg, has a particle P of mass m attached to its mid-point. One end of the string is attached to a fixed point A and the other end is attached to a fixed point B which is at a distance $4a$ vertically below A.

(*a*) Show that P hangs in equilibrium at the point E where $AE = \frac{5}{2}a$.

(5)

The particle P is held at a distance $3a$ vertically below A and is released from rest at time $t = 0$. When the speed of the particle is v, there is a resistance to motion of magnitude $2mkv$, where $k = \sqrt{\left(\frac{g}{a}\right)}$. At time t the particle is at a distance $(\frac{5}{2}a + x)$ from A.

(*b*) Show that

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + 2k^2x = 0.$$

(5)

(*c*) Hence find x in terms of t.

(7)

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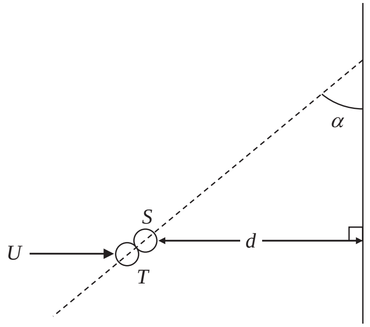
6. **Figure 3**



A small smooth uniform sphere S is at rest on a smooth horizontal floor at a distance d from a straight vertical wall. An identical sphere T is projected along the floor with speed U towards S and in a direction which is perpendicular to the wall. At the instant when T strikes S the line joining their centres makes an angle α with the wall, as shown in Fig. 3.

Each sphere is modelled as having negligible diameter in comparison with d. The coefficient of restitution between the spheres is e.

(*a*) Show that the components of the velocity of T after the impact, parallel and perpendicular to the line of centres, are $\frac{1}{2}U(1-e)\sin\alpha$ and $U\cos\alpha$ respectively.
(7)

(*b*) Show that the components of the velocity of T after the impact, parallel and perpendicular to the wall, are $\frac{1}{2}U(1+e)\cos\alpha\sin\alpha$ and $\frac{1}{2}U[2-(1+e)\sin^2\alpha]$ respectively.
(6)

The spheres S and T strike the wall at the points A and B respectively.

Given that $e = \frac{2}{3}$ and $\tan\alpha = \frac{3}{4}$,

(*c*) find, in terms of d, the distance AB.
(5)

END

Paper Reference(s)

6680
Edexcel GCE
Mechanics M4
Advanced/Advanced Subsidiary
Wednesday 21 January 2004 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination
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Graph Paper (ASG2)

Items included with question papers
Nil

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N13900A

1. A particle P of mass 3 kg moves in a straight line on a smooth horizontal plane. When the speed of P is v m s^{-1}, the resultant force acting on P is a resistance to motion of magnitude $2v$ N. Find the distance moved by P while slowing down from 5 m s^{-1} to 2 m s^{-1}.

(5)

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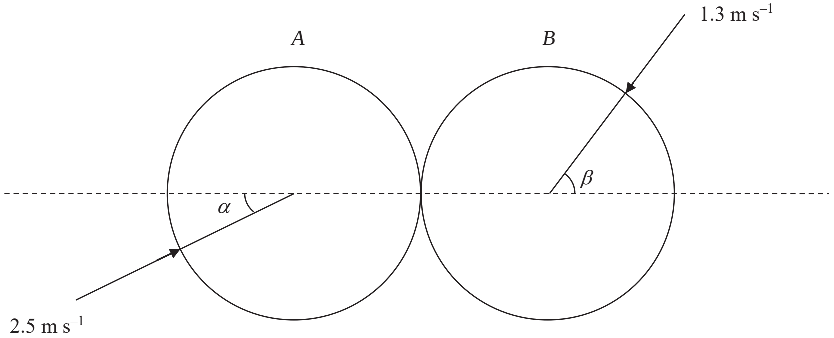
2. **Figure 1**



Two smooth uniform spheres A and B of equal radius have masses 2 kg and 1 kg respectively. They are moving on a smooth horizontal plane when they collide. Immediately before the collision the speed of A is 2.5 m s^{-1} and the speed of B is 1.3 m s^{-1}. When they collide the line joining their centres makes an angle α with the direction of motion of A and an angle β with the direction of motion of B, where $\tan \alpha = \frac{4}{3}$ and $\tan \beta = \frac{12}{5}$ as shown in Fig. 1.

(*a*) Find the components of the velocities of A and B perpendicular and parallel to the line of centres immediately before the collision.

(4)

The coefficient of restitution between A and B is $\frac{1}{2}$.

(*b*) Find, to one decimal place, the speed of each sphere after the collision.

(9)

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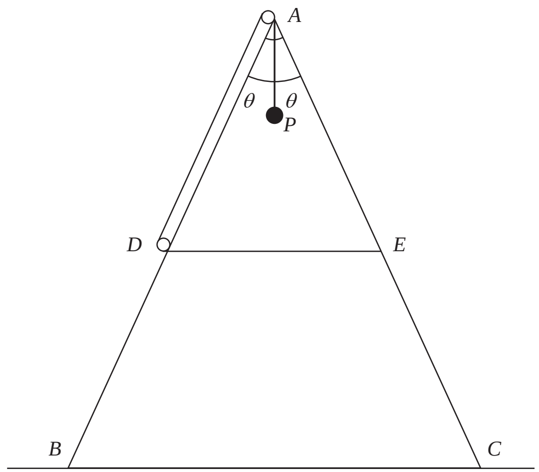
3. **Figure 2**



Two uniform rods AB and AC, each of mass $2m$ and length $2L$, are freely jointed at A. The mid-points of the rods are D and E respectively. A light inextensible string of length s is fixed to E and passes round small, smooth light pulleys at D and A. A particle P of mass m is attached to the other end of the string and hangs vertically. The points A, B and C lie in the same vertical plane with B and C on a smooth horizontal surface. The angles PAB and PAC are each equal to θ ($\theta > 0$), as shown in Fig. 2.

(*a*) Find the length of AP in terms of s, L and θ.

(2)

(*b*) Show that the potential energy V of the system is given by

$$V = 2mgL\,(3\cos\theta + \sin\theta) + \text{constant}.$$

(4)

(*c*) Hence find the value of θ for which the system is in equilibrium.

(4)

(*d*) Determine whether this position of equilibrium is stable or unstable.

(4)

---

4. A particle P of mass m is attached to the mid-point of a light elastic string, of natural length $2L$ and modulus of elasticity $2mk^2L$, where k is a positive constant. The ends of the string are attached to points A and B on a smooth horizontal surface, where $AB = 3L$. The particle is released from rest at the point C, where $AC = 2L$ and ACB is a straight line. During the subsequent motion P experiences air resistance of magnitude $2mkv$, where v is the speed of P. At time t, $AP = 1.5L + x$.

(*a*) Show that $\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + 4k^2x = 0.$

(6)

(*b*) Find an expression, in terms of t, k and L, for the distance AP at time t.

(8)

---

5. **Figure 3**

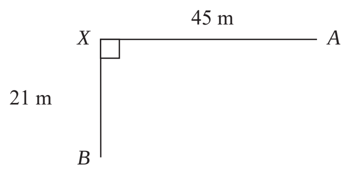


Figure 3 represents the scene of a road accident. A car of mass 600 kg collided at the point X with a stationary van of mass 800 kg. After the collision the van came to rest at the point A having travelled a horizontal distance of 45 m, and the car came to rest at the point B having travelled a horizontal distance of 21 m. The angle AXB is 90°.

The accident investigators are trying to establish the speed of the car before the collision and they model both vehicles as small spheres.

(*a*) Find the coefficient of restitution between the car and the van.

(5)

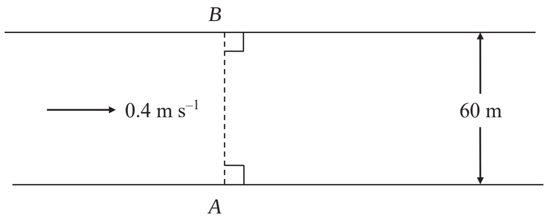
The investigators assume that after the collision, and until the vehicles came to rest, the van was subject to a constant horizontal force of 500 N acting along AX and the car to a constant horizontal force of 300 N along BX.

(*b*) Find the speed of the car immediately before the collision.

(9)

---

6. **Figure 4**



Mary swims in still water at 0.85 m s^{-1}. She swims across a straight river which is 60 m wide and flowing at 0.4 m s^{-1}. She sets off from a point A on the near bank and lands at a point B, which is directly opposite A on the far bank, as shown in Fig. 4.

Find

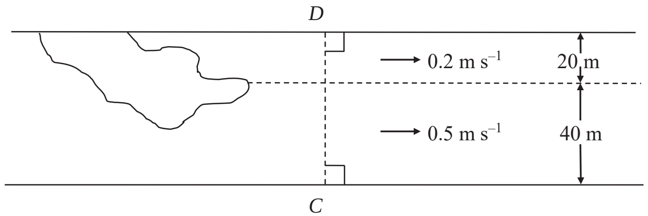
(*a*) the angle between the near bank and the direction in which Mary swims,

(3)

(*b*) the time she takes to cross the river.

(3)

Figure 5



A little further downstream a large tree has fallen from the far bank into the river. The river is modelled as flowing at 0.5 m s^{-1} for a width of 40 m from the near bank, and 0.2 m s^{-1} for the 20 m beyond this. Nassim swims at 0.85 m s^{-1} in still water. He swims across the river from a point C on the near bank. The point D on the far bank is directly opposite C, as shown in Fig. 5. Nassim swims at the same angle to the near bank as Mary.

(*c*) Find the maximum distance, downstream from CD, of Nassim during the crossing.

(5)

(*d*) Show that he will land at the point D.

(4)

Paper Reference(s)
6680

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Thursday 1 July 2004 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has six questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*.]

