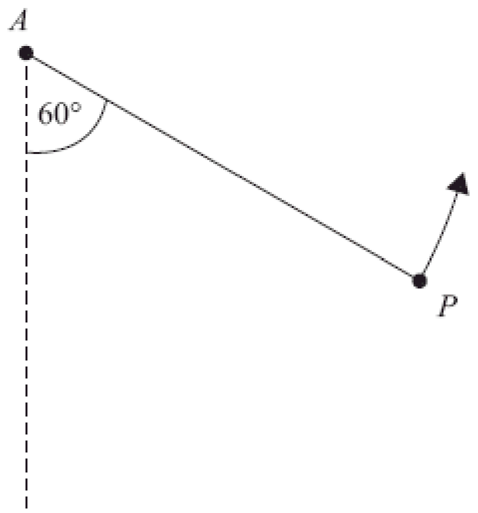
1. One end of a light inextensible string of length l is attached to a fixed point A. The other end is attached to a particle P of mass m, which is held at a point B with the string taut and AP making an angle arccos $\frac{1}{4}$ with the downward vertical. The particle is released from rest. When AP makes an angle θ with the downward vertical, the string is taut and the tension in the string is T.

(a) Show that

$$T = 3mg\cos\theta - \frac{mg}{2}.$$

(6)

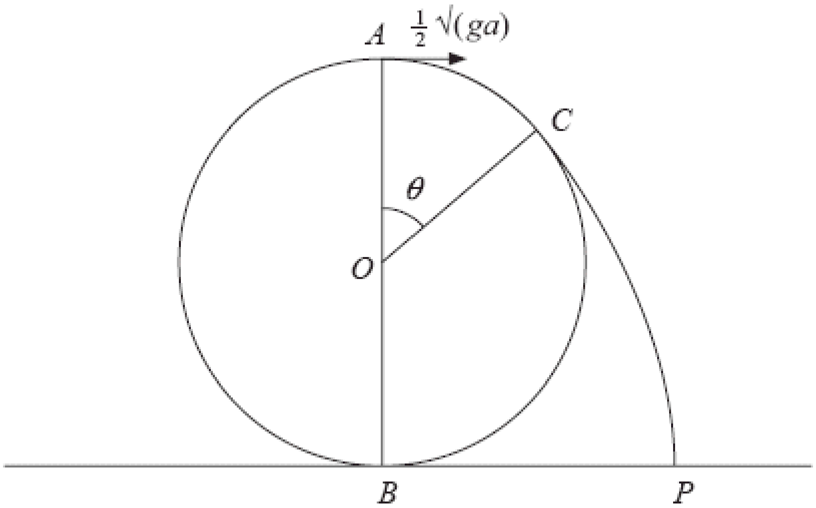


At an instant when AP makes an angle of 60° to the downward vertical, P is moving upwards, as shown in the diagram. At this instant the string breaks. At the highest point reached in the subsequent motion, P is at a distance d below the horizontal through A.

(b) Find d in terms of l.

(5)
(Total 11 marks)

2.



A particle is projected from the highest point A on the outer surface of a fixed smooth sphere of radius a and centre O. The lowest point B of the sphere is fixed to a horizontal plane. The particle is projected horizontally from A with speed $\frac{1}{2}\sqrt{(ga)}$. The particle leaves the surface of the sphere at the point C, where $\angle AOC = \theta,$ and strikes the plane at the point P, as shown in the diagram above.

(a) Show that $\cos \theta = \frac{3}{4}$.

(7)

(b) Find the angle that the velocity of the particle makes with the horizontal as it reaches P.

(8)

(Total 15 marks)

3. A smooth solid hemisphere is fixed with its plane face on a horizontal table and its curved surface uppermost. The plane face of the hemisphere has centre O and radius a. The point A is the highest point on the hemisphere. A particle P is placed on the hemisphere at A. It is then given an initial horizontal speed u, where $u^2 = \frac{1}{2}(ag)$. When OP makes an angle θ with OA, and while P remains on the hemisphere, the speed of P is v.

(a) Find an expression for v^2.

(3)

(b) Show that, when $\theta = \arccos 0.9$, P is still on the hemisphere.

(5)

(c) Find the value of $\cos\theta$ when P leaves the hemisphere.

(2)

(d) Find the value of v when P leaves the hemisphere.

(2)

After leaving the hemisphere P strikes the table at B.

