Edexcel Maths M3

Topic Questions from Papers

Circular Motion

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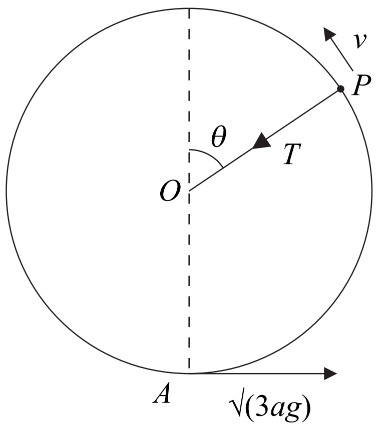
4. **Figure 2**



A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a point O. The point A is vertically below O, and $OA = a$. The particle is projected horizontally from A with speed $\sqrt{(3ag)}$. When OP makes an angle θ with the upward vertical through O and the string is still taut, the tension in the string is T and the speed of P is v, as shown in Figure 2.

(a) Find, in terms of a, g and θ, an expression for v^2.

(3)

(b) Show that $T = (1 - 3\cos\theta)mg$.

(3)

The string becomes slack when P is at the point B.

(c) Find, in terms of a, the vertical height of B above A.

(2)

After the string becomes slack, the highest point reached by P is C.

(d) Find, in terms of a, the vertical height of C above B.

(5)

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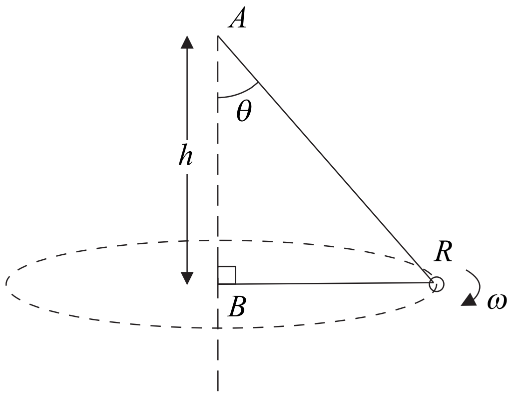
5. **Figure 3**



One end of a light inextensible string is attached to a fixed point A. The other end of the string is attached to a fixed point B, vertically below A, where $AB = h$. A small smooth ring R of mass m is threaded on the string. The ring R moves in a horizontal circle with centre B, as shown in Figure 3. The upper section of the string makes a constant angle θ with the downward vertical and R moves with constant angular speed ω. The ring is modelled as a particle.

(a) Show that $\omega^2 = \frac{g}{h}\left(\frac{1+\sin\theta}{\sin\theta}\right)$.

(7)

(b) Deduce that $\omega > \sqrt{\frac{2g}{h}}$.

(2)

Given that $\omega = \sqrt{\frac{3g}{h}}$,

(c) find, in terms of m and g, the tension in the string.

(4)

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Question 5 continued

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Question 7 continued

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4. A light inextensible string of length l has one end attached to a fixed point A. The other end is attached to a particle P of mass m. The particle moves with constant speed v in a horizontal circle with the string taut. The centre of the circle is vertically below A and the radius of the circle is r.

Show that

$$gr^2 = v^2\sqrt{(l^2 - r^2)}.$$

(9)

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6. A particle P is free to move on the smooth inner surface of a fixed thin hollow sphere of internal radius a and centre O. The particle passes through the lowest point of the spherical surface with speed U. The particle loses contact with the surface when OP is inclined at an angle α to the upward vertical.

(a) Show that $$U^2 = ag(2 + 3\cos\alpha).$$
(7)

The particle has speed W as it passes through the level of O. Given that $\cos\alpha = \frac{1}{\sqrt{3}}$,

(b) show that $$W^2 = ag\sqrt{3}.$$
(5)

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Question 6 continued

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5. A car of mass m moves in a circular path of radius 75 m round a bend in a road. The maximum speed at which it can move without slipping sideways on the road is 21 m s^{-1}. Given that this section of the road is horizontal,

(a) show that the coefficient of friction between the car and the road is 0.6.

(3)

The car comes to another bend in the road. The car's path now forms an arc of a horizontal circle of radius 44 m. The road is banked at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the car and the road is again 0.6. The car moves at its maximum speed without slipping sideways.