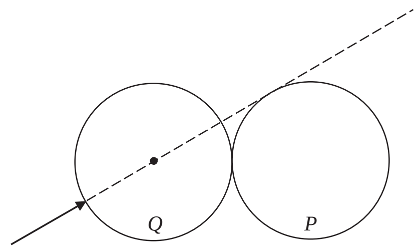
An aeroplane makes a journey from a point P to a point Q which is due east of P. The wind velocity is $w(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$, where w is a positive constant. The velocity of the aeroplane relative to the wind is $v(\cos\phi\,\mathbf{i} - \sin\phi\,\mathbf{j})$, where v is a constant and $v > w$. Given that θ and ϕ are both acute angles,

(*a*) show that $v \sin\phi = w \sin\theta$,
(2)

(*b*) find, in terms of v, w and θ, the speed of the aeroplane relative to the ground.
(4)

2.

Figure 1



A smooth uniform sphere P is at rest on a smooth horizontal plane, when it is struck by an identical sphere Q moving on the plane. Immediately before the impact, the line of motion of the centre of Q is tangential to the sphere P, as shown in Fig. 1. The direction of motion of Q is turned through 30° by the impact.

Find the coefficient of restitution between the spheres.
(11)

3. At noon, two boats A and B are 6 km apart with A due east of B. Boat B is moving due north at a constant speed of 13 km h^{-1}. Boat A is moving with constant speed 12 km h^{-1} and sets a course so as to pass as close as possible to boat B. Find

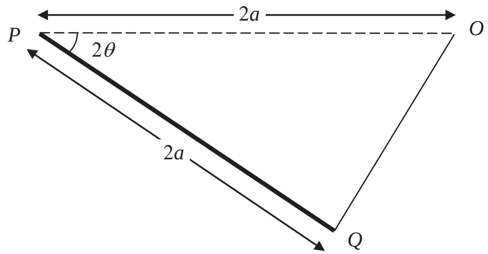
(*a*) the direction of motion of A, giving your answer as a bearing,
(4)

(*b*) the time when the boats are closest,
(5)

(*c*) the shortest distance between the boats.
(2)

4.

Figure 2



A uniform rod PQ, of length $2a$ and mass m, is free to rotate in a vertical plane about a fixed smooth horizontal axis through the end P. The end Q is attached to one end of a light elastic string, of natural length a and modulus of elasticity $\frac{mg}{2\sqrt{3}}$. The other end of the string is attached to a fixed point O, where OP is horizontal and $OP = 2a$, as shown in Fig. 2. $\angle OPQ$ is denoted by 2θ.

(*a*) Show that, when the string is taut, the potential energy of the system is

$$-\frac{mga}{\sqrt{3}}(2\cos 2\theta + \sqrt{3}\sin 2\theta + 2\sin\theta) + \text{constant.}$$

(7)

(*b*) Verify that there is a position of equilibrium at $\theta = \frac{\pi}{6}$.

(4)

(*c*) Determine whether this is a position of stable equilibrium.

(4)

5. A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $2mak^2$, where k is a positive constant. The other end of the string is attached to a fixed point A. At time $t = 0$, P is released from rest from a point which is a distance $2a$ vertically below A. When P is moving with speed v, the air resistance has magnitude $2mkv$. At time t, the extension of the string is x.

(*a*) Show that, while the string is taut,

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + 2k^2x = g.$$

(4)

You are given that the general solution of this differential equation is

$$x = e^{-kt}(C\sin kt + D\cos kt) + \frac{g}{2k^2}, \quad \text{where } C \text{ and } D \text{ are constants.}$$

(*b*) Find the value of C and the value of D.

(5)

Assuming that the string remains taut,

(*c*) find the value of t when P first comes to rest,

(3)

(*d*) show that $2k^2a < g(1 + e^{\pi})$.

(4)

6. A particle P of mass m is attached to one end of a light inextensible string and hangs at rest at time $t = 0$. The other end of the string is then raised vertically by an engine which is working at a constant rate kmg, where $k > 0$. At time t, the distance of P above its initial position is x, and P is moving upwards with speed v.

(*a*) Show that $\quad v^2\frac{dv}{dx} = (k - v)g.$

(4)

(*b*) Show that $\quad gx = k^2\ln\left(\frac{k}{k-v}\right) - kv - \frac{1}{2}v^2.$

(7)

(*c*) Hence, or otherwise, find t in terms of k, v and g.

(5)

END

Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Thursday 30 June 2005 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Lilac or Green)	Nil
Answer Book (AB16)	
Graph paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take g = 9.8 m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and parts of questions are shown in round brackets: e.g. **(2)**.
This paper has 7 questions. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A small smooth ball of mass $\frac{1}{2}$ kg is falling vertically. The ball strikes a smooth plane which is inclined at an angle α to the horizontal, where tan $\alpha = \frac{3}{4}$. Immediately before striking the plane the ball has speed 10 m s^{-1}. The coefficient of restitution between ball and plane is $\frac{1}{2}$. Find

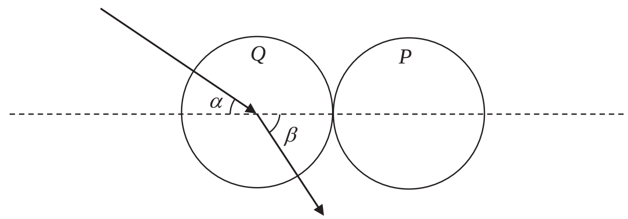
(*a*) the speed, to 3 significant figures, of the ball immediately after the impact,
(5)

(*b*) the magnitude of the impulse received by the ball as it strikes the plane.
(2)

2. A cyclist P is cycling due north at a constant speed of 20 km h^{-1}. At 12 noon another cyclist Q is due west of P. The speed of Q is constant at 10 km h^{-1}. Find the course which Q should set in order to pass as close to P as possible, giving your answer as a bearing.
(5)

3.

Figure 1



A smooth sphere P lies at rest on a smooth horizontal plane. A second identical sphere Q, moving on the plane, collides with the sphere P. Immediately before the collision the direction of motion of Q makes an angle α with the line joining the centres of the spheres. Immediately after the collision the direction of motion of Q makes an angle β with the line joining the centres of spheres, as shown in Figure 1. The coefficient of restitution between the spheres is e.

Show that $(1 - e)$ tan $\beta = 2$ tan α.
(11)

4. A lorry of mass M is moving along a straight horizontal road. The engine produces a constant driving force of magnitude F. The total resistance to motion is modelled as having magnitude kv^2, where k is a constant, and v is the speed of the lorry.

Given the lorry moves with constant speed V,

(*a*) show that $V = \sqrt{\dfrac{F}{k}}$.

(2)

Given instead that the lorry starts from rest,

(*b*) show that the distance travelled by the lorry in attaining a speed of $\frac{1}{2}V$ is

$$\frac{M}{2k}\ln\left(\frac{4}{3}\right).$$

(9)

---

5. A non-uniform rod BC has mass m and length $3l$. The centre of mass of the rod is at distance l from B. The rod can turn freely about a fixed smooth horizontal axis through B. One end of a light elastic string, of natural length l and modulus of elasticity $\dfrac{mg}{6}$, is attached to C. The other end of the string is attached to a point P which is at a height $3l$ vertically above B.

(*a*) Show that, while the string is stretched, the potential energy of the system is

$$mgl(\cos^2\theta - \cos\theta) + \text{constant},$$

where θ is the angle between the string and the downward vertical and $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$.

(6)

(*b*) Find the values of θ for which the system is in equilibrium with the string stretched.

(6)

---

6. A ship A has maximum speed 30 km h^{-1}. At time $t = 0$, A is 70 km due west of B which is moving at a constant speed of 36 km h^{-1} on a bearing of 300°. Ship A moves on a straight course at a constant speed and intercepts B. The course of A makes an angle θ with due north.

(*a*) Show that $-\arctan\frac{4}{3} \leq \theta \leq \arctan\frac{4}{3}$.

(7)

(*b*) Find the least time for A to intercept B.

(5)

---

7. A light elastic string, of natural length a and modulus of elasticity $5ma\omega^2$, lies unstretched along a straight line on a smooth horizontal plane. A particle of mass m is attached to one end of the spring. At time $t = 0$, the other end of the spring starts to move with constant speed U along the line of the spring and away from the particle. As the particle moves along the plane it is subject to a resistance of magnitude $2m\omega v$, where v is its speed. At time t, the extension of the spring is x and the displacement of the particle from its initial position is y. Show that

(*a*) $x + y = Ut$,

(2)

(*b*) $\dfrac{d^2x}{dt^2} + 2\omega\dfrac{dx}{dt} + 5\omega^2 x = 2\omega U$.

(7)

(*c*) Find x in terms of ω, U and t.

(8)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced Subsidiary

Wednesday 18 January 2006 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Lilac or Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

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Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 6 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N22334A

1. A particle P of mass 0.5 kg is released from rest at time $t = 0$ and falls vertically through a liquid. The motion of P is resisted by a force of magnitude $2v$ N, where v m s^{-1} is the speed of v at time t seconds.

(*a*) Show that $5\dfrac{\mathrm{d}v}{\mathrm{d}t} = 49 - 20v$.

(2)

(*b*) Find the speed of P when $t = 1$.

(5)

---

2. A small smooth sphere S of mass m is attached to one end of a light inextensible string of length $2a$. The other end of the string is attached to a fixed point A which is at a distance $a\sqrt{3}$ from a smooth vertical wall. The sphere S hangs at rest in equilibrium. It is then projected horizontally towards the wall with a speed $\sqrt{\left(\dfrac{37ga}{5}\right)}$.

(*a*) Show that S strikes the wall with speed $\sqrt{\left(\dfrac{27ga}{5}\right)}$.

(4)

Given that the loss in kinetic energy due to the impact with the wall is $\dfrac{3mga}{5}$,

(*b*) find the coefficient of restitution between S and the wall.

(7)

---

3. Two ships P and Q are moving with constant velocity. At 3 p.m., P is 20 km due north of Q and is moving at 16 km h^{-1} due west. To an observer on ship P, ship Q appears to be moving on a bearing of 030° at 10 km h^{-1}. Find

(*a*) (i) the speed of Q,

(ii) the direction in which Q is moving, giving your answer as a bearing to the nearest degree,

(6)

(*b*) the shortest distance between the ships,

(3)

(*c*) the time at which the two ships are closest together.