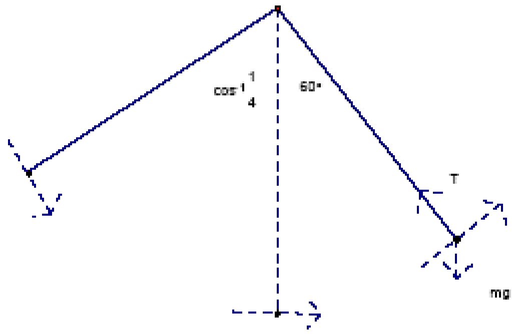
(e) Find the speed of P at B.

(2)

(f) Find the angle at which P strikes the table.

(3)
(Total 17 marks)

1. (a)



Energy:

$\left(\frac{1}{2}mu^2 + \right)mgl\left(\cos\theta - \frac{1}{4}\right) = \frac{1}{2}mv^2$ M1A1

Resolving:

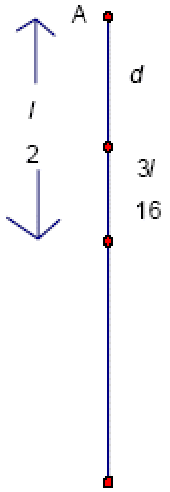
$T - mg\cos\theta = \frac{mv^2}{l}$ M1A1

Eliminate v^2:

$T = mg\cos\ \theta + \frac{1}{l}(2mgl(\cos\ \theta - \frac{1}{l}))$ M1

$T = 3mg\cos\ \theta - \frac{mg}{2}$ * A1

(b)



$\theta = 60° \Rightarrow mv^2 = 2mgl(\frac{1}{2} - \frac{1}{4})$ M1

$\Rightarrow v^2 = \frac{gl}{2}$

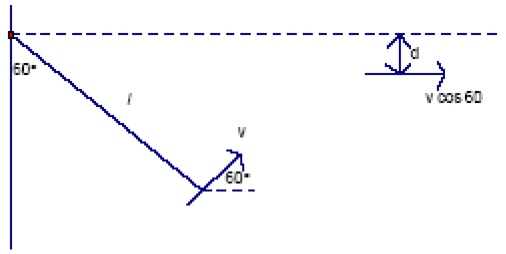
vertical motion under gravity:

$\updownarrow\ 0 = (v \cos 30°)^2 - 2gs$ M1

$0 = \frac{gl}{2} \times \frac{3}{2} - 2gs \Rightarrow s = \frac{3l}{16}$ A1

Distance below A = $\frac{l}{2} - \frac{3l}{16} = \frac{5l}{16}$ M1A1

<u>Alternative</u>



$\frac{1}{2}mv^2 - mgl\cos 60 = \frac{1}{2}m(v\cos 60)^2 - mgd$ M1A1

$\frac{gl}{4} - \frac{gl}{2} = \frac{gl}{4} \times \frac{1}{4} - gd$ M1

$d = \frac{1-4+8}{16}l = \frac{5l}{16}$ A1

[11]

2. (a) Let speed at C be u

CE $\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mga\,(1-\cos\theta)$ M1 A1

$u^2 = \frac{9ga}{4} - 2ga\cos\theta$

$mg\cos\theta\,(+R) = \frac{mu^2}{a}$ M1 A1

$mg\cos\theta = \frac{9mg}{4} - 2mg\cos\theta$ eliminating u M1

Leading to $\cos\theta = \frac{3}{4}*$ M1 A1 7

(b) At C $u^2 = \frac{9ga}{4} - 2ga \times \frac{3}{4} = \frac{3}{4}ga$ B1

$(\rightarrow)$ $u_x = u\cos\theta = \sqrt{\left(\frac{3ga}{4}\right)} \times \frac{3}{4} = \sqrt{\left(\frac{27ga}{64}\right)} = 2.033\sqrt{a}$ M1 A1ft

$(\downarrow)$ $u_y = u\cos\theta = \sqrt{\left(\frac{3ga}{4}\right)} \times \frac{\sqrt{7}}{4} = \sqrt{\left(\frac{21ga}{64}\right)} = 1.792\sqrt{a}$ M1

$v_y^2 = u_y^2 + 2gh \;\Rightarrow\; v_y^2 = \frac{21}{64}ga + 2g \times \frac{7}{4}a = \frac{245}{64}ga$ M1 A1

$\tan\psi = \frac{v_y}{u_x} = \sqrt{\left(\frac{245}{27}\right)} \approx 3.012...$ M1

$\psi \approx 72°$ awrt 72° A1 8

Or 1.3° (1.2502°) awrt 1.3°

Alternative for the last five marks

Let speed at P be v.

