1. A particle P of mass 0.5 kg moves along the positive x-axis. It moves away from the origin O under the action of a single force directed away from O. When $OP = x$ metres, the magnitude of the force is $\frac{3}{(x+1)^3}$ N and the speed of P is v m s^{-1}.

Initially P is at rest at O.

(a) Show that $v^2 = 6\left(1 - \frac{1}{(x+1)^2}\right)$.

(6)

(b) Show that the speed of P never reaches $\sqrt{6}$ m s^{-1}.

(1)

(c) Find x when P has been moving for 2 seconds.

(7)
(Total 14 marks)

1.	(a)	$F = ma\ (\rightarrow)\ \dfrac{3}{(x+1)^3} = 0.5a = 0.5\,v\dfrac{\mathrm{d}v}{\mathrm{d}x}$	M1A1	
		$\displaystyle\int \frac{3}{(x+1)^3}\mathrm{d}x = 0.5\int v\,\mathrm{d}v$ Separate and $\int$	M1	
		$-\dfrac{3}{2(x+1)^2} = \dfrac{1}{4}v^2\ (+c)$	A1	
		$x = 0,\ v = 0 \Rightarrow c' = -\dfrac{3}{2}\ \therefore\ v^2 = 6\left(1 - \dfrac{1}{(x+1)^2}\right)$*	M1A1cso	6
	(b)	$\forall x\ v^2 < 6\ \therefore\ v < \sqrt{6}\ \ (\because (x+1)^2 \text{ always} > 0)$	B1	1
	(c)	$v = \dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{\sqrt{6}\sqrt{(x+1)^2 - 1}}{x+1}$	M1	
		$\displaystyle\int \frac{x+1}{\sqrt{(x+1)^2 - 1}}\mathrm{d}x = \sqrt{6}\int \mathrm{d}t$	M1	
		$\sqrt{(x+1)^2 - 1} = \sqrt{6}t + c'$	M1A1	
		$t = 0,\ x = 0 \Rightarrow c' = 0$	M1	
		$t = 2 \Rightarrow (x+1)^2 - 1 = (2\sqrt{6})^2$	M1	
		$(x+1)^2 = 25 \Rightarrow x = 4$ (c' need not have been found)	A1 cao	7

[14]

1. This was a genuine "applied" mathematics question in the sense that most of the work was pure mathematics following from a little mechanics setting up the initial equations. Part (a) was usually completed successfully although a few forgot to include the mass in their original equation. The point of the question in (b) was lost on most candidates. They showed that v was never $\sqrt{6}$, by indicating that x had to be infinite for v to equal $\sqrt{6}$. However, they were asked to show that v never reached $\sqrt{6}$ so a correct inequality was needed to show that it was always less than $\sqrt{6}$. Part (c) was rarely completed correctly. Some candidates gave up at this point. Many candidates had no difficulty in seeing the necessity for replacing the acceleration in part (a) with $v\frac{dv}{dx}$ but then could not see the necessity for replacing v with $\frac{dx}{dt}$ in (c). Even if this hurdle was overcome separating the variables was beyond many – square root term by term and reciprocal term by term should not be seen at this level. Those who did the integration were fairly evenly divided between those who did it by inspection and those who used substitution.