(3)

---

4. A particle P of mass m is suspended from a fixed point by a light elastic spring. The spring has natural length a and modulus of elasticity $2m\omega^2 a$, where ω is a positive constant. At time $t = 0$ the particle is projected vertically downwards with speed U from its equilibrium position. The motion of the particle is resisted by a force of magnitude $2m\omega v$, where v is the speed of the particle. At time t, the displacement of P downwards from its equilibrium position is x.

(*a*) Show that $\frac{d^2x}{dt^2} + 2\omega\frac{dx}{dt} + 2\omega^2 x = 0.$

(5)

Given that the solution of this differential equation is $x = e^{-\omega t}(A\cos\omega t + B\sin\omega t)$, where A and B are constants,

(*b*) find A and B.

(4)

(*c*) Find an expression for the time at which P first comes to rest.

(3)

---

5. Two smooth uniform spheres A and B have equal radii. Sphere A has mass m and sphere B has mass km. The spheres are at rest on a smooth horizontal table. Sphere A is then projected along the table with speed u and collides with B. Immediately before the collision, the direction of motion of A makes an angle of 60° with the line joining the centres of the two spheres. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(*a*) Show that the speed of B immediately after the collision is $\frac{3u}{4(k+1)}$.

(6)

Immediately after the collision the direction of motion of A makes an angle arctan $(2\sqrt{3})$ with the direction of motion of B.

(*b*) Show that $k = \frac{1}{2}$.

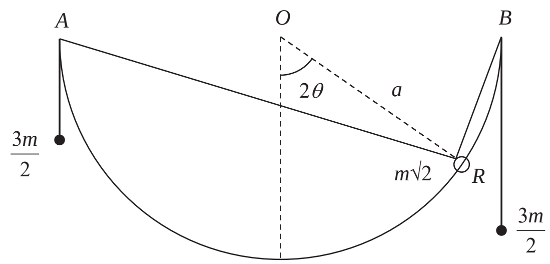
(6)

(*c*) Find the loss of kinetic energy due to the collision.

(4)

6.

Figure 1



A smooth wire with ends A and B is in the shape of a semi-circle of radius a. The mid-point of AB is O. The wire is fixed in a vertical plane and hangs below AB which is horizontal. A small ring R, of mass $m\sqrt{2}$, is threaded on the wire and is attached to two light inextensible strings. The other end of each string is attached to a particle of mass $\frac{3m}{2}$. The particles hang vertically under gravity, as shown in Figure 1.

(*a*) Show that, when the radius OR makes an angle 2θ with the vertical, the potential energy, V, of the system is given by

$$V = \sqrt{2}mga(3\cos\theta - \cos 2\theta) + \text{constant}.$$

(7)

(*b*) Find the values of θ for which the system is in equilibrium.

(6)

(*c*) Determine the stability of the position of equilibrium for which $\theta > 0$.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced Level

Thursday 15 June 2006 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

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Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N22908RA

1. At noon, a boat P is on a bearing of 120° from boat Q. Boat P is moving due east at a constant speed of 12 km h^{-1}. Boat Q is moving in a straight line with a constant speed of 15 km h^{-1} on a course to intercept P. Find the direction of motion of Q, giving your answer as a bearing.
(5)

---

2. A smooth uniform sphere S of mass m is moving on a smooth horizontal plane when it collides with a fixed smooth vertical wall. Immediately before the collision, the speed of S is U and its direction of motion makes an angle α with the wall. The coefficient of restitution between S and the wall is e. Find the kinetic energy of S immediately after the collision.
(6)

---

3. A cyclist C is moving with a constant speed of 10 m s^{-1} due south. Cyclist D is moving with a constant speed of 16 m s^{-1} on a bearing of 240°.

(*a*) Show that the magnitude of the velocity of C relative to D is 14 m s^{-1}.
(3)

At 2 pm, D is 4 km due east of C.

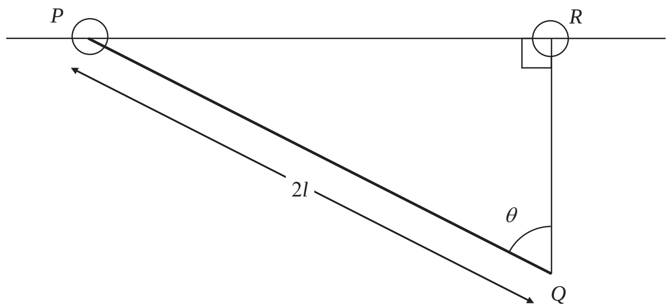
(*b*) Find

(i) the shortest distance between C and D during the subsequent motion,

(ii) the time, to the nearest minute, at which this shortest distance occurs.
(7)

---

4. **Figure 1**



A uniform rod PQ has mass m and length $2l$. A small smooth light ring is fixed to the end P of the rod. This ring is threaded on to a fixed horizontal smooth straight wire. A second small smooth light ring R is threaded on to the wire and is attached by a light elastic string, of natural length l and modulus of elasticity kmg, to the end Q of the rod, where k is a constant.

(*a*) Show that, when the rod PQ makes an angle θ with the vertical, where $0 < \theta \leq \frac{\pi}{3}$, and Q is vertically below R, as shown in Figure 1, the potential energy of the system is

$$mgl\,[2k\cos^2\theta - (2k+1)\cos\theta] + \text{constant}.$$

(7)

Given that there is a position of equilibrium with $\theta > 0$,

(*b*) show that $k > \frac{1}{2}$.

(5)

---

5. A train of mass m is moving along a straight horizontal railway line. A time t, the train is moving with speed v and the resistance to motion has magnitude kv, where k is a constant. The engine of the train is working at a constant rate P.

(*a*) Show that, when $v > 0$, $\quad mv\frac{\mathrm{d}v}{\mathrm{d}t} + kv^2 = P$.

(3)

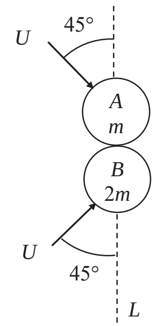
When $t = 0$, the speed of the train is $\frac{1}{3}\sqrt{\left(\frac{P}{k}\right)}$.

(*b*) Find, in terms of m and k, the time taken for the train to double its initial speed.

(8)

---

6. **Figure 2**

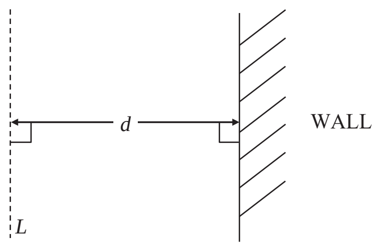


Two small smooth spheres A and B, of equal size and of mass m and $2m$ respectively, are moving initially with the same speed U on a smooth horizontal floor. The spheres collide when their centres are on a line L. Before the collision the spheres are moving towards each other, with their directions of motion perpendicular to each other and each inclined at an angle of 45° to the line L, as shown in Figure 2. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(*a*) Find the magnitude of the impulse which acts on A in the collision.

(9)

Figure 3



The line L is parallel to and a distance d from a smooth vertical wall, as shown in Figure 3.

(*b*) Find, in terms of d, the distance between the points at which the spheres first strike the wall.

(5)

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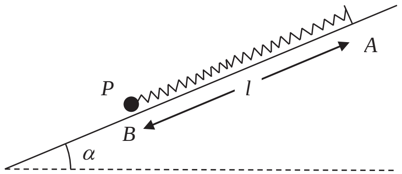
7. **Figure 4**



A light elastic spring has natural length l and modulus of elasticity $4mg$. One end of the spring is attached to a point A on a plane that is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$. The other end of the spring is attached to a particle P of mass m. The plane is rough and the coefficient of friction between P and the plane is $\frac{1}{2}$. The particle P is held at a point B on the plane where B is below A and $AB = l$, with the spring lying along a line of greatest slope of the plane, as shown in Figure 4. At time $t = 0$, the particle is projected up the plane towards A with speed $\frac{1}{2}\sqrt{(gl)}$. At time t, the compression of the spring is x.

(*a*) Show that $\dfrac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 4\omega^2 x = -g$, where $\omega = \sqrt{\left(\dfrac{g}{l}\right)}$.

(6)

(*b*) Find x in terms of l, ω and t.

(7)

(*c*) Find the distance that P travels up the plane before first coming to rest.

(4)

TOTAL FOR PAPER: 75 MARKS

END

N22908RA 5

Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced Level

Monday 18 June 2007 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 6 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N26117A

1. A small ball is moving on a horizontal plane when it strikes a smooth vertical wall. The coefficient of restitution between the ball and the wall is e. Immediately before the impact the direction of motion of the ball makes an angle of 60° with the wall. Immediately after the impact the direction of motion of the ball makes an angle of 30° with the wall.

(*a*) Find the fraction of the kinetic energy of the ball which is lost in the impact.
(6)

(*b*) Find the value of e.
(4)

---

2. A lorry of mass M moves along a straight horizontal road against a constant resistance of magnitude R. The engine of the lorry works at a constant rate RU, where U is a constant. At time t, the lorry is moving with speed v.

(*a*) Show that $Mv\frac{\mathrm{d}v}{\mathrm{d}t} = R(U - v)$.
(3)

At time $t = 0$, the lorry has speed $\frac{1}{4}U$ and the time taken by the lorry to attain a speed of $\frac{1}{3}U$ is $\frac{kMU}{R}$, where k is a constant.

(*b*) Find the exact value of k.
(7)

3.

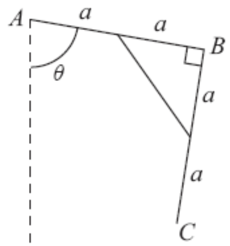


Figure 1

A framework consists of two uniform rods AB and BC, each of mass m and length $2a$, joined at B. The mid-points of the rods are joined by a light rod of length $a\sqrt{2}$, so that angle ABC is a right angle. The framework is free to rotate in a vertical plane about a fixed smooth horizontal axis. This axis passes through the point A and is perpendicular to the plane of the framework. The angle between the rod AB and the downward vertical is denoted by θ, as shown in Fig. 1.