CE $\quad \frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mg \times 2a$ or equivalent M1

$v^2 = \frac{17mga}{4}$ M1 A1

$\cos\psi = \frac{u_x}{v} = \sqrt{\left(\frac{27}{64} \times \frac{4}{17}\right)} = \sqrt{\left(\frac{27}{272}\right)} \approx 0.315$ M1

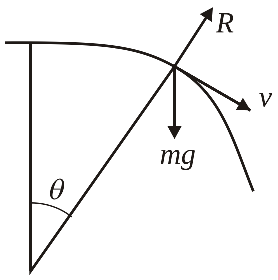
$\psi \approx 72°$ awrt 72° A1

<u>Note</u>

The time of flight from C to P is $\frac{\sqrt{235} - \sqrt{21}}{8}\sqrt{\left(\frac{a}{g}\right)} \approx 1.38373\sqrt{\left(\frac{a}{g}\right)}$

[15]

3. (a)



Energy $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(1 - \cos\theta)$ M1 A1

$v^2 = \frac{1}{2}ga + 2ga(1 - \cos\theta)$

$= \frac{1}{2}ga(5 - 4\cos\theta)$ A1 3

(b) $R(\nearrow)$: $mg\cos\theta - R = \dfrac{mv^2}{a}$ M1 A1

so $R = mg(3\cos\theta - \frac{5}{2})$ A1

$\cos\theta = 0.9 \Rightarrow R = mg(2.7 - 2.5)$ M1

$= 0.2\,mg > 0 \Rightarrow P$ still on hemisphere

P leaves hemisphere when $R = 0 \Rightarrow 3\cos\theta - \frac{5}{2} = 0 \Rightarrow \cos\theta = \frac{5}{6}$ A1 5

$\cos\theta = \frac{5}{6} \Rightarrow v^2 = \frac{1}{2}ga(5 - 4 \times \frac{5}{6})$ M1 A1 2

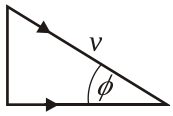
$= \dfrac{5ga}{6}, \quad v = \sqrt{\dfrac{5ga}{6}}$ A1 2

At B, speed v is given by $v^2 = u^2 + 2ga = \frac{5}{2}ga, \quad v = \sqrt{\dfrac{5ga}{2}}$ A1 2

A fter leaving hemisphere, horizontal component of velocity remains

constant $= \sqrt{\dfrac{5ga}{6}}\,\dfrac{5}{6}$ B1

$\cos\phi = \dfrac{\frac{5}{6}\sqrt{\frac{5ga}{6}}}{\sqrt{\frac{5ga}{2}}} = \dfrac{5}{6\sqrt{3}}$



M1

$\Rightarrow \phi = 61.2°$ or $61°$ to horizontal A1 3

[17]

1. Part (a) of this question was a fairly standard vertical circular motion question and many candidates could produce two equations by considering change in kinetic and potential energies and resolving along the radius. Some tried to resolve vertically with little success. The need to find a difference in potential energy between two points created problems for some candidates who seemed to think the particle either started at the top of the circle or from the horizontal level of A. The presence of a printed answer did enable some candidates to retrace their steps and correct the signs in their working. This was not a problem as long as the result was consistently correct. Part (b) produced a different set of problems. Some candidates thought that the string broke at the point where $T = 0$ and so found an incorrect value for the initial velocity of the projectile. The projectile problem could be solved by using vertical motion under gravity or by energy considerations. Many who opted for energy forgot that the particle was still moving (horizontally) at the highest point of its path; some of those who opted for vertical motion under gravity forgot that the initial velocity they had found needed to be resolved vertically before use in the appropriate equation. Most who had worked to this point in the question remembered to finish off by finding the distance below the horizontal through A.

2. Vertical circle questions usually present problems for many candidates and this one was no exception. Not all seem to be aware that an energy equation and an equation of motion along a radius (in this case at C) should be sufficient to make a sound start on the question. There were incorrect signs in the energy equation which were then adjusted later to arrive at the given result. Similarly, incorrect trigonometric functions became correct ones. Part (b) was attempted by most of those who had achieved success with (a) but the projectile motion defeated many. Some could not relate θ correctly to the horizontal and vertical components of the velocity at C. A variety of methods were seen in (b). Some used the separate components, finding the components at P and finishing off with the tangent of the required angle. Others used an energy approach, working from either A or C to obtain the final velocity at P and finished off with the horizontal component and the cosine. Some made extra work for themselves by finding the final velocity and the final vertical component, finishing off with the sine of the required angle. The usual mistakes such as using $v^2 = u^2 - 2as$ when energy was required occurred.

3. No Report available for this question.