(b) Find, as a multiple of mg, the normal reaction between the car and road as the car moves round this bend.

(4)

(c) Find the speed of the car as it goes round this bend.

(5)

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Question 5 continued

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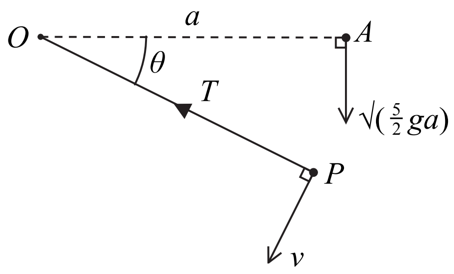
6. **Figure 2**



A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. At time $t = 0$, P is projected vertically downwards with speed $\sqrt{(\frac{5}{2}ga)}$ from a point A which is at the same level as O and a distance a from O. When the string has turned through an angle θ and the string is still taut, the speed of P is v and the tension in the string is T, as shown in Figure 2.

(a) Show that $v^2 = \dfrac{ga}{2}(5 + 4\sin\theta)$.

(3)

(b) Find T in terms of m, g and θ.

(3)

The string becomes slack when $\theta = \alpha$.

(c) Find the value of α.

(3)

The particle is projected again from A with the same velocity as before. When P is at the same level as O for the first time after leaving A, the string meets a small smooth peg B which has been fixed at a distance $\frac{1}{2}a$ from O. The particle now moves on an arc of a circle centre B. Given that the particle reaches the point C, which is $\frac{1}{2}a$ vertically above the point B, without the string going slack,

(d) find the tension in the string when P is at the point C.

(6)

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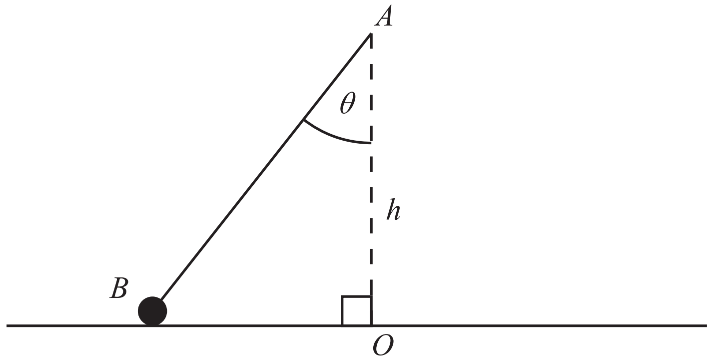

Figure 2

Figure 2 shows a particle B, of mass m, attached to one end of a light elastic string. The other end of the string is attached to a fixed point A, at a distance h vertically above a smooth horizontal table. The particle moves on the table in a horizontal circle with centre O, where O is vertically below A. The string makes a constant angle θ with the downward vertical and B moves with constant angular speed ω about OA.

(a) Show that $\omega^2 \leqslant \dfrac{g}{h}$.

(8)

The elastic string has natural length h and modulus of elasticity $2mg$.

Given that $\tan\theta = \dfrac{3}{4}$,

(b) find ω in terms of g and h.

(5)

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Question 3 continued

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5. A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is released from rest with the string taut and OP horizontal.

(a) Find the tension in the string when OP makes an angle of 60° with the downward vertical.

(6)

A particle Q of mass $3m$ is at rest at a distance a vertically below O. When P strikes Q the particles join together and the combined particle of mass $4m$ starts to move in a vertical circle with initial speed u.

(b) Show that $u = \sqrt{\left(\frac{ga}{8}\right)}$.

(3)

The combined particle comes to instantaneous rest at A.

(c) Find

(i) the angle that the string makes with the downward vertical when the combined particle is at A,

(ii) the tension in the string when the combined particle is at A.

(6)

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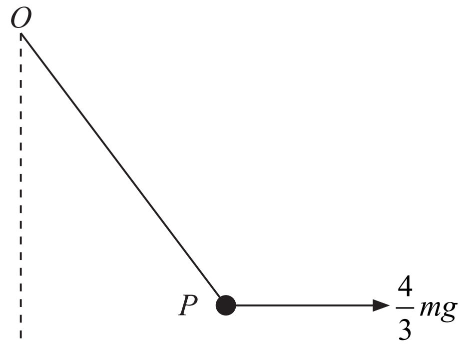

Figure 1

A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $3mg$. The other end of the string is attached to a fixed point O.