(*a*) Show that the potential energy of the framework is

$$-mga(3\cos\theta + \sin\theta) + \text{constant}.$$
(4)

(*b*) Find the value of θ when the framework is in equilibrium, with B below the level of A.
(4)

(*c*) Determine the stability of this position of equilibrium.
(4)

---

4. At 12 noon, ship A is 20 km from ship B, on a bearing of 300°. Ship A is moving at a constant speed of 15 km h^{-1} on a bearing of 070°. Ship B moves in a straight line with constant speed V km h^{-1} and intercepts A.

(*a*) Find, giving your answer to 3 significant figures, the minimum possible for V.
(3)

It is now given that $V = 13$.

(*b*) Explain why there are two possible times at which ship B can intercept ship A.
(2)

(*c*) Find, giving your answer to the nearest minute, the earlier time at which ship B can intercept ship A.
(8)

---

5. A smooth uniform sphere A has mass $2m$ kg and another smooth uniform sphere B, with the same radius as A, has mass m kg. The spheres are moving on a smooth horizontal plane when they collide. At the instant of collision the line joining the centres of the spheres is parallel to $\mathbf{j}$. Immediately **after** the collision, the velocity of A is $(3\mathbf{i} - \mathbf{j})$ m s^{-1} and the velocity of B is $(2\mathbf{i} + \mathbf{j})$ m s^{-1}. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(*a*) Find the velocities of the two spheres immediately before the collision.
(7)

(*b*) Find the magnitude of the impulse in the collision.
(2)

(*c*) Find, to the nearest degree, the angle through which the direction of motion of A is deflected by the collision.
(4)

---

6. A small ball is attached to one end of a spring. The ball is modelled as a particle of mass 0.1 kg and the spring is modelled as a light elastic spring AB, of natural length 0.5 m and modulus of elasticity 2.45 N. The particle is attached to the end B of the spring. Initially, at time $t = 0$, the end A is held at rest and the particle hangs at rest in equilibrium below A at the point E. The end A then begins to move along the line of the spring in such a way that, at time t seconds, $t \leq 1$, the downward displacement of A from its initial position is $2 \sin 2t$ metres. At time t seconds, the extension of the spring is x metres and the displacement of the particle below E is y metres.

(*a*) Show, by referring to a simple diagram, that $y + 0.2 = x + 2 \sin 2t$.
(3)

(*b*) Hence show that $\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 49y = 98 \sin 2t$.
(5)

Given that $y = \frac{98}{45} \sin 2t$ is a particular integral of this differential equation,

(*c*) find y in terms of t.
(5)

(*d*) Find the time at which the particle first comes to instantaneous rest.
(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced Level

Thursday 12 June 2008 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

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The total mark for this paper is 75.

Advice to Candidates

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H30094RA

1. [*In this question* **i** *and* **j** *are unit vectors due east and due north respectively*.]

A ship P is moving with velocity $(5\mathbf{i} - 4\mathbf{j})$ km h^{-1} and a ship Q is moving with velocity $(3\mathbf{i} + 7\mathbf{j})$ km h^{-1}.

Find the direction that ship Q appears to be moving in, to an observer on ship P, giving your answer as a bearing.

(5)

---

2. Two small smooth spheres A and B have equal radii. The mass of A is $2m$ kg and the mass of B is m kg. The spheres are moving on a smooth horizontal plane and they collide. Immediately before the collision the velocity of A is $(2\mathbf{i} - 2\mathbf{j})$ m s^{-1} and the velocity of B is $(-3\mathbf{i} - \mathbf{j})$ m s^{-1}. Immediately after the collision the velocity of A is $(\mathbf{i} - 3\mathbf{j})$ m s^{-1}.

Find the speed of B immediately after the collision.

(5)

---

3. At time $t = 0$, a particle of mass m is projected vertically downwards with speed U from a point above the ground. At time t the speed of the particle is v and the magnitude of the air resistance is modelled as being mkv, where k is a constant.

Given that $U < \frac{g}{2k}$, find, in terms of k, U and g, the time taken for the particle to double its speed.

(8)

4.

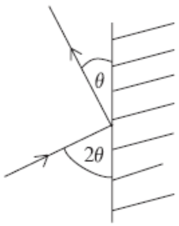

Figure 1

A small smooth ball B, moving on a horizontal plane, collides with a fixed vertical wall. Immediately before the collision the angle between the direction of motion of B and the wall is 2θ, where $0° < \theta < 45°$. Immediately after the collision the angle between the direction of motion of B and the wall is θ, as shown in Figure 1.

Given that the coefficient of restitution between B and the wall is $\frac{3}{8}$, find the value of $\tan\theta$.

(8)

---

5. A light elastic spring has natural length l and modulus of elasticity mg. One end of the spring is fixed to a point O on a rough horizontal table. The other end is attached to a particle P of mass m which is at rest on the table with $OP = l$. At time $t = 0$ the particle is projected with speed $\sqrt{(gl)}$ along the table in the direction OP. At time t the displacement of P from its initial position is x and its speed is v. The motion of P is subject to air resistance of magnitude $2mv\omega$, where $\omega = \sqrt{\frac{g}{l}}$. The coefficient of friction between P and the table is 0.5.

(*a*) Show that, until P first comes to rest,

$$\frac{d^2x}{dt^2} + 2\omega\frac{dx}{dt} + \omega^2 x = -0.5g.$$

(6)

(*b*) Find x in terms of t, l and ω.

(6)

(*c*) Hence find, in terms of ω, the time taken for P to first come to instantaneous rest.

(3)

6.

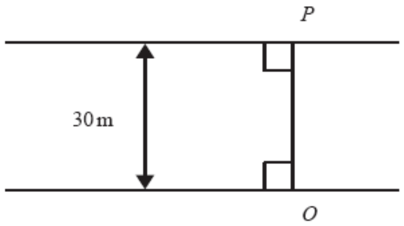

Figure 2

A river is 30 m wide and flows between two straight parallel banks. At each point of the river, the direction of flow is parallel to the banks. At time $t = 0$, a boat leaves a point O on one bank and moves in a straight line across the river to a point P on the opposite bank. Its path OP is perpendicular to both banks and $OP = 30$ m, as shown in Figure 2. The speed of flow of the river, r m s^{-1}, at a point on OP which is at a distance x m from O, is modelled as

$$r = \frac{1}{10}x, \quad 0 \leq x \leq 30.$$

The speed of the boat relative to the water is constant at 5 m s^{-1}. At time t seconds the boat is at a distance x m from O and is moving with speed v m s^{-1} in the direction OP.

(*a*) Show that

$$100v^2 = 2500 - x^2.$$

(3)

(*b*) Hence show that

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{x}{100} = 0.$$

(4)

(*c*) Find the total time taken for the boat to cross the river from O to P.

(9)

7.

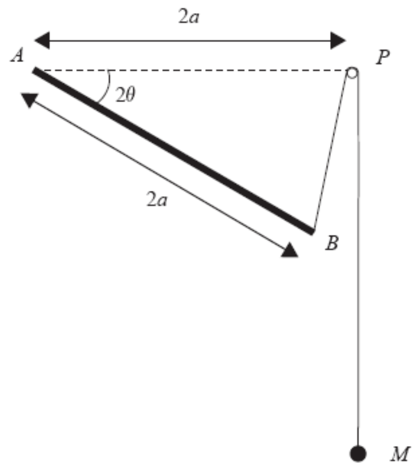

Figure 3

A uniform rod AB, of length $2a$ and mass kM where k is a constant, is free to rotate in a vertical plane about the fixed point A. One end of a light inextensible string of length $6a$ is attached to the end B of the rod and passes over a small smooth pulley which is fixed at the point P. The line AP is horizontal and of length $2a$. The other end of the string is attached to a particle of mass M which hangs vertically below the point P, as shown in Figure 3. The angle PAB is 2θ, where $0° \leq \theta \leq 180°$.

(*a*) Show that the potential energy of the system is

$$Mga(4\sin\theta - k\sin 2\theta) + \text{constant}.$$

(5)

The system has a position of equilibrium when $\cos\theta = \frac{3}{4}$.

(*b*) Find the value of k.

(5)

(*c*) Hence find the value of $\cos\theta$ at the other position of equilibrium.

(3)

(*d*) Determine the stability of each of the two positions of equilibrium.

(5)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6680

Edexcel GCE

Mechanics M4

Advanced

Monday 15 June 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Orange or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

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When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

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Full marks may be obtained for answers to ALL questions.
There are 6 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

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M34276A

1.

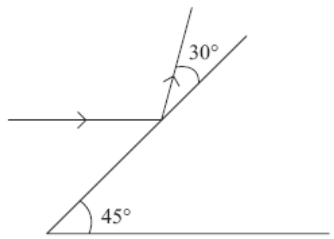

Figure 1

A fixed smooth plane is inclined to the horizontal at an angle of 45°. A particle P is moving horizontally and strikes the plane. Immediately before the impact, P is moving in a vertical plane containing a line of greatest slope of the inclined plane. Immediately after the impact, P is moving in a direction which makes an angle of 30° with the inclined plane, as shown in Figure 1.

Find the fraction of the kinetic energy of P which is lost in the impact.

(6)

---

2. At time $t = 0$, a particle P of mass m is projected vertically upwards with speed $\sqrt{\frac{g}{k}}$, where k is a constant. At time t the speed of P is v. The particle P moves against air resistance whose magnitude is modelled as being mkv^2 when the speed of P is v. Find, in terms of k, the distance travelled by P until its speed first becomes half of its initial speed.

(9)

---

3. At noon a motorboat P is 2 km north-west of another motorboat Q. The motorboat P is moving due south at 20 m s^{-1}. The motorboat Q is pursuing motorboat P at a speed of 12 m s^{-1} and sets a course in order to get as close to motorboat P as possible.

