Edexcel Maths M3

Topic Questions from Papers

Statics

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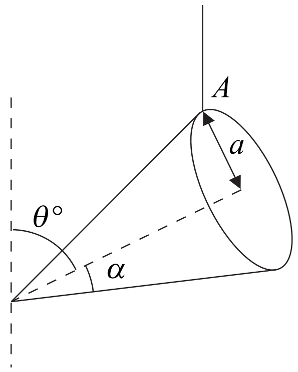
2. **Figure 1**



A uniform solid right circular cone has base radius a and semi-vertical angle α, where $\tan\alpha = \frac{1}{3}$. The cone is freely suspended by a string attached at a point A on the rim of its base, and hangs in equilibrium with its axis of symmetry making an angle of $\theta°$ with the upward vertical, as shown in Figure 1.

Find, to one decimal place, the value of θ.

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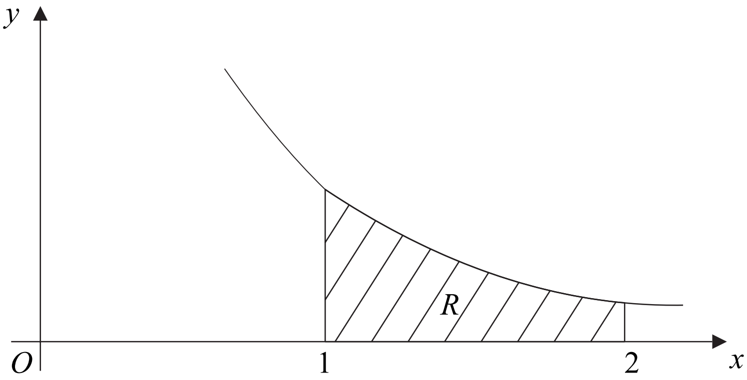
6. **Figure 4**

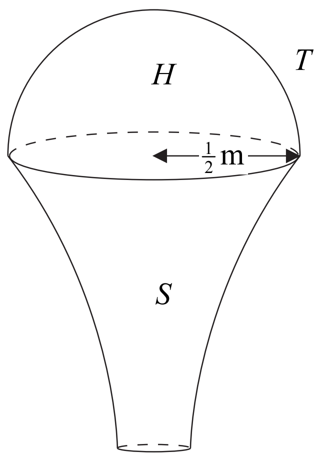


The shaded region R is bounded by the curve with equation $y = \frac{1}{2x^2}$, the x-axis and the lines $x = 1$ and $x = 2$, as shown in Figure 4. The unit of length on each axis is 1 m. A uniform solid S has the shape made by rotating R through 360° about the x-axis.

(a) Show that the centre of mass of S is $\frac{2}{7}$ m from its larger plane face.

(6)

Figure 5



A sporting trophy T is a uniform solid hemisphere H joined to the solid S. The hemisphere has radius $\frac{1}{2}$ m and its plane face coincides with the larger plane face of S, as shown in Figure 5. Both H and S are made of the same material.

(b) Find the distance of the centre of mass of T from its plane face.

(7)

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Question 6 continued

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1. The rudder on a ship is modelled as a uniform plane lamina having the same shape as the region R which is enclosed between the curve with equation $y = 2x - x^2$ and the x-axis.

(a) Show that the area of R is $\frac{4}{3}$.

(4)

(b) Find the coordinates of the centre of mass of the lamina.

(5)

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2. An open container C is modelled as a thin uniform hollow cylinder of radius h and height h with a base but no lid. The centre of the base is O.

(a) Show that the distance of the centre of mass of C from O is $\frac{1}{3}h$.

(5)

The container is filled with uniform liquid. Given that the mass of the container is M and the mass of the liquid is M,

(b) find the distance of the centre of mass of the filled container from O.