The particle P is held in equilibrium by a horizontal force of magnitude $\frac{4}{3}mg$ applied to P.

This force acts in the vertical plane containing the string, as shown in Figure 1. Find

(a) the tension in the string,

(5)

(b) the elastic energy stored in the string.

(4)

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3. A rough disc rotates about its centre in a horizontal plane with constant angular speed 80 revolutions per minute. A particle P lies on the disc at a distance 8 cm from the centre of the disc. The coefficient of friction between P and the disc is μ. Given that P remains at rest relative to the disc, find the least possible value of μ.

(7)

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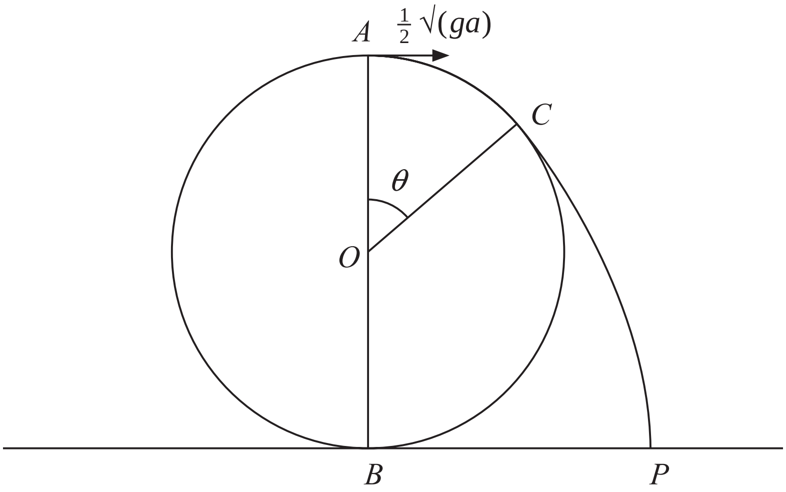

Figure 5

A particle is projected from the highest point A on the outer surface of a fixed smooth sphere of radius a and centre O. The lowest point B of the sphere is fixed to a horizontal plane. The particle is projected horizontally from A with speed $\frac{1}{2}\sqrt{(ga)}$. The particle leaves the surface of the sphere at the point C, where $\angle AOC = \theta$, and strikes the plane at the point P, as shown in Figure 5.

(a) Show that $\cos\theta = \frac{3}{4}$.

(7)

(b) Find the angle that the velocity of the particle makes with the horizontal as it reaches P.

(8)

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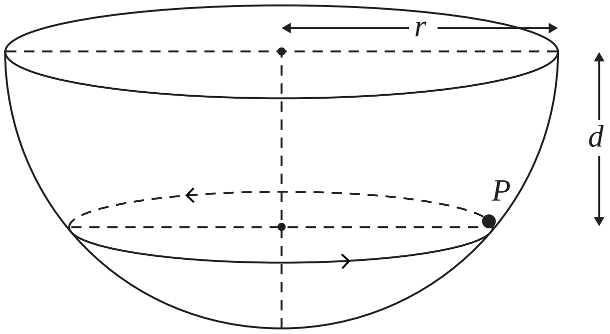

Figure 2

A particle P of mass m moves on the smooth inner surface of a hemispherical bowl of radius r. The bowl is fixed with its rim horizontal as shown in Figure 2. The particle moves with constant angular speed $\sqrt{\left(\frac{3g}{2r}\right)}$ in a horizontal circle at depth d below the centre of the bowl.

(a) Find, in terms of m and g, the magnitude of the normal reaction of the bowl on P.
(4)

(b) Find d in terms of r.
(4)

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Question 3 continued

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5. One end of a light inextensible string of length l is attached to a fixed point A. The other end is attached to a particle P of mass m, which is held at a point B with the string taut and AP making an angle $\arccos\frac{1}{4}$ with the downward vertical. The particle is released from rest. When AP makes an angle θ with the downward vertical, the string is taut and the tension in the string is T.

