Edexcel Maths M3

Topic Questions from Papers

Strings & Springs

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3. A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $3.6mg$. The other end of the string is fixed at a point O on a rough horizontal table. The particle is projected along the surface of the table from O with speed $\sqrt{(2ag)}$. At its furthest point from O, the particle is at the point A, where $OA = \frac{4}{3}a$.

(a) Find, in terms of m, g and a, the elastic energy stored in the string when P is at A.

(3)

(b) Using the work-energy principle, or otherwise, find the coefficient of friction between P and the table.

(6)

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1. A light elastic string of natural length 0.4 m has one end A attached to a fixed point. The other end of the string is attached to a particle P of mass 2 kg. When P hangs in equilibrium vertically below A, the length of the string is 0.56 m.

(a) Find the modulus of elasticity of the string.

(3)

A horizontal force is applied to P so that it is held in equilibrium with the string making an angle θ with the downward vertical. The length of the string is now 0.72 m.

(b) Find the angle θ.

(3)

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4. A particle P of mass m lies on a smooth plane inclined at an angle 30º to the horizontal. The particle is attached to one end of a light elastic string, of natural length a and modulus of elasticity $2mg$. The other end of the string is attached to a fixed point O on the plane. The particle P is in equilibrium at the point A on the plane and the extension of the string is $\frac{1}{4}a$. The particle P is now projected from A down a line of greatest slope of the plane with speed V. It comes to instantaneous rest after moving a distance $\frac{1}{2}a$.

By using the principle of conservation of energy,

(a) find V in terms of a and g,

(6)

(b) find, in terms of a and g, the speed of P when the string first becomes slack.

(4)

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7. A particle P of mass 2 kg is attached to one end of a light elastic string, of natural length 1 m and modulus of elasticity 98 N. The other end of the string is attached to a fixed point A. When P hangs freely below A in equilibrium, P is at the point E, 1.2 m below A. The particle is now pulled down to a point B which is 0.4 m vertically below E and released from rest.

(a) Prove that, while the string is taut, P moves with simple harmonic motion about E with period $\frac{2\pi}{7}$ s.

(5)

(b) Find the greatest magnitude of the acceleration of P while the string is taut.

(1)

(c) Find the speed of P when the string first becomes slack.

(3)

(d) Find, to 3 significant figures, the time taken, from release, for P to return to B for the first time.

(7)

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Question 7 continued

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1.

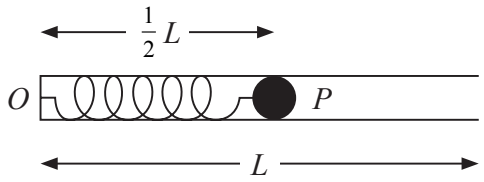

Figure 1

A light elastic spring, of natural length L and modulus of elasticity λ, has a particle P of mass m attached to one end. The other end of the spring is fixed to a point O on the closed end of a fixed smooth hollow tube of length L.

The tube is placed horizontally and P is held inside the tube with $OP = \frac{1}{2}L$, as shown in Figure 1. The particle P is released and passes through the open end of the tube with speed $\sqrt{(2gL)}$.

(a) Show that $\lambda = 8mg$.

(4)

The tube is now fixed vertically and P is held inside the tube with $OP = \frac{1}{2}L$ and P above O. The particle P is released and passes through the open top of the tube with speed u.

(b) Find u.

(5)

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5.

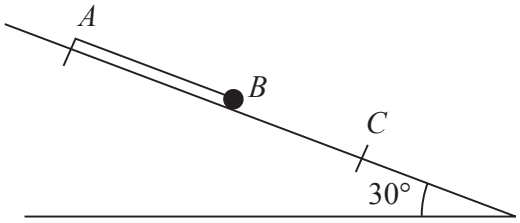

Figure 2

One end A of a light elastic string, of natural length a and modulus of elasticity $6mg$, is fixed at a point on a smooth plane inclined at 30° to the horizontal. A small ball B of mass m is attached to the other end of the string. Initially B is held at rest with the string lying along a line of greatest slope of the plane, with B below A and $AB = a$. The ball is released and comes to instantaneous rest at a point C on the plane, as shown in Figure 2. Find

(a) the length AC,

(5)

(b) the greatest speed attained by B as it moves from its initial position to C.

(7)

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Question 5 continued

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1. A light elastic string has natural length 8 m and modulus of elasticity 80 N.