(5)

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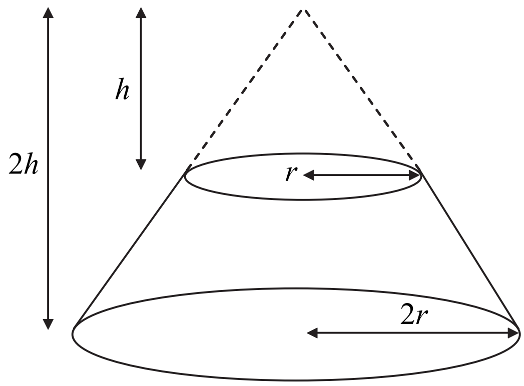
3. **Figure 1**



A uniform solid S is formed by taking a uniform solid right circular cone, of base radius $2r$ and height $2h$, and removing the cone, with base radius r and height h, which has the same vertex as the original cone, as shown in Figure 1.

(a) Show that the distance of the centre of mass of S from its larger plane face is $\frac{11}{28}h$.
(5)

The solid S lies with its larger plane face on a rough table which is inclined at an angle $\theta°$ to the horizontal. The table is sufficiently rough to prevent S from slipping.
Given that $h = 2r$,

(b) find the greatest value of θ for which S does not topple.
(3)

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Question 3 continued

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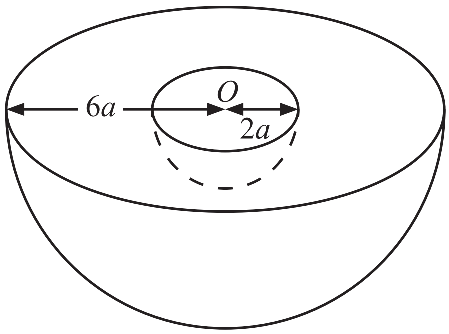

Figure 3

A uniform solid hemisphere, of radius $6a$ and centre O, has a solid hemisphere of radius $2a$, and centre O, removed to form a bowl B as shown in Figure 3.

(a) Show that the centre of mass of B is $\frac{30}{13}a$ from O.
(5)

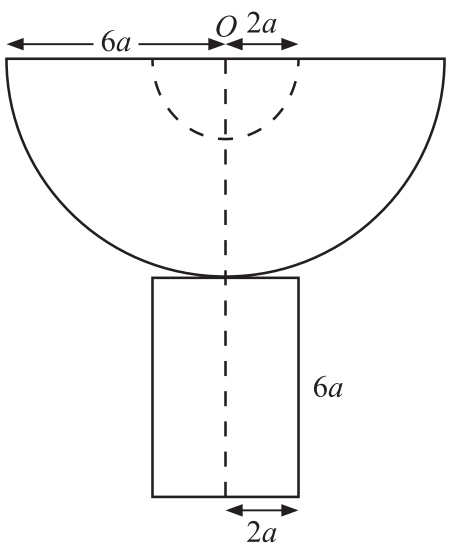

Figure 4

The bowl B is fixed to a plane face of a uniform solid cylinder made from the same material as B. The cylinder has radius $2a$ and height $6a$ and the combined solid S has an axis of symmetry which passes through O, as shown in Figure 4.

(b) Show that the centre of mass of S is $\frac{201}{61}a$ from O.
(4)

The plane surface of the cylindrical base of S is placed on a rough plane inclined at 12° to the horizontal. The plane is sufficiently rough to prevent slipping.

(c) Determine whether or not S will topple.
(4)

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Question 4 continued

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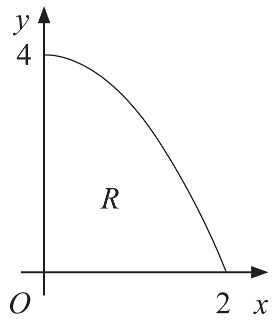

Figure 3

The region R is bounded by part of the curve with equation $y = 4 - x^2$, the positive x-axis and the positive y-axis, as shown in Figure 3. The unit of length on both axes is one metre. A uniform solid S is formed by rotating R through 360° about the x-axis.

(a) Show that the centre of mass of S is $\frac{5}{8}$ m from O.