(a) Show that

$$T = 3mg\cos\theta - \frac{mg}{2}.$$

(6)

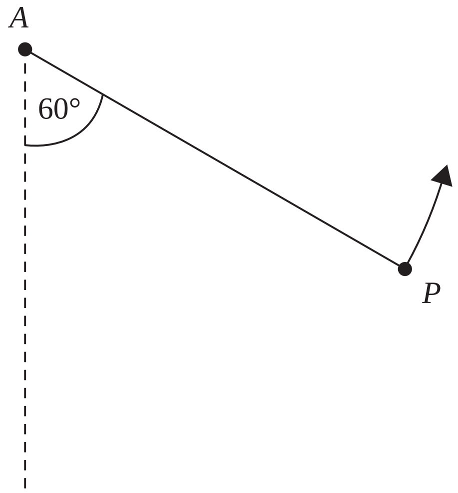

Figure 3

At an instant when AP makes an angle of 60° to the downward vertical, P is moving upwards, as shown in Figure 3. At this instant the string breaks. At the highest point reached in the subsequent motion, P is at a distance d below the horizontal through A.

(b) Find d in terms of l.

(5)

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Question 5 continued

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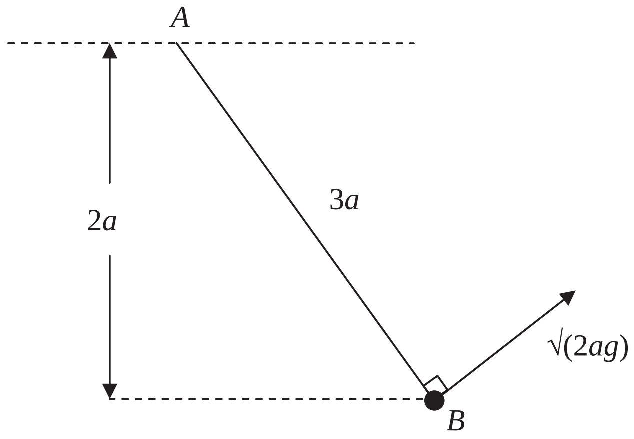

Figure 4

One end A of a light inextensible string of length $3a$ is attached to a fixed point. A particle of mass m is attached to the other end B of the string. The particle is held in equilibrium at a distance $2a$ below the horizontal through A, with the string taut. The particle is then projected with speed $\sqrt{(2ag)}$, in the direction perpendicular to AB, in the vertical plane containing A and B, as shown in Figure 4. In the subsequent motion the string remains taut. When AB is at an angle θ below the horizontal, the speed of the particle is v and the tension in the string is T.

(a) Show that $v^2 = 2ag(3\sin\theta - 1)$.

(5)

(b) Find the range of values of T.

(6)

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Question 5 continued

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6. A bend of a race track is modelled as an arc of a horizontal circle of radius 120 m. The track is not banked at the bend. The maximum speed at which a motorcycle can be ridden round the bend without slipping sideways is $28\,\mathrm{m\,s^{-1}}$. The motorcycle and its rider are modelled as a particle and air resistance is assumed to be negligible.

(a) Show that the coefficient of friction between the motorcycle and the track is $\frac{2}{3}$.

(6)

The bend is now reconstructed so that the track is banked at an angle α to the horizontal. The maximum speed at which the motorcycle can now be ridden round the bend without slipping sideways is $35\,\mathrm{m\,s^{-1}}$. The radius of the bend and the coefficient of friction between the motorcycle and the track are unchanged.

(b) Find the value of $\tan\alpha$.

(8)

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Question 6 continued

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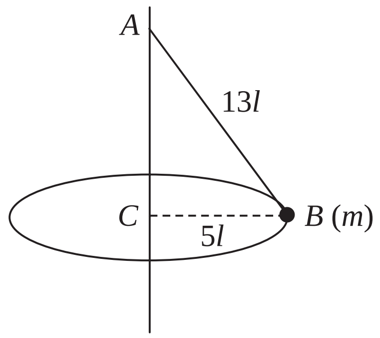

Figure 1

A garden game is played with a small ball B of mass m attached to one end of a light inextensible string of length $13l$. The other end of the string is fixed to a point A on a vertical pole as shown in Figure 1. The ball is hit and moves with constant speed in a horizontal circle of radius $5l$ and centre C, where C is vertically below A. Modelling the ball as a particle, find

(a) the tension in the string,

(3)

(b) the speed of the ball.