(*a*) Find the course set by Q, giving your answer as a bearing to the nearest degree.

(4)

(*b*) Find the shortest distance between P and Q.

(3)

(*c*) Find the distance travelled by Q from its position at noon to the point of closest approach.

(5)

4.

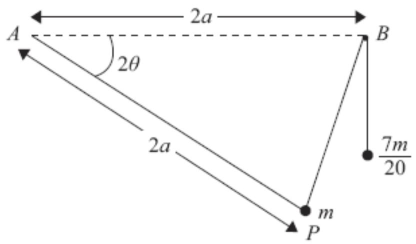

Figure 2

A light inextensible string of length $2a$ has one end attached to a fixed point A. The other end of the string is attached to a particle P of mass m. A second light inextensible string of length L, where $L > \frac{12a}{5}$, has one of its ends attached to P and passes over a small smooth peg fixed at a point B. The line AB is horizontal and $AB = 2a$. The other end of the second string is attached to a particle of mass $\frac{7m}{20}$, which hangs vertically below B, as shown in Figure 2.

(*a*) Show that the potential energy of the system, when the angle $PAB = 2\theta$, is

$$\tfrac{1}{5}mga(7\sin\theta - 10\sin 2\theta) + \text{constant}.$$

(4)

(*b*) Show that there is only one value of $\cos\theta$ for which the system is in equilibrium and find this value.

(8)

(*c*) Determine the stability of the position of equilibrium.

(4)

---

5. Two small smooth spheres A and B, of mass 2 kg and 1 kg respectively, are moving on a smooth horizontal plane when they collide. Immediately before the collision the velocity of A is $(\mathbf{i} + 2\mathbf{j})$ m s^{-1} and the velocity of B is $-2\mathbf{i}$ m s^{-1}. Immediately after the collision the velocity of A is $\mathbf{j}$ m s^{-1}.

(*a*) Show that the velocity of B immediately after the collision is $2\mathbf{j}$ m s^{-1}.

(3)

(*b*) Find the impulse of B on A in the collision, giving your answer as a vector, and hence show that the line of centres is parallel to $\mathbf{i} + \mathbf{j}$.

(4)

(*c*) Find the coefficient of restitution between A and B.

(6)

---

6. A light elastic spring AB has natural length $2a$ and modulus of elasticity $2mn^2a$, where n is a constant. A particle P of mass m is attached to the end A of the spring. At time $t = 0$, the spring, with P attached, lies at rest and unstretched on a smooth horizontal plane. The other end B of the spring is then pulled along the plane in the direction AB with constant acceleration f. At time t the extension of the spring is x.

(*a*) Show that $\frac{d^2x}{dt^2} + n^2x = f$.

(6)

(*b*) Find x in terms of n, f and t.

(8)

Hence find

(*c*) the maximum extension of the spring,

(3)

(*d*) the speed of P when the spring first reaches its maximum extension.

(2)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6680/01
Edexcel GCE
Mechanics M4
Advanced
Tuesday 22 June 2010 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

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The total mark for this paper is 75.

Advice to Candidates

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N35393A

1. [*In this question* **i** *and* **j** *are unit vectors due east and due north respectively*.]

A man cycles at a constant speed u m s^{-1} on level ground and finds that when his velocity is $u\mathbf{j}$ m s^{-1} the velocity of the wind appears to be $v(3\mathbf{i} - 4\mathbf{j})$ m s^{-1}, where v is a positive constant.

When the man cycles with velocity $\frac{1}{5}u(-3\mathbf{i} + 4\mathbf{j})$ m s^{-1}, the velocity of the wind appears to be $w\mathbf{i}$ m s^{-1}, where w is a positive constant.

Find, in terms of u, the true velocity of the wind.
(7)

---

2. Two smooth uniform spheres S and T have equal radii. The mass of S is 0.3 kg and the mass of T is 0.6 kg. The spheres are moving on a smooth horizontal plane and collide obliquely. Immediately before the collision the velocity of S is $\mathbf{u}_1$ m s^{-1} and the velocity of T is $\mathbf{u}_2$ m s^{-1}. The coefficient of restitution between the spheres is 0.5. Immediately after the collision the velocity of S is $(-\mathbf{i} + 2\mathbf{j})$ m s^{-1} and the velocity of T is $(\mathbf{i} + \mathbf{j})$ m s^{-1}.

Given that when the spheres collide the line joining their centres is parallel to **i** ,

(*a*) find

(i) $\mathbf{u}_1$,

(ii) $\mathbf{u}_2$.
(6)

After the collision, T goes on to collide with a smooth vertical wall which is parallel to **j**.

Given that the coefficient of restitution between T and the wall is also 0.5, find

(*b*) the angle through which the direction of motion of T is deflected as a result of the collision with the wall,
(5)

(c) the loss in kinetic energy of T caused by the collision with the wall.
(3)

---

3. At 12 noon, ship A is 8 km due west of ship B. Ship A is moving due north at a constant speed of 10 km h^{-1}. Ship B is moving at a constant speed of 6 km h^{-1} on a bearing so that it passes as close to A as possible.

(*a*) Find the bearing on which ship B moves.
(4)

(*b*) Find the shortest distance between the two ships.
(3)

(*c*) Find the time when the two ships are closest.
(3)

---

4. A particle of mass m is projected vertically upwards, at time $t = 0$, with speed U. The particle is subject to air resistance of magnitude $\frac{mgv^2}{k^2}$, where v is the speed of the particle at time t and k is a positive constant.

(*a*) Show that the particle reaches its greatest height above the point of projection at time $\frac{k}{g}\tan^{-1}\left(\frac{U}{k}\right)$

(6)

(*b*) Find the greatest height above the point of projection attained by the particle.

(6)

5.

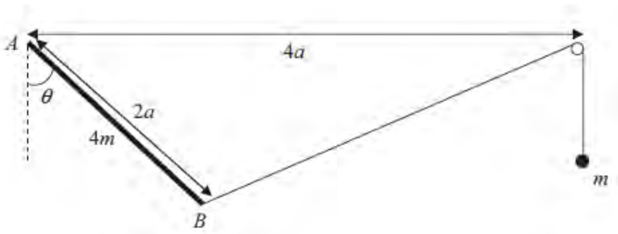

Figure 1

The end A of a uniform rod AB, of length $2a$ and mass $4m$, is smoothly hinged to a fixed point. The end B is attached to one end of a light inextensible string which passes over a small smooth pulley, fixed at the same level as A. The distance from A to the pulley is $4a$. The other end of the string carries a particle of mass m which hangs freely, vertically below the pulley, with the string taut. The angle between the rod and the downward vertical is θ, where $0 < \theta < \frac{\pi}{2}$, as shown in Figure 1.

(*a*) Show that the potential energy of the system is

$$2mga(\sqrt{(5 - 4\sin\theta)} - 2\cos\theta) + \text{constant}.$$

(5)

(*b*) Hence, or otherwise, show that any value of θ which corresponds to a position of equilibrium of the system satisfies the equation

$$4\sin^3\theta - 6\sin^2\theta + 1 = 0.$$

(5)

(*c*) Given that $\theta = \frac{\pi}{6}$ corresponds to a position of equilibrium, determine its stability.

(5)

6. Two points A and B lie on a smooth horizontal table with $AB = 4a$. One end of a light elastic spring, of natural length a and modulus of elasticity $2mg$, is attached to A. The other end of the spring is attached to a particle P of mass m. Another light elastic spring, of natural length a and modulus of elasticity mg, has one end attached to B and the other end attached to P. The particle P is on the table at rest and in equilibrium.

(*a*) Show that $AP = \frac{5a}{3}$.

(4)

The particle P is now moved along the table from its equilibrium position through a distance $0.5a$ towards B and released from rest at time $t = 0$. At time t, P is moving with speed v and has displacement x from its equilibrium position. There is a resistance to motion of magnitude $4m\omega vg$ where $\omega = \sqrt{\left(\frac{g}{a}\right)}$.

(*b*) Show that $\frac{d^2x}{dt^2} + 4\omega\frac{dx}{dt} + 3\omega^2 x = 0$.

(5)

(*c*) Find the velocity, $\frac{dx}{dt}$, of P in terms of a, ω and t.

(8)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced Level

Wednesday 22 June 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

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1.

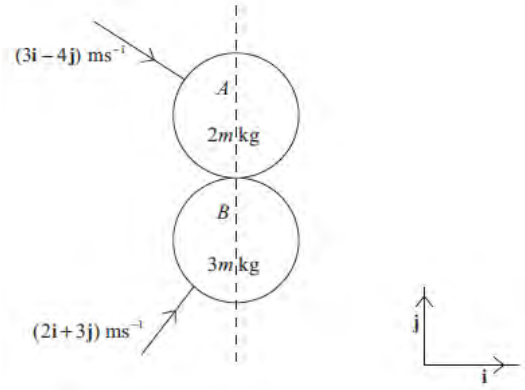

Figure 1

Two smooth uniform spheres A and B have masses $2m$ kg and $3m$ kg respectively and equal radii. The spheres are moving on a smooth horizontal surface. Initially, sphere A has velocity $(3\mathbf{i} - 4\mathbf{j})$ m s^{-1} and sphere B has velocity $(2\mathbf{i} + 3\mathbf{j})$ m s^{-1}. When the spheres collide, the line joining their centres is parallel to $\mathbf{j}$, as shown in Figure 1. The coefficient of restitution between the spheres is $\frac{3}{7}$.

Find, in terms of m, the total kinetic energy lost in the collision.

(10)

2.

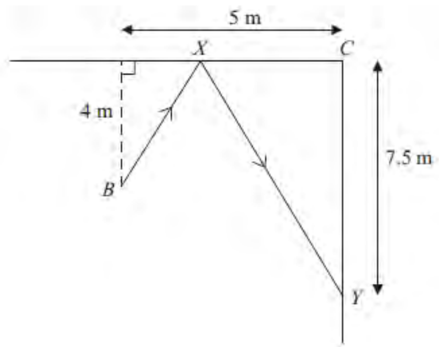

Figure 1

Figure 2 represents part of the smooth rectangular floor of a sports hall. A ball is at *B*, 4 m from one wall of the hall and 5 m from an adjacent wall. These two walls are smooth and meet at the corner *C*. The ball is kicked so that it travels along the floor, bounces off the first wall at the point *X* and hits the second wall at the point *Y*. The point *Y* is 7.5 m from the corner *C*.