(10)

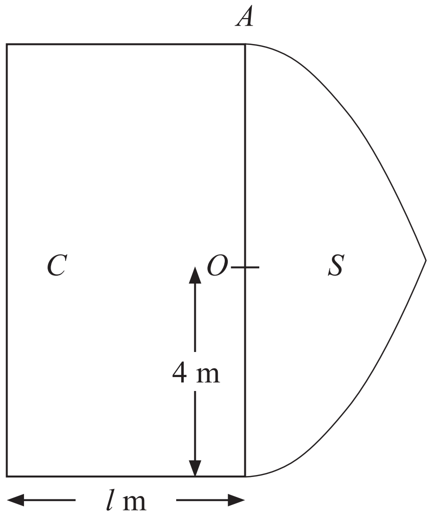

Figure 4

Figure 4 shows a cross section of a uniform solid P consisting of two components, a solid cylinder C and the solid S. The cylinder C has radius 4 m and length l metres. One end of C coincides with the plane circular face of S. The point A is on the circumference of the circular face common to C and S. When the solid P is freely suspended from A, the solid P hangs with its axis of symmetry horizontal.

(b) Find the value of l.

(4)

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Question 6 continued

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2. [*The centre of mass of a uniform hollow cone of height h is* $\frac{1}{3}h$ *above the base on the line from the centre of the base to the vertex*.]

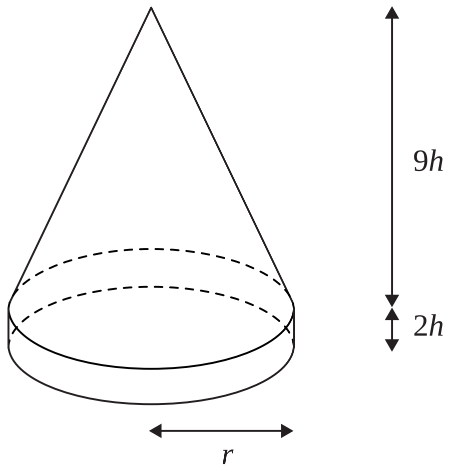

Figure 1

A marker for the route of a charity walk consists of a uniform hollow cone fixed on to a uniform solid cylindrical ring, as shown in Figure 1. The hollow cone has base radius r, height $9h$ and mass m. The solid cylindrical ring has outer radius r, height $2h$ and mass $3m$. The marker stands with its base on a horizontal surface.

(a) Find, in terms of h, the distance of the centre of mass of the marker from the horizontal surface.

(5)

When the marker stands on a plane inclined at $\arctan\frac{1}{12}$ to the horizontal it is on the point of toppling over. The coefficient of friction between the marker and the plane is large enough to be certain that the marker will not slip.

(b) Find h in terms of r.

(3)

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Question 2 continued

Q2

(Total 8 marks)

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4. The finite region bounded by the x-axis, the curve $y = \frac{1}{x^2}$, the line $x = \frac{1}{4}$ and the line $x = 1$, is rotated through one complete revolution about the x-axis to form a uniform solid of revolution.

(a) Show that the volume of the solid is 21π.

(4)

(b) Find the coordinates of the centre of mass of the solid.

(5)

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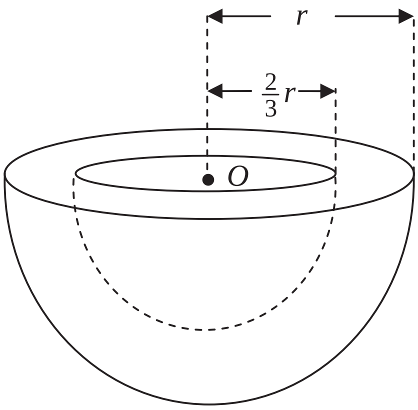

Figure 1

A bowl B consists of a uniform solid hemisphere, of radius r and centre O, from which is removed a solid hemisphere, of radius $\frac{2}{3}r$ and centre O, as shown in Figure 1.

(a) Show that the distance of the centre of mass of B from O is $\frac{65}{152}r$.