(4)

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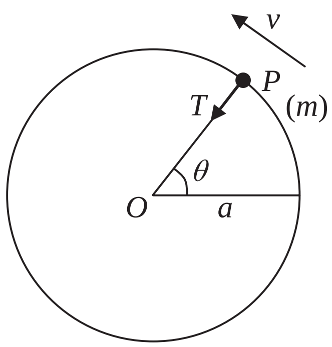

Figure 5

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is fixed at the point O. The particle is initially held with OP horizontal and the string taut. It is then projected vertically upwards with speed u, where $u^2 = 5ag$. When OP has turned through an angle θ the speed of P is v and the tension in the string is T, as shown in Figure 5.

(a) Find, in terms of a, g and θ, an expression for v^2.

(3)

(b) Find, in terms of m, g and θ, an expression for T.

(4)

(c) Prove that P moves in a complete circle.

(3)

(d) Find the maximum speed of P.

(2)

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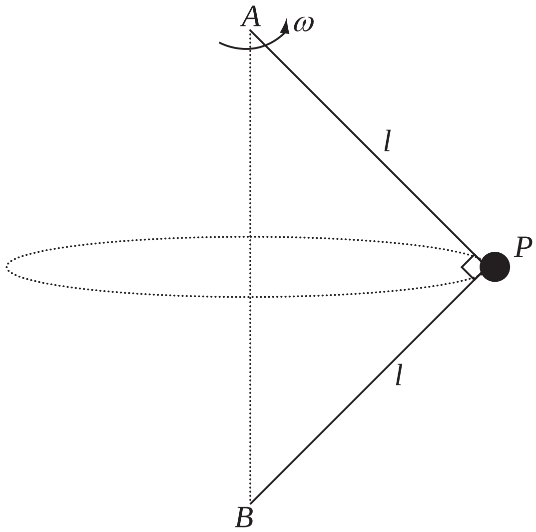

Figure 3

A small ball P of mass m is attached to the ends of two light inextensible strings of length l. The other ends of the strings are attached to fixed points A and B, where A is vertically above B. Both strings are taut and AP is perpendicular to BP as shown in Figure 3. The system rotates about the line AB with constant angular speed ω. The ball moves in a horizontal circle.

(a) Find, in terms of m, g, l and ω, the tension in AP and the tension in BP.

(8)

(b) Show that $\omega^2 > \dfrac{g\sqrt{2}}{l}$.

(2)

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Question 5 continued

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Question 6 continued

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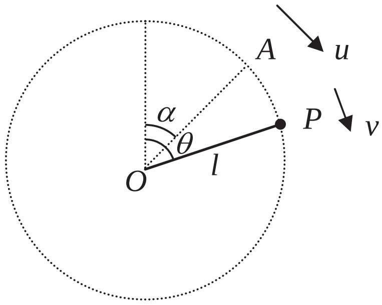

Figure 5

A particle P of mass m is attached to one end of a light rod of length l. The other end of the rod is attached to a fixed point O. The rod can turn freely in a vertical plane about O. The particle is projected with speed u from a point A, where OA makes an angle α with the upward vertical through O and $0 < \alpha < \frac{\pi}{2}$. When OP makes an angle θ with the upward vertical through O the speed of P is v as shown in Figure 5.

(a) Show that $v^2 = u^2 + 2gl(\cos\alpha - \cos\theta)$.

(4)

It is given that $\cos\alpha = \frac{3}{5}$ and that P moves in a complete vertical circle.

(b) Show that $u > 2\sqrt{\left(\frac{gl}{5}\right)}$.

(4)

As the rod rotates the least tension in the rod is T and the greatest tension is $5T$.

(c) Show that $u^2 = \frac{33}{10}gl$.

(9)

Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	7	9	/	0	1	Signature	

Paper Reference(s)

6679/01

Edexcel GCE

Mechanics M3

Advanced/Advanced Subsidiary

Thursday 16 June 2011 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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Question 3 continued

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5. A particle P of mass m is attached to one end of a light elastic string of natural length l and modulus of elasticity $3mg$. The other end of the string is attached to a fixed point O on a rough horizontal table. The particle lies at rest at the point A on the table, where $OA = \frac{7}{6}l$. The coefficient of friction between P and the table is μ.