The coefficient of restitution between the ball and the first wall is $\frac{3}{4}$.

Modelling the ball as a particle, find the distance *CX*.

(9)

3. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A coastguard patrol boat *C* is moving with constant velocity $(8\mathbf{i} + u\mathbf{j})$ km h^{-1}. Another ship *S* is moving with constant velocity $(12\mathbf{i} + 16\mathbf{j})$ km h^{-1}.

(*a*) Find, in terms of *u*, the velocity of *C* relative to *S*.

(2)

At noon, *S* is 10 km due west of *C*.

If *C* is to intercept *S*,

(*b*) (i) find the value of *u*.

(ii) Using this value of *u*, find the time at which *C* would intercept *S*.

(4)

If instead, at noon, *C* is moving with velocity $(8\mathbf{i} + 8\mathbf{j})$ km h^{-1} and continues at this constant velocity,

(*c*) find the distance of closest approach of *C* to *S*.

(5)

4. A hiker walking due east at a steady speed of 5 km h^{-1} notices that the wind appears to come from a direction with bearing 050. At the same time, another hiker moving on a bearing of 320, and also walking at 5 km h^{-1}, notices that the wind appears to come from due north.

Find

(*a*) the direction from which the wind is blowing,

(3)

(*b*) the wind speed.

(4)

5. A particle Q of mass 6 kg is moving along the x-axis. At time t seconds the displacement of Q from the origin O is x metres and the speed of Q is v m s^{-1}. The particle moves under the action of a retarding force of magnitude $(a + bv^2)$ N, where a and b are positive constants. At time $t = 0$, Q is at O and moving with speed U m s^{-1} in the positive x-direction. The particle Q comes to instantaneous rest at the point X.

(*a*) Show that the distance OX is

$$\frac{3}{b}\ln\left(1+\frac{bU^2}{a}\right) \text{ m.}$$

(6)

Given that $a = 12$ and $b = 3$,

(*b*) find, in terms of U, the time taken to move from O to X.

(5)

---

6. A particle P of mass 4 kg moves along a horizontal straight line under the action of a force directed towards a fixed point O on the line. At time t seconds, P is x metres from O and the force towards O has magnitude $9x$ newtons. The particle P is also subject to air resistance, which has magnitude $12v$ newtons when P is moving with speed v m s–1.

(*a*) Show that the equation of motion of P is

$$4\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 9x = 0.$$

(4)

It is given that the solution of this differential equation is of the form $x = e^{-\lambda t}(At + B)$.

When $t = 0$ the particle is released from rest at the point R, where $OR = 4$ m.

Find,

(*b*) the values of the constants λ, A and B,

(4)

(*c*) the greatest speed of P in the subsequent motion.

(5)

7.

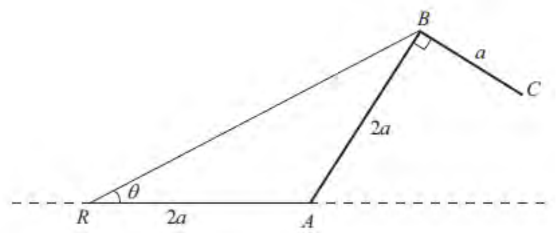

Figure 3

Figure 3 shows a framework ABC, consisting of two uniform rods rigidly joined together at B so that $\angle ABC = 90°$. The rod AB has length $2a$ and mass $4m$, and the rod BC has length a and mass $2m$. The framework is smoothly hinged at A to a fixed point, so that the framework can rotate in a fixed vertical plane. One end of a light elastic string, of natural length $2a$ and modulus of elasticity $3mg$, is attached to A. The string passes through a small smooth ring R fixed at a distance $2a$ from A, on the same horizontal level as A and in the same vertical plane as the framework. The other end of the string is attached to B.

The angle ARB is θ, where $0 < \theta < \frac{\pi}{2}$.

(*a*) Show that the potential energy V of the system is given by

$$V = 8amg \sin 2\theta + 5amg \cos 2\theta + \text{constant}.$$

(7)

(*b*) Find the value of θ for which the system is in equilibrium.

(4)

(*c*) Determine the stability of this position of equilibrium.

(3)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced Level

Friday 1 June 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 6 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P40110A

1. A smooth uniform sphere S, of mass m, is moving on a smooth horizontal plane when it collides obliquely with another smooth uniform sphere T, of the same radius as S but of mass $2m$, which is at rest on the plane. Immediately before the collision the velocity of S makes an angle α, where $\tan \alpha = \frac{3}{4}$, with the line joining the centres of the spheres.

Immediately after the collision the speed of T is V. The coefficient of restitution between the two spheres is $\frac{3}{4}$.

(*a*) Find, in terms of V, the **speed** of S

(i) immediately before the collision,

(ii) immediately after the collision.

(9)

(*b*) Find the angle through which the direction of motion of S is deflected as a result of the collision.

(4)

---

2. A ship A is moving at a constant speed of 8 km h^{-1} on a bearing of 150°. At noon a second ship B is 6 km from A, on a bearing of 210°. Ship B is moving due east at a constant speed. At a later time, B is $2\sqrt{3}$ km dues south of A.

Find

(*a*) the time at which B will be due east of A,

(*b*) the distance between the ships at that time.

(13)

---

3. Two particles, of masses m and $2m$, are connected to the ends of a long light inextensible string. The string passes over a small smooth fixed pulley and hangs vertically on either side. The particles are released from rest with the string taut. Each particle is subject to air resistance of magnitude kv^2, where v is the speed of each particle after it has moved a distance x from rest and k is a positive constant.

(*a*) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(v^2) + \frac{4k}{3m}v^2 = \frac{2g}{3}$.

(6)

(*b*) Find v^2 in terms of x.

(5)

(*c*) Deduce that the tension in the string, T, satisfies

$$\frac{4mg}{3} \leq T < \frac{3mg}{2}.$$

(5)

---

4. A rescue boat, whose maximum speed is 20 km h^{-1}, receives a signal which indicates that a yacht is in distress near a fixed point P. The rescue boat is 15 km south-west of P. There is a constant current of 5 km h^{-1} flowing uniformly from west to east. The rescue boat sets the course needed to get to P as quickly as possible. Find

(*a*) the course the rescue boat sets,

(4)

(*b*) the time, to the nearest minute, to get to P.

(4)

When the rescue boat arrives at P, the yacht is just visible 4 km due north of P and is drifting with the current. Find

(*c*) the course that the rescue boat should set to get to the yacht as quickly as possible,

(1)

(*d*) the time taken by the rescue boat to reach the yacht from P.

(1)

5.

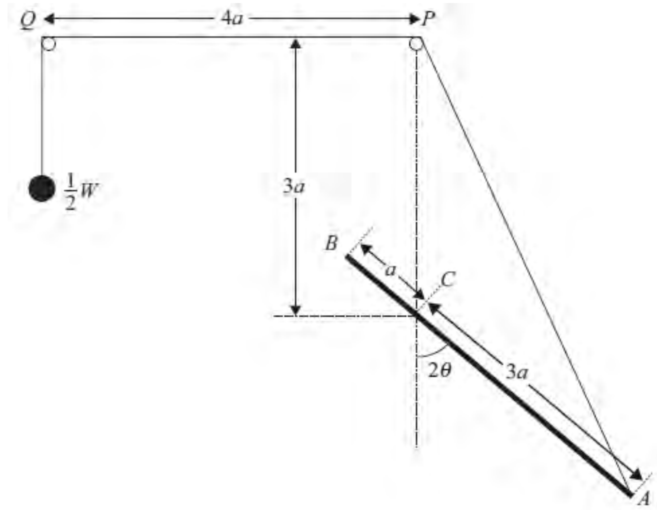

Figure 1

A uniform rod AB, of length $4a$ and weight W, is free to rotate in a vertical plane about a fixed smooth horizontal axis which passes through the point C of the rod, where $AC = 3a$. One end of a light inextensible string of length L, where $L > 10a$, is attached to the end A of the rod and passes over a small, smooth fixed peg at P and another small, smooth fixed peg at Q. The point Q lies in the same vertical plane as P, A and B. The point P is at a distance $3a$ vertically above C and PQ is horizontal with $PQ = 4a$. A particle of weight $\frac{1}{2}W$ is attached to the other end of the string and hangs vertically below Q. The rod is inclined at an angle 2θ to the vertical, where $-\pi < 2\theta < \pi$, as shown in Figure 1.

(*a*) Show that the potential energy of the system is

$$Wa(3\cos\theta - \cos 2\theta) + \text{constant}.$$

(4)

(*b*) Find the positions of equilibrium and determine their stability.

(8)

6. Two points A and B are in a vertical line, with A above B and $AB = 4a$. One end of a light elastic spring, of natural length a and modulus of elasticity $3mg$, is attached to A. The other end of the spring is attached to a particle P of mass m. Another light elastic spring, of natural length a and modulus of elasticity mg, has one end attached to B and the other end attached to P. The particle P hangs at rest in equilibrium.

(*a*) Show that $AP = \dfrac{7a}{4}$.

(3)

The particle P is now pulled down vertically from its equilibrium position towards B and at time $t = 0$ it is released from rest. At time t, the particle P is moving with speed v and has displacement x from its equilibrium position. The particle P is subject to air resistance of magnitude mkv, where k is a positive constant.

(*b*) Show that

$$\frac{d^2x}{dt^2} + k\frac{dx}{dt} + \frac{4g}{a}x = 0.$$

(5)

(*c*) Find the range of values of k which would result in the motion of P being a damped oscillation.

(3)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6680/01R
Edexcel GCE

Mechanics M4 (R)

Advanced/Advanced Subsidiary

Tuesday 18 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

This paper is strictly for students outside the UK.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P42959A

1. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*.]