(5)

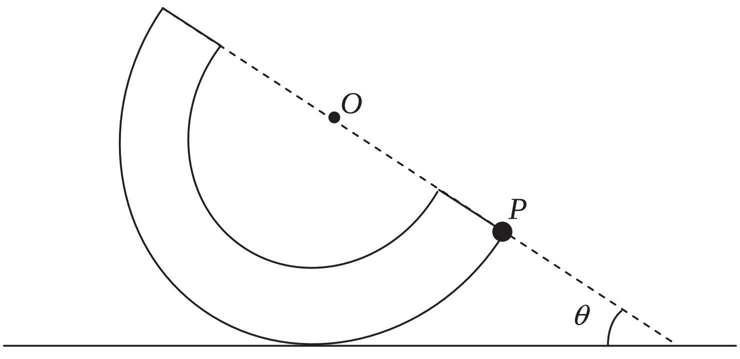

Figure 2

The bowl B has mass M. A particle of mass kM is attached to a point P on the outer rim of B. The system is placed with a point C on its outer curved surface in contact with a horizontal plane. The system is in equilibrium with P, O and C in the same vertical plane. The line OP makes an angle θ with the horizontal as shown in Figure 2. Given that

$\tan\theta = \frac{4}{5}$,

(b) find the exact value of k.

(5)

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Question 3 continued

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Question 3 continued

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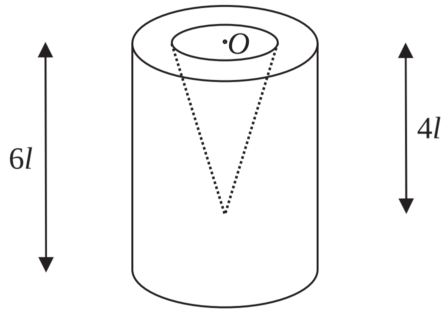

Figure 3

A container is formed by removing a right circular solid cone of height $4l$ from a uniform solid right circular cylinder of height $6l$. The centre O of the plane face of the cone coincides with the centre of a plane face of the cylinder and the axis of the cone coincides with the axis of the cylinder, as shown in Figure 3. The cylinder has radius $2l$ and the base of the cone has radius l.

(a) Find the distance of the centre of mass of the container from O.

(6)

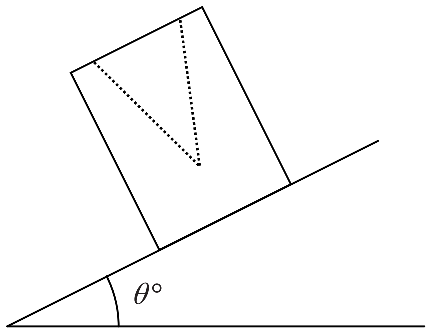

Figure 4

The container is placed on a plane which is inclined at an angle $\theta°$ to the horizontal. The open face is uppermost, as shown in Figure 4. The plane is sufficiently rough to prevent the container from sliding. The container is on the point of toppling.

(b) Find the value of θ.

(4)

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Question 4 continued

N 3 5 3 9 2 A 0 1 3 2 8

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2.

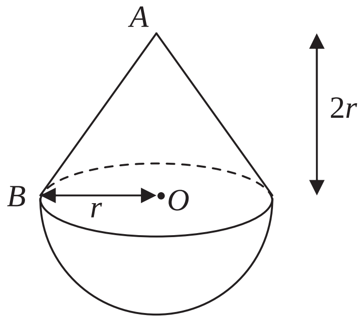

Figure 1

A toy is formed by joining a uniform solid hemisphere, of radius r and mass $4m$, to a uniform right circular solid cone of mass km. The cone has vertex A, base radius r and height $2r$. The plane face of the cone coincides with the plane face of the hemisphere. The centre of the plane face of the hemisphere is O and OB is a radius of its plane face as shown in Figure 1. The centre of mass of the toy is at O.

(a) Find the value of k.

(4)

A metal stud of mass λm is attached to the toy at A. The toy is now suspended by a light string attached to B and hangs freely at rest. The angle between OB and the vertical is 30°.

(b) Find the value of λ.