(a) Show that $\mu \geqslant \frac{1}{2}$.

(4)

The particle is now moved along the table to the point B, where $OB = \frac{3}{2}l$, and released from rest. Given that $\mu = \frac{1}{2}$, find

(b) the speed of P at the instant when the string becomes slack,

(5)

(c) the total distance moved by P before it comes to rest again.

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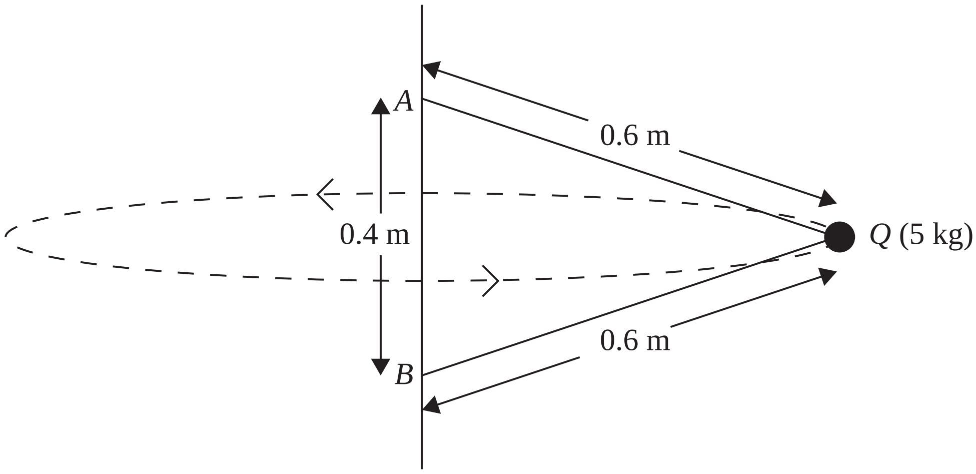

Figure 1

A particle Q of mass 5 kg is attached by two light inextensible strings to two fixed points A and B on a vertical pole. Each string has length 0.6 m and A is 0.4 m vertically above B, as shown in Figure 1.

Both strings are taut and Q is moving in a horizontal circle with constant angular speed 10 rad s^{-1}.

Find the tension in

(i) AQ,

(ii) BQ.

(10)

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Question 3 continued

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5. A fixed smooth sphere has centre O and radius a. A particle P is placed on the surface of the sphere at the point A, where OA makes an angle α with the upward vertical through O. The particle is released from rest at A. When OP makes an angle θ to the upward vertical through O, P is on the surface of the sphere and the speed of P is v.

Given that $\cos\alpha = \frac{3}{5}$

(a) show that

$$v^2 = \frac{2ga}{5}(3 - 5\cos\theta)$$

(4)

(b) find the speed of P at the instant when it loses contact with the sphere.

(8)

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Question 5 continued

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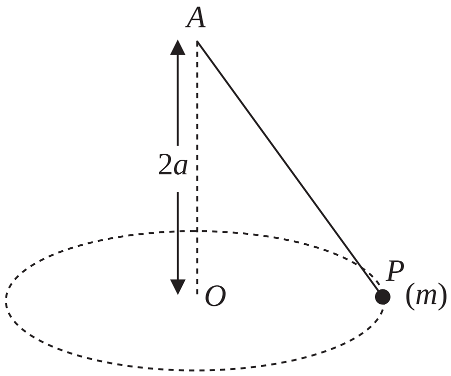

Figure 2

A particle P of mass m is attached to one end of a light elastic string, of natural length $2a$ and modulus of elasticity $6mg$. The other end of the string is attached to a fixed point A. The particle moves with constant speed v in a horizontal circle with centre O, where O is vertically below A and $OA = 2a$, as shown in Figure 2.

(a) Show that the extension in the string is $\frac{2}{5}a$.

(6)

(b) Find v^2 in terms of a and g.