Boat A is moving with velocity $(3\mathbf{i} + 4\mathbf{j})$ km h^{-1} and boat B is moving with velocity $(6\mathbf{i} - 5\mathbf{j})$ km h^{-1}. Find

(*a*) the magnitude of the velocity of A relative to B,
(3)

(*b*) the direction of the velocity of A relative to B, giving your answer as a bearing.
(2)

2.

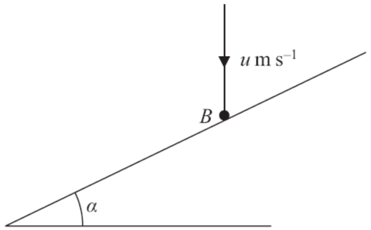

Figure 1

A smooth fixed plane is inclined at an angle α to the horizontal. A smooth ball B falls vertically and hits the plane. Immediately before the impact the speed of B is u m s^{-1}, as shown in Figure 1. Immediately after the impact the direction of motion of B is horizontal. The coefficient of restitution between B and the plane is $\frac{1}{3}$.

Find the size of angle α.
(6)

---

3. A smooth uniform sphere A, of mass $5m$ and radius r, is at rest on a smooth horizontal plane. A second smooth uniform sphere B, of mass $3m$ and radius r, is moving in a straight line on the plane with speed u m s^{-1} and strikes A. Immediately before the impact the direction of motion of B makes an angle of 60° with the line of centres of the spheres. The direction of motion of B is turned through an angle of 30° by the impact.

Find

(*a*) the speed of B immediately after the impact,
(3)

(*b*) the coefficient of restitution between the spheres.
(6)

---

4. At 10 a.m. two walkers A and B are 4 km apart with A due north of B. Walker A is moving due east at a constant speed of 6 km h^{-1}. Walker B is moving with constant speed 5 km h^{-1} and walks in the straight line which allows him to pass as close as possible to A.

Find

(*a*) the direction of motion of B, giving your answer as a bearing,
(4)

(*b*) the least distance between A and B,
(2)

(*c*) the time when the distance between A and B is least.
(4)

---

5. A van of mass 1200 kg travels along a straight horizontal road against a resistance to motion which is proportional to the speed of the van. The engine of the van is working at a constant rate of 40 kW. The van starts from rest at time $t = 0$. At time t seconds, the speed of the van is v m s^{-1}. When the speed of the van is 40 m s^{-1}, the acceleration of the van is 0.3 m s^{-2}.

(*a*) Show that

$$75v\frac{\mathrm{d}v}{\mathrm{d}t} = 2500 - v^2$$

(6)

(*b*) Find v in terms of t.
(6)

6.

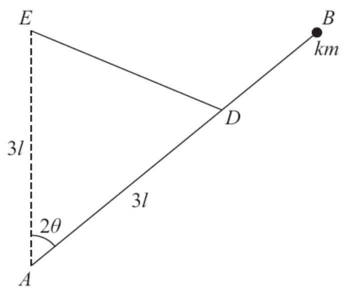

Figure 2

A uniform rod AB has mass $4m$ and length $4l$. The rod can turn freely in a vertical plane about a fixed smooth horizontal axis through A. A particle of mass km, where $k < 7$, is attached to the rod at B. One end of a light elastic string, of natural length l and modulus of elasticity $4mg$, is attached to the point D of the rod, where $AD = 3l$. The other end of the string is attached to a fixed point E which is vertically above A, where $AE = 3l$, as shown in Figure 2.

The angle between the rod and the upward vertical is 2θ, where $\arcsin\left(\frac{1}{6}\right) < \theta \leq \frac{\pi}{2}$.

(*a*) Show that, while the string is stretched, the potential energy of the system is

$$8mgl\{(7-k)\sin^2\theta - 3\sin\theta\} + \text{constant}$$

(6)

There is a position of equilibrium with $\theta \leq \frac{\pi}{6}$.

(*b*) Show that $k \leq 4$.

(5)

Given that $k = 4$,

(*c*) show that this position of equilibrium is stable.

(5)

---

7. A particle P of mass 0.5 kg is attached to the end A of a light elastic spring AB, of natural length 0.6 m and modulus of elasticity 2.7 N. At time $t = 0$ the end B of the spring is held at rest and P hangs at rest at the point C which is vertically below B. The end B is then moved along the line of the spring so that, at time t seconds, the downwards displacement of B from its initial position is $4\sin 2t$ metres. At time t seconds, the extension of the spring is x metres and the displacement of P below C is y metres.

(*a*) Show that

$$y + \frac{49}{45} = x + 4\sin 2t$$

(3)

(*b*) Hence show that

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 9y = 36\sin 2t$$

(5)

Given that $y = \frac{36}{5}\sin 2t$ is a particular integral of this differential equation,

(*c*) find y in terms of t,

(5)

(*d*) find the speed of P when $t = \frac{1}{3}\pi$.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6680/01

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Tuesday 18 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

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Advice to Candidates

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1. A particle P of mass 0.5 kg falls vertically from rest. After t seconds it has speed v m s^{-1}. A resisting force of magnitude $1.5v$ newtons acts on P as it falls.

(*a*) Show that $3v = 9.8(1 - e^{-3t})$.
(8)

(*b*) Find the distance that P falls in the first two seconds of its motion.
(5)

2.

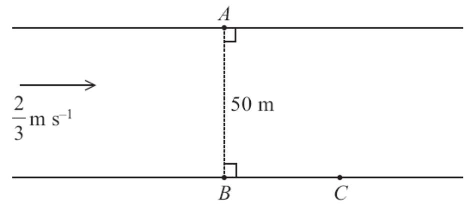

Figure 1

A river is 50 m wide and flows between two straight parallel banks. The river flows with a uniform speed of $\frac{2}{3}$ m s^{-1} parallel to the banks. The points A and B are on opposite banks of the river and AB is perpendicular to both banks of the river, as shown in Figure 1.

Keith and Ian decide to swim across the river. The speed relative to the water of both swimmers is $\frac{10}{9}$ m s^{-1}.

Keith sets out from A and crosses the river in the least possible time, reaching the opposite bank at the point C. Find

(*a*) the time taken by Keith to reach C,
(2)

(*b*) the distance BC.
(2)

Ian sets out from A and swims in a straight line so as to land on the opposite bank at B.

(*c*) Find the time taken by Ian to reach B.
(4)

3.

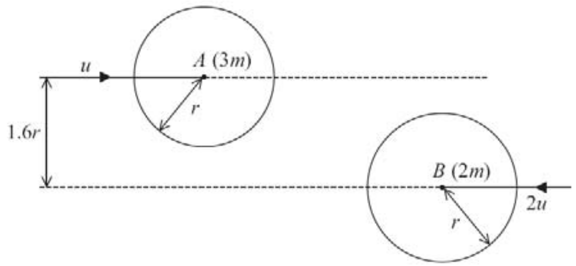

Figure 2

Two smooth uniform spheres *A* and *B*, of equal radius *r*, have masses $3m$ and $2m$ respectively. The spheres are moving on a smooth horizontal plane when they collide. Immediately before the collision they are moving with speeds *u* and $2u$ respectively. The centres of the spheres are moving towards each other along parallel paths at a distance $1.6r$ apart, as shown in Figure 2.

The coefficient of restitution between the two spheres is $\frac{1}{6}$.

Find, in terms of *m* and *u*, the magnitude of the impulse received by *B* in the collision.

(10)

4.

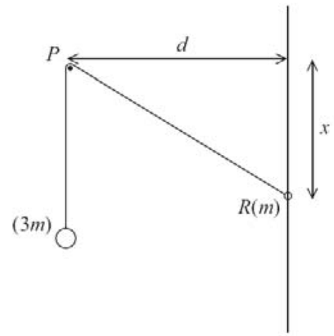

Figure 3

A small smooth peg *P* is fixed at a distance *d* from a fixed smooth vertical wire. A particle of mass $3m$ is attached to one end of a light inextensible string which passes over *P*. The particle hangs vertically below *P*. The other end of the string is attached to a small ring *R* of mass *m*, which is threaded on the wire, as shown in Figure 3.

(*a*) Show that when *R* is at a distance *x* below the level of *P* the potential energy of the system is

$$3mg\sqrt{(x^2 + d^2)} - mgx + \text{constant}$$

(4)

(*b*) Hence find *x*, in terms of *d*, when the system is in equilibrium.

(3)

(*c*) Determine the stability of the position of equilibrium.

(3)

5. A coastguard ship *C* is due south of a ship *S*. Ship *S* is moving at a constant speed of 12 km h^{-1} on a bearing of 140°. Ship *C* moves in a straight line with constant speed *V* km h^{-1} in order to intercept *S*.

(*a*) Find, giving your answer to 3 significant figures, the minimum possible value for *V*.

(3)

It is now given that $V = 14$.

(*b*) Find the bearing of the course that *C* takes to intercept *S*.

(5)

6. A particle P of mass m kg is attached to the end A of a light elastic string AB, of natural length a metres and modulus of elasticity $9ma$ newtons. Initially the particle and the string lie at rest on a smooth horizontal plane with $AB = a$ metres. At time $t = 0$ the end B of the string is set in motion and moves at a constant speed U m s^{-1} in the direction AB. The air resistance acting on P has magnitude $6mv$ newtons, where v m s^{-1} is the speed of P. At time t seconds, the extension of the string is x metres and the displacement of P from its initial position is y metres.

Show that, while the string is taut,

(*a*) $x + y = Ut$

(2)

(*b*) $\dfrac{d^2x}{dt^2} + 6\dfrac{dx}{dt} + 9x = 6U$

(5)

You are given that the general solution of the differential equation in (*b*) is

$$x = (A + Bt)Ue^{-3t} + \frac{2U}{3}$$

where A and B are arbitrary constants.

(*c*) Find the value of A and the value of B.

(5)

(*d*) Find the speed of P at time t seconds.