The ends of the string are attached to fixed points P and Q which are on the same horizontal level and 12 m apart. A particle is attached to the mid-point of the string and hangs in equilibrium at a point 4.5 m below PQ.

(a) Calculate the weight of the particle.

(6)

(b) Calculate the elastic energy in the string when the particle is in this position.

(3)

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7. A light elastic string has natural length a and modulus of elasticity $\frac{3}{2}mg$. A particle P of mass m is attached to one end of the string. The other end of the string is attached to a fixed point A. The particle is released from rest at A and falls vertically. When P has fallen a distance $a + x$, where $x > 0$, the speed of P is v.

(a) Show that $v^2 = 2g(a+x) - \frac{3gx^2}{2a}$.

(4)

(b) Find the greatest speed attained by P as it falls.

(4)

After release, P next comes to instantaneous rest at a point D.

(c) Find the magnitude of the acceleration of P at D.

(6)

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Question 7 continued

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3.

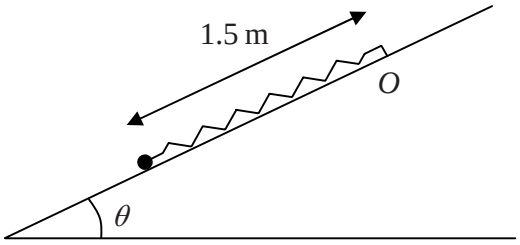

Figure 2

A particle of mass 0.5 kg is attached to one end of a light elastic spring of natural length 0.9 m and modulus of elasticity λ newtons. The other end of the spring is attached to a fixed point O on a rough plane which is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{3}{5}$. The coefficient of friction between the particle and the plane is 0.15. The particle is held on the plane at a point which is 1.5 m down the line of greatest slope from O, as shown in Figure 2. The particle is released from rest and first comes to rest again after moving 0.7 m up the plane.

Find the value of λ.

(9)

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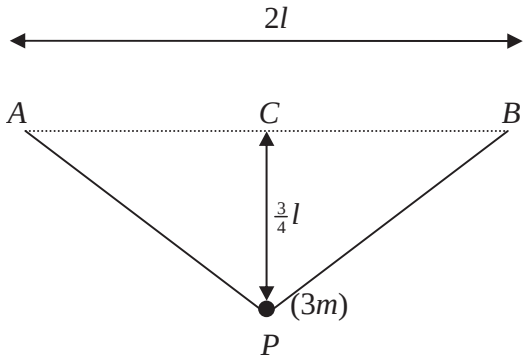

Figure 4

A small ball of mass $3m$ is attached to the ends of two light elastic strings AP and BP, each of natural length l and modulus of elasticity kmg. The ends A and B of the strings are attached to fixed points on the same horizontal level, with $AB = 2l$. The mid-point of AB is C. The ball hangs in equilibrium at a distance $\frac{3}{4}l$ vertically below C as shown in Figure 4.

(a) Show that $k = 10$

(7)

The ball is now pulled vertically downwards until it is at a distance $\frac{12}{5}l$ below C. The ball is released from rest.

(b) Find the speed of the ball as it reaches C.

(6)

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4.

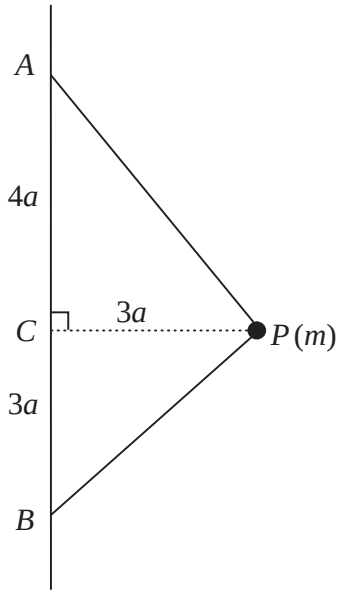

Figure 4

A light inextensible string has its ends attached to two fixed points A and B. The point A is vertically above B and $AB = 7a$. A particle P of mass m is fixed to the string and moves in a horizontal circle of radius $3a$ with angular speed ω. The centre of the circle is C where C lies on AB and $AC = 4a$, as shown in Figure 4. Both parts of the string are taut.

(a) Show that the tension in AP is $\frac{5}{7}m\left(3a\omega^2 + g\right)$.

(8)

(b) Find the tension in BP.