(5)

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Question 4 continued

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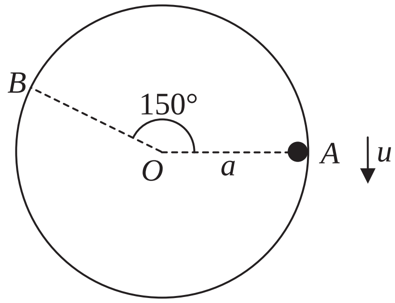

Figure 3

A smooth hollow cylinder of internal radius a is fixed with its axis horizontal. A particle P moves on the inner surface of the cylinder in a vertical circle with radius a and centre O, where O lies on the axis of the cylinder. The particle is projected vertically downwards with speed u from point A on the circle, where OA is horizontal. The particle first loses contact with the cylinder at the point B, where $\angle AOB = 150°$, as shown in Figure 3. Given that air resistance can be ignored,

(a) show that the speed of P at B is $\sqrt{\left(\frac{ag}{2}\right)}$,

(3)

(b) find u in terms of a and g.

(4)

After losing contact with the cylinder, P crosses the diameter through A at the point D. At D the velocity of P makes an angle $\theta°$ with the horizontal.

(c) Find the value of θ.

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Question 6 continued

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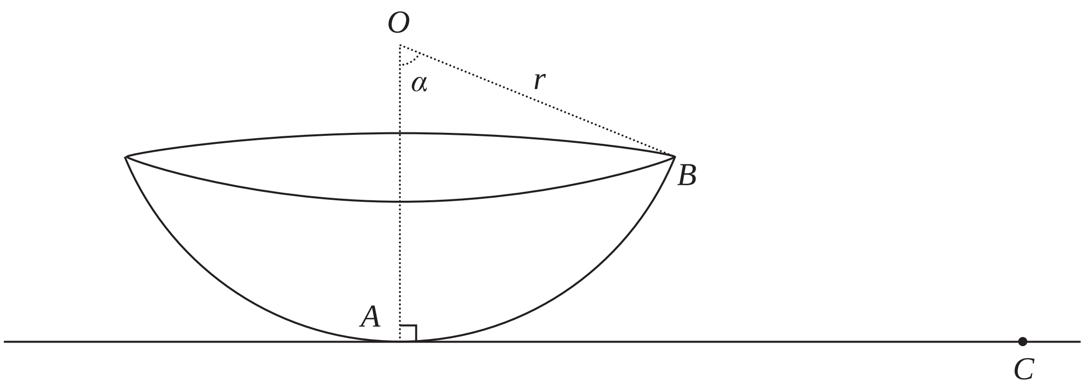

Figure 2

Part of a hollow spherical shell, centre O and radius r, forms a bowl with a plane circular rim. The bowl is fixed to a horizontal surface at A with the rim uppermost and horizontal.

The point A is the lowest point of the bowl. The point B, where $\angle AOB = \alpha$ and $\tan \alpha = \frac{3}{4}$, is on the rim of the bowl, as shown in Figure 2. A small smooth marble M is placed inside the bowl at A, and given an initial horizontal speed $\sqrt{(gr)}$. The motion of M takes place in the vertical plane OAB.

(a) Show that the speed of M as it reaches B is $\sqrt{\left(\frac{3}{5}gr\right)}$.

(4)

After leaving the surface of the bowl at B, M moves freely under gravity and first strikes the horizontal surface at the point C. Given that $r = 0.4$ m,

(b) find the distance AC.

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Question 5 continued

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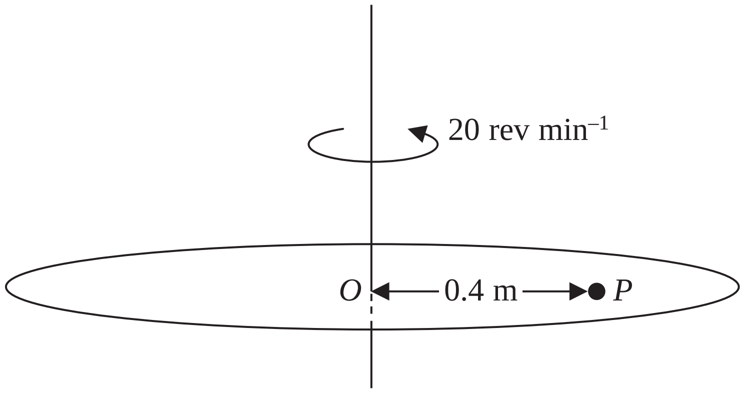

Figure 1

A rough disc is rotating in a horizontal plane with constant angular speed 20 revolutions per minute about a fixed vertical axis through its centre O. A particle P rests on the disc at a distance 0.4 m from O, as shown in Figure 1. The coefficient of friction between P and the disc is μ. The particle P is on the point of slipping.