(4)

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Question 2 continued

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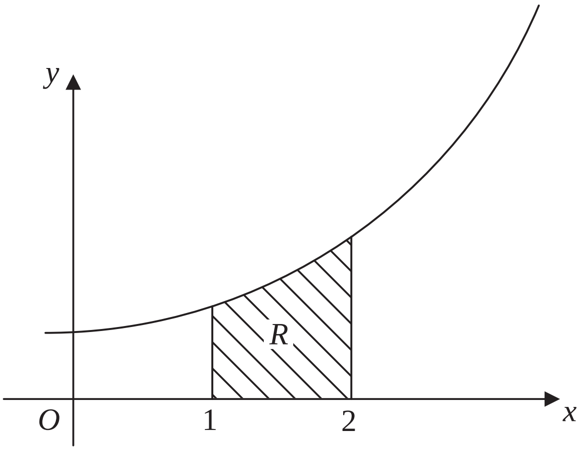

Figure 2

The region R is bounded by the curve with equation $y = e^x$, the line $x = 1$, the line $x = 2$ and the x-axis as shown in Figure 2. A uniform solid S is formed by rotating R through 2π about the x-axis.

(a) Show that the volume of S is $\frac{1}{2}\pi(e^4 - e^2)$.

(4)

(b) Find, to 3 significant figures, the x-coordinate of the centre of mass of S.

(6)

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Question 3 continued

N 3 5 4 0 9 A 0 9 2 8

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1. A particle P of mass 0.5 kg moves on the positive x-axis under the action of a single force directed towards the origin O. At time t seconds the distance of P from O is x metres, the magnitude of the force is $0.375x^2$ N and the speed of P is v m s^{-1}.

When $t = 0$, $OP = 8$ m and P is moving towards O with speed 2 m s^{-1}.

(a) Show that $v^2 = 260 - \frac{1}{2}x^3$.

(4)

(b) Find the distance of P from O at the instant when $v = 5$.

(2)

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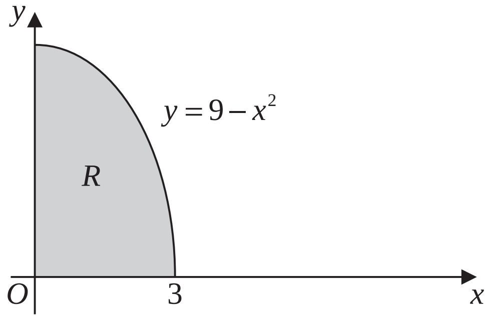

Figure 1

The shaded region R is bounded by the curve with equation $y = 9 - x^2$, the positive x-axis and the positive y-axis, as shown in Figure 1. A uniform solid S is formed by rotating R through 360° about the x-axis.

Find the x-coordinate of the centre of mass of S.

(9)

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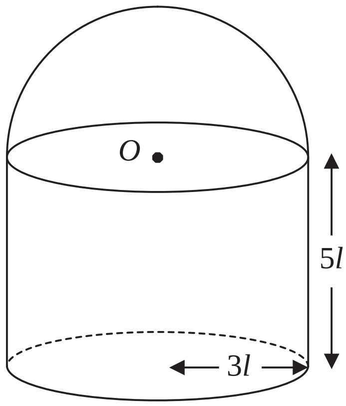

Figure 2

A solid consists of a uniform solid right cylinder of height $5l$ and radius $3l$ joined to a uniform solid hemisphere of radius $3l$. The plane face of the hemisphere coincides with a circular end of the cylinder and has centre O, as shown in Figure 2.

The density of the hemisphere is **twice** the density of the cylinder.

(a) Find the distance of the centre of mass of the solid from O.