(2)

(c) Deduce that $\omega \geqslant \frac{1}{2}\sqrt{\left(\frac{g}{a}\right)}$.

(2)

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Question 4 continued

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6.

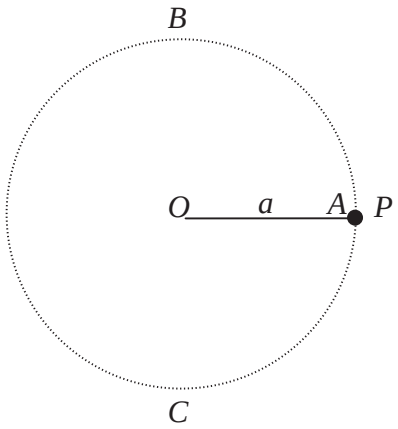

Figure 5

A particle P is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held at the point A, where $OA = a$ and OA is horizontal. The point B is vertically above O and the point C is vertically below O, with $OB = OC = a$, as shown in Figure 5. The particle is projected vertically upwards with speed $3\sqrt{(ag)}$.

(a) Show that P will pass through B.

(6)

(b) Find the speed of P as it reaches C.

(2)

As P passes through C it receives an impulse. Immediately after this, the speed of P is $\frac{5}{12}\sqrt{(11ag)}$ and the direction of motion of P is unchanged.

(c) Find the angle between the string and the downward vertical when P comes to instantaneous rest.

(4)

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Question 6 continued

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1. A particle of mass 0.8 kg is attached to one end of a light elastic string of natural length 0.6 m. The other end of the string is attached to a fixed point *A*. The particle is released from rest at *A* and comes to instantaneous rest 1.1 m below *A*.

Find the modulus of elasticity of the string.

(4)

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4. A light elastic string AB has natural length 0.8 m and modulus of elasticity 19.6 N. The end A is attached to a fixed point. A particle of mass 0.5 kg is attached to the end B. The particle is moving with constant angular speed ω rad s^{-1} in a horizontal circle whose centre is vertically below A. The string is inclined at 60° to the vertical.

(a) Show that the extension of the string is 0.4 m.

(5)

(b) Find the value of ω.

(5)

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Question 4 continued

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6. A particle P of mass m is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point O. The particle is hanging in equilibrium at the point A, vertically below O, when it is set in motion with a horizontal speed $\frac{1}{2}\sqrt{(11gl)}$. When the string has turned through an angle θ and the string is still taut, the tension in the string is T.

(a) Show that $T = 3mg\left(\cos\theta + \frac{1}{4}\right)$.

(8)

At the instant when P reaches the point B, the string becomes slack.
Find

(b) the speed of P at B,

(3)

(c) the maximum height above B reached by P before it starts to fall.

(4)

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Question 6 continued

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7. A particle B of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.75 m and modulus of elasticity 24.5 N. The other end of the string is attached to a fixed point A. The particle is hanging in equilibrium at the point E, vertically below A.

(a) Show that $AE = 0.9$ m.

(3)

The particle is held at A and released from rest. The particle first comes to instantaneous rest at the point C.

(b) Find the distance AC.

(5)

(c) Show that while the string is taut, B is moving with simple harmonic motion with centre E.

(4)

(d) Calculate the maximum speed of B.

(2)

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Question 7 continued

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7. A particle P of mass 1.5 kg is attached to the mid-point of a light elastic string of natural length 0.30 m and modulus of elasticity λ newtons. The ends of the string are attached to two fixed points A and B, where AB is horizontal and $AB = 0.48$ m. Initially P is held at rest at the mid-point, M, of the line AB and the tension in the string is 240 N.

(a) Show that $\lambda = 400$

(3)

The particle is now held at rest at the point C, where C is 0.07 m vertically below M. The particle is released from rest at C.

(b) Find the magnitude of the initial acceleration of P.

(6)

(c) Find the speed of P as it passes through M.

(6)

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Question 7 continued

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4. A particle P of mass 2 kg is attached to one end of a light elastic string of natural length 1.2 m. The other end of the string is attached to a fixed point O on a rough horizontal plane. The coefficient of friction between P and the plane is $\frac{2}{5}$. The particle is held at rest at a point B on the plane, where $OB = 1.5$ m. When P is at B, the tension in the string is 20 N. The particle is released from rest.

(a) Find the speed of P when $OP = 1.2$ m.

(7)

The particle comes to rest at the point C.

(b) Find the distance BC.

(2)

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Question 4 continued