(2)

7. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*]

A small smooth ball of mass m kg is moving on a smooth horizontal plane and strikes a fixed smooth vertical wall. The plane and the wall intersect in a straight line which is parallel to the vector $2\mathbf{i} + \mathbf{j}$. The velocity of the ball immediately before the impact is $b\mathbf{i}$ m s^{-1}, where b is positive. The velocity of the ball immediately after the impact is $a(\mathbf{i} + \mathbf{j})$ m s^{-1}, where a is positive.

(*a*) Show that the impulse received by the ball when it strikes the wall is parallel to $(-\mathbf{i} + 2\mathbf{j})$.

(1)

Find

(*b*) the coefficient of restitution between the ball and the wall,

(8)

(*c*) the fraction of the kinetic energy of the ball that is lost due to the impact.

(3)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6680/01R

Edexcel GCE

Mechanics M4 (R)

Advanced/Advanced Subsidiary

Monday 16 June 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

This paper is strictly for students outside the UK.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680R), your surname, initials and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 6 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A small smooth ball of mass m is falling vertically when it strikes a fixed smooth plane which is inclined to the horizontal at an angle α, where $0° < \alpha < 45°$. Immediately before striking the plane the ball has speed u. Immediately after striking the plane the ball moves in a direction which makes an angle of 45° with the plane. The coefficient of restitution between the ball and the plane is e. Find, in terms of m, u and e, the magnitude of the impulse of the plane on the ball.

(11)

---

2. A ship A is travelling at a constant speed of 30 km h^{-1} on a bearing of 050°. Another ship B is travelling at a constant speed of v km h^{-1} and sets a course to intercept A. At 1400 hours B is 20 km from A and the bearing of A from B is 290°.

(*a*) Find the least possible value of v.

(3)

Given that $v = 32$,

(*b*) find the time at which B intercepts A.

(8)

---

3. A small ball of mass m is projected vertically upwards from a point O with speed U. The ball is subject to air resistance of magnitude mkv, where v is the speed of the ball and k is a positive constant.

Find, in terms of U, g and k, the maximum height above O reached by the ball.

(8)

---

4. A smooth uniform sphere S is moving on a smooth horizontal plane when it collides obliquely with an identical sphere T which is at rest on the plane. Immediately before the collision S is moving with speed U in a direction which makes an angle of 60° with the line joining the centres of the spheres. The coefficient of restitution between the spheres is e.

(*a*) Find, in terms of e and U where necessary,

(i) the speed and direction of motion of S immediately after the collision,

(ii) the speed and direction of motion of T immediately after the collision.

(12)

The angle through which the direction of motion of S is deflected is δ°.

(*b*) Find

(i) the value of e for which δ takes the largest possible value,

(ii) the value of δ in this case.

(3)

5.

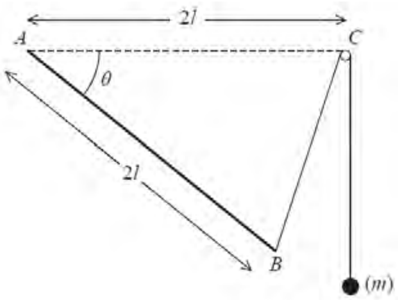

Figure 1

A uniform rod AB, of length $2l$ and mass $12m$, has its end A smoothly hinged to a fixed point. One end of a light inextensible string is attached to the other end B of the rod. The string passes over a small smooth pulley which is fixed at the point C, where AC is horizontal and $AC = 2l$. A particle of mass m is attached to the other end of the string and the particle hangs vertically below C.

The angle BAC is θ, where $0 < \theta < \frac{\pi}{2}$ as shown in Figure 1.

(*a*) Show that the potential energy of the system is

$$4mgl\left(\sin\frac{\theta}{2} - 3\sin\theta\right) + \text{constant}$$

(4)

(*b*) Find the value of θ when the system is in equilibrium and determine the stability of this equilibrium position.

(10)

6.

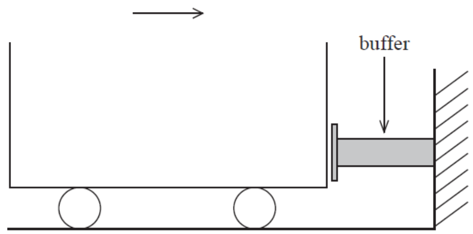

Figure 2

A railway truck of mass M approaches the end of a straight horizontal track and strikes a buffer. The buffer is parallel to the track, as shown in Figure 2. The buffer is modelled as a light horizontal spring PQ, which is fixed at the end P. The spring has a natural length a and modulus of elasticity Mn^2a, where n is a positive constant. At time $t = 0$, the spring has length a and the truck strikes the end Q with speed U. A resistive force whose magnitude is Mkv, where v is the speed of the truck at time t, and k is a positive constant, also opposes the motion of the truck. At time t, the truck is in contact with the buffer and the compression of the buffer is x.

(*a*) Show that, while the truck is compressing the buffer

$$\frac{d^2x}{dt^2} + k\frac{dx}{dt} + n^2x = 0$$

(4)

It is given that $k = \frac{5n}{2}$.

(*b*) Find x in terms of U, n and t.

(7)

(*c*) Find, in terms of U and n, the greatest value of x.

(5)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6680/01

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Monday 16 June 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A particle A has constant velocity $(3\mathbf{i} + \mathbf{j})$ m s^{-1} and a particle B has constant velocity $(\mathbf{i} - \mathbf{k})$ m s^{-1}. At time $t = 0$ seconds, the position vectors of the particles A and B with respect to a fixed origin O are $(-6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k})$ m and $(-2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ m respectively.

(*a*) Show that, in the subsequent motion, the minimum distance between A and B is $4\sqrt{2}$ m.
(6)

(*b*) Find the position vector of A at the instant when the distance between A and B is a minimum.
(2)

---

2. A car of mass 1000 kg is moving along a straight horizontal road. The engine of the car is working at a constant rate of 25 kW. When the speed of the car is v m s^{-1}, the resistance to motion has magnitude $10v$ newtons.

(*a*) Show that, at the instant when $v = 20$, the acceleration of the car is 1.05 m s^{-2}.
(3)

(*b*) Find the distance travelled by the car as it accelerates from a speed of 10 m s^{-1} to a speed of 20 m s^{-1}.
(8)

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3. A small ball is moving on a smooth horizontal plane when it collides obliquely with a smooth plane vertical wall. The coefficient of restitution between the ball and the wall is $\frac{1}{3}$. The speed of the ball immediately after the collision is half the speed of the ball immediately before the collision.

Find the angle through which the path of the ball is deflected by the collision.
(8)

---

4. At noon two ships A and B are 20 km apart with A on a bearing of 230° from B. Ship B is moving at 6 km h^{-1} on a bearing of 015°. The maximum speed of A is 12 km h^{-1}. Ship A sets a course to intercept B as soon as possible.

(*a*) Find the course set by A, giving your answer as a bearing to the nearest degree.
(4)

(*b*) Find the time at which A intercepts B.
(4)

5.

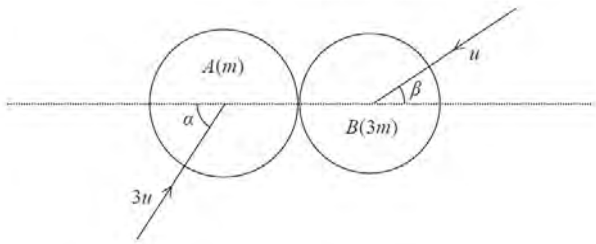

Figure 1

Two smooth uniform spheres A and B have equal radii. The mass of A is m and the mass of B is $3m$. The spheres are moving on a smooth horizontal plane when they collide obliquely. Immediately before the collision, A is moving with speed $3u$ at angle α to the line of centres and B is moving with speed u at angle β to the line of centres, as shown in Figure 1. The coefficient of restitution between the two spheres is $\frac{1}{5}$. It is given that $\cos\alpha = \frac{1}{3}$ and $\cos\beta = \frac{2}{3}$ and that α and β are both acute angles.

(*a*) Find the magnitude of the impulse on A due to the collision in terms of m and u.
(8)

(*b*) Express the kinetic energy lost by A in the collision as a fraction of its initial kinetic energy.
(4)

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6. A particle of mass m kg is attached to one end of a light elastic string of natural length a metres and modulus of elasticity $5ma$ newtons. The other end of the string is attached to a fixed point O on a smooth horizontal plane. The particle is held at rest on the plane with the string stretched to a length $2a$ metres and then released at time $t = 0$. During the subsequent motion, when the particle is moving with speed v m s^{-1}, the particle experiences a resistance of magnitude $4mv$ newtons. At time t seconds after the particle is released, the length of the string is $(a + x)$ metres, where $0 \le x \le a$.

(*a*) Show that, from $t = 0$ until the string becomes slack,

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = 0$$

(3)

(*b*) Hence express x in terms of a and t.
(6)

(*c*) Find the speed of the particle at the instant when the string first becomes slack, giving your answer in the form ka, where k is a constant to be found correct to 2 significant figures.
(4)

7.

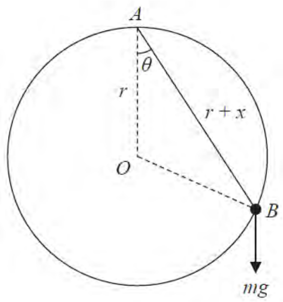

Figure 2

A bead B of mass m is threaded on a smooth circular wire of radius r, which is fixed in a vertical plane. The centre of the circle is O, and the highest point of the circle is A. A light elastic string of natural length r and modulus of elasticity kmg has one end attached to the bead and the other end attached to A. The angle between the string and the downward vertical is θ, and the extension in the string is x, as shown in Figure 2.

Given that the string is taut,

(*a*) show that the potential energy of the system is

$$2mgr\{(k-1)\cos^2\theta - k\cos\theta\} + \text{constant}$$

(6)

Given also that $k = 3$,

(*b*) find the positions of equilibrium and determine their stability.
(9)

TOTAL FOR PAPER: 75 MARKS

END