(5)

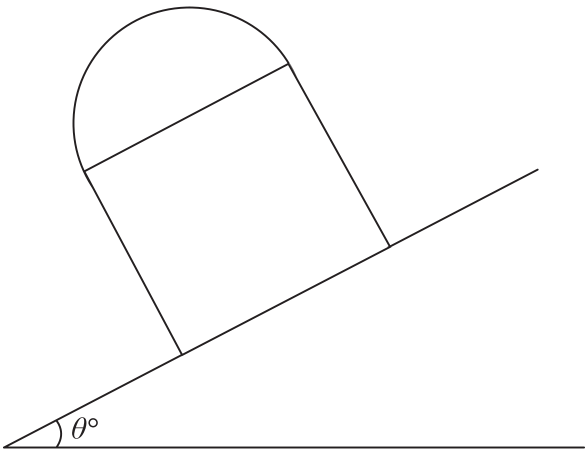

Figure 3

The solid is now placed with its circular face on a plane inclined at an angle $\theta°$ to the horizontal, as shown in Figure 3. The plane is sufficiently rough to prevent the solid slipping. The solid is on the point of toppling.

(b) Find the value of θ.

(4)

P 3 8 1 6 3 A 0 6 2 8

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7.

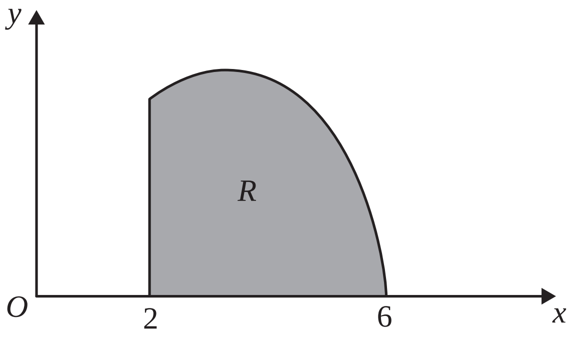

Diagram **NOT** accurately drawn

Figure 1

The shaded region R is bounded by the curve with equation $y = \frac{1}{2}x(6 - x)$, the x-axis and the line $x = 2$, as shown in Figure 1. The unit of length on both axes is 1 cm. A uniform solid P is formed by rotating R through 360° about the x-axis.

(a) Show that the centre of mass of P is, to 3 significant figures, 1.42 cm from its plane face.

(9)

The uniform solid P is placed with its plane face on an inclined plane which makes an angle θ with the horizontal. Given that the plane is sufficiently rough to prevent P from sliding and that P is on the point of toppling when $\theta = \alpha$,

(b) find the angle α.

(4)

Given instead that P is on the point of sliding down the plane when $\theta = \beta$ and that the coefficient of friction between P and the plane is 0.3,

(c) find the angle β.

(3)

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Question 7 continued

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4.

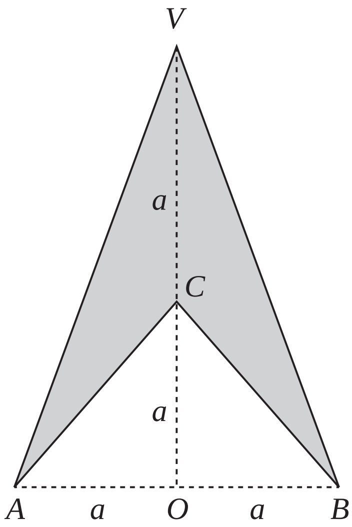

Figure 2

Figure 2 shows the cross-section *AVBC* of the solid *S* formed when a uniform right circular cone of base radius *a* and height *a*, is removed from a uniform right circular cone of base radius *a* and height 2*a*. Both cones have the same axis *VCO*, where *O* is the centre of the base of each cone.

(a) Show that the distance of the centre of mass of *S* from the vertex *V* is $\frac{5}{4}a$.

(5)

The mass of *S* is *M*. A particle of mass *kM* is attached to *S* at *B*. The system is suspended by a string attached to the vertex *V*, and hangs freely in equilibrium. Given that *VA* is at an angle 45° to the vertical through *V*,

(b) find the value of *k*.