Find the value of μ.

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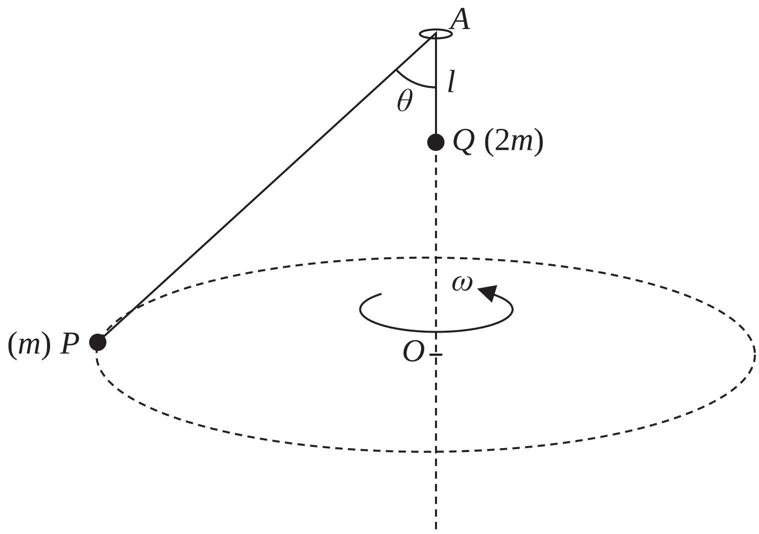

Figure 2

Two particles P and Q, of mass m and $2m$ respectively, are attached to the ends of a light inextensible string of length $6l$. The string passes through a small smooth fixed ring at the point A. The particle Q is hanging freely at a distance l vertically below A. The particle P is moving in a horizontal circle with constant angular speed ω. The centre O of the circle is vertically below A. The particle Q does not move and AP makes a constant angle θ with the downward vertical, as shown in Figure 2.

Show that

(i) $\theta = 60°$

(ii) $\omega = \sqrt{\left(\dfrac{2g}{5l}\right)}$

(8)

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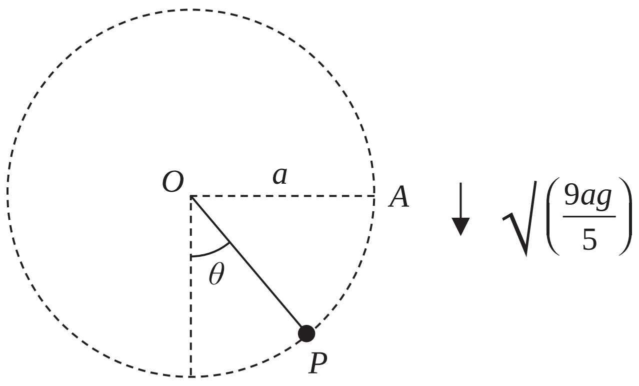

Figure 6

A particle P of mass $5m$ is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held at the point A, where $OA = a$ and OA is horizontal, as shown in Figure 6. The particle is projected vertically downwards with speed $\sqrt{\left(\frac{9ag}{5}\right)}$. When the string makes an angle θ with the downward vertical through O and the string is still taut, the tension in the string is T.

(a) Show that $T = 3mg\,(5\cos\theta + 3)$.

(6)

At the instant when the particle reaches the point B the string becomes slack.

(b) Find the speed of P at B.

(3)

At time $t = 0$, P is at B.

At time t, before the string becomes taut once more, the coordinates of P are (x, y) referred to horizontal and vertical axes with origin O. The x-axis is directed along OA produced and the y-axis is vertically upward.

(c) Find

(i) x in terms of t, a and g,

(ii) y in terms of t, a and g.

(7)

Leave blank

Question 7 continued

P 4 3 0 2 1 A 0 2 5 2 8