(5)

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Question 4 continued

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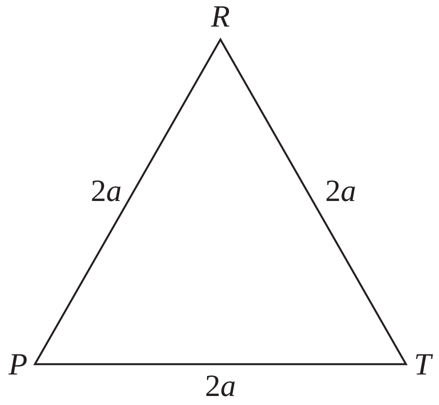

Figure 3

Figure 3 shows a uniform equilateral triangular lamina *PRT* with sides of length $2a$.

(a) Using calculus, prove that the centre of mass of *PRT* is at a distance $\frac{2\sqrt{3}}{3}a$ from *R*.
(6)

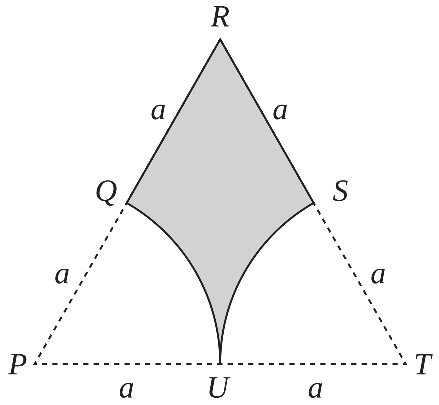

Figure 4

The circular sector *PQU*, of radius a and centre *P*, and the circular sector *TUS*, of radius a and centre *T*, are removed from *PRT* to form the uniform lamina *QRSU* shown in Figure 4.

(b) Show that the distance of the centre of mass of *QRSU* from *U* is $\frac{2a}{3\sqrt{3}-\pi}$
(6)

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Question 6 continued

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2.

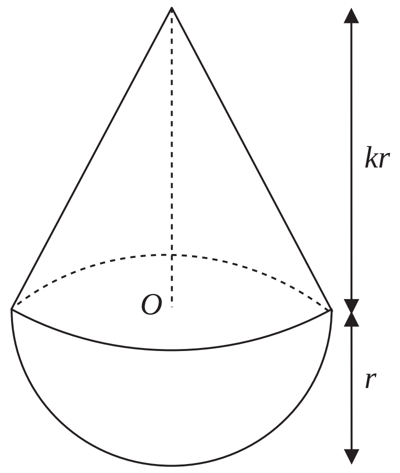

Figure 1

A uniform solid consists of a right circular cone of radius r and height kr, where $k > \sqrt{3}$, fixed to a hemisphere of radius r. The centre of the plane face of the hemisphere is O and this plane face coincides with the base of the cone, as shown in Figure 1.

(a) Show that the distance of the centre of mass of the solid from O is

$$\frac{(k^2 - 3)r}{4(k+2)}$$

(5)

The point A lies on the circumference of the base of the cone. The solid is suspended by a string attached at A and hangs freely in equilibrium. The angle between AO and the vertical is θ, where $\tan\theta = \frac{11}{14}$

(b) Find the value of k.

(4)

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Question 2 continued

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6. (a) A uniform lamina is in the shape of a quadrant of a circle of radius a. Show, by integration, that the centre of mass of the lamina is at a distance of $\frac{4a}{3\pi}$ from each of its straight edges.

(7)

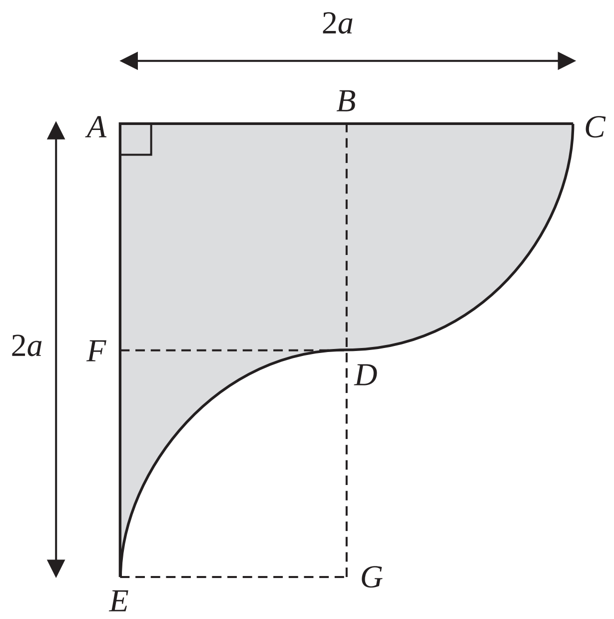

Figure 3

A second uniform lamina *ABCDEFA* is shown shaded in Figure 3. The straight sides *AC* and *AE* are perpendicular and $AC = AE = 2a$. In the figure, the midpoint of *AC* is *B*, the midpoint of *AE* is *F*, and *ABDF* and *DGEF* are squares of side a. *BCD* is a quadrant of a circle with centre *B*. *DGE* is a quadrant of a circle with centre *G*.

(b) Find the distance of the centre of mass of the lamina from the side *AE*.

(5)

The lamina is smoothly hinged to a horizontal axis which passes through *E* and is perpendicular to the plane of the lamina. The lamina has weight *W* newtons. The lamina is held in equilibrium in a vertical plane, with *A* vertically above *E*, by a horizontal force of magnitude *X* newtons applied at *C*.

(c) Find *X* in terms of *W*.

(3)

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Question 6 continued

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Question 6 continued

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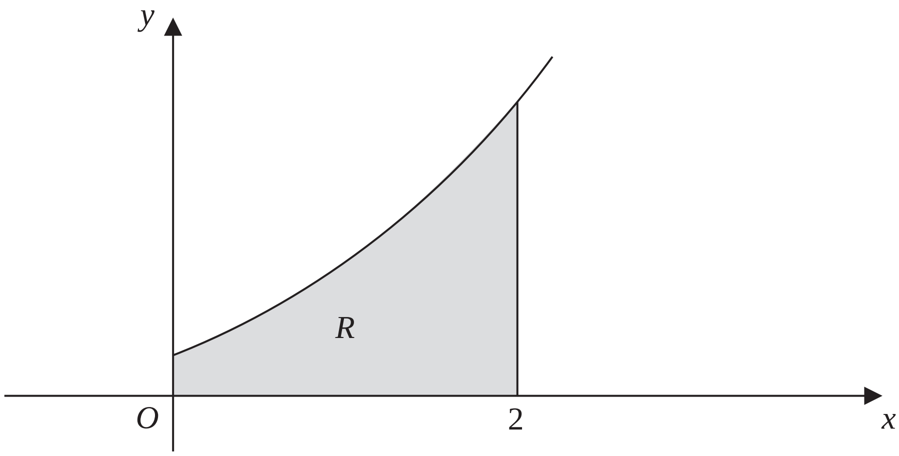

Figure 3

The shaded region R is bounded by the curve with equation $y = (x + 1)^2$, the x-axis, the y-axis and the line with equation $x = 2$, as shown in Figure 3. The region R is rotated through 2π radians about the x-axis to form a uniform solid S.

(a) Use algebraic integration to find the x coordinate of the centre of mass of S.

(8)

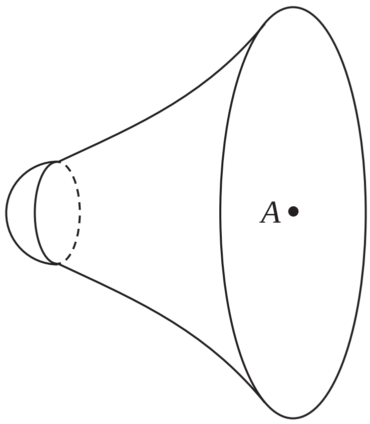

Figure 4

A uniform solid hemisphere is fixed to S to form a solid T. The hemisphere has the same radius as the smaller plane face of S and its plane face coincides with the smaller plane face of S, as shown in Figure 4. The mass per unit volume of the hemisphere is 10 times the mass per unit volume of S. The centre of the circular plane face of T is A. All lengths are measured in centimetres.

(b) Find the distance of the centre of mass of T from A.

(5)

Leave blank

Question 5 continued

Turn over