1. A light elastic string, of natural length $3a$ and modulus of elasticity $6mg$, has one end attached to a fixed point A. A particle P of mass $2m$ is attached to the other end of the string and hangs in equilibrium at the point O, vertically below A.

(a) Find the distance AO.

(3)

The particle is now raised to point C vertically below A, where $AC > 3a$, and is released from rest.

(b) Show that P moves with simple harmonic motion of period $2\pi\sqrt{\left(\frac{a}{g}\right)}$.

(5)

It is given that $OC = \frac{1}{4}a$.

(c) Find the greatest speed of P during the motion.

(3)

The point D is vertically above O and $OD = \frac{1}{8}a$. The string is cut as P passes through D, moving upwards.

(d) Find the greatest height of P above O in the subsequent motion.

(4)
(Total 15 marks)

2. A particle P moves in a straight line with simple harmonic motion of period 2.4 s about a fixed origin O. At time t seconds the speed of P is v ms^{-1}. When $t = 0$, P is at O. When $t = 0.4$, $v = 4$. Find

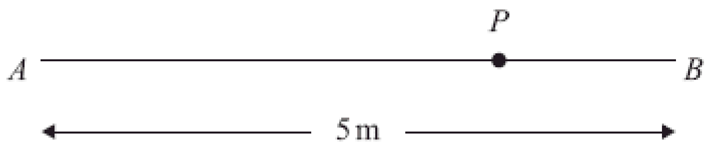
(a) the greatest speed of P,

(7)

(b) the magnitude of the greatest acceleration of P.

(2)
(Total 9 marks)

3.



A and B are two points on a smooth horizontal floor, where $AB = 5$ m.

A particle P has mass 0.5 kg. One end of a light elastic spring, of natural length 2 m and modulus of elasticity 16 N, is attached to P and the other end is attached to A. The ends of another light elastic spring, of natural length 1 m and modulus of elasticity 12 N, are attached to P and B, as shown in the diagram above.

(a) Find the extensions in the two springs when the particle is at rest in equilibrium.

(5)

Initially P is at rest in equilibrium. It is then set in motion and starts to move towards B. In the subsequent motion P does not reach A or B.

(b) Show that P oscillates with simple harmonic motion about the equilibrium position.

(4)

(c) Given that the initial speed of P is $\sqrt{10}$ m s^{-1}, find the proportion of time in each complete oscillation for which P stays within 0.25 m of the equilibrium position.

(7)
(Total 16 marks)

4. A small shellfish is attached to a wall in a harbour. The rise and fall of the water level is modelled as simple harmonic motion and the shellfish as a particle. On a particular day the minimum depth of water occurs at 10 00 hours and the next time that this minimum depth occurs is at 22 30 hours. The shellfish is fixed in a position 5 m above the level of the minimum depth of the water and 11 m below the level of the maximum depth of the water. Find

(a) the speed, in metres per hour, at which the water level is rising when it reaches the shellfish,

(7)

(b) the earliest time after 10 00 hours on this day at which the water reaches the shellfish.

(4)
(Total 11 marks)

5. A particle P moves with simple harmonic motion and comes to rest at two points A and B which are 0.24 m apart on a horizontal line. The time for P to travel from A to B is 1.5 s. The midpoint of AB is O. At time $t = 0$, P is moving through O, towards A, with speed u m s_{-1}.

(a) Find the value of u.

(4)

(b) Find the distance of P from B when $t = 2$ s.

(5)

(c) Find the speed of P when $t = 2$ s.

(2)
(Total 11 marks)

6. A particle P of mass 2 kg is attached to one end of a light elastic string, of natural length 1 m and modulus of elasticity 98 N. The other end of the string is attached to a fixed point A. When P hangs freely below A in equilibrium, P is at the point E, 1.2 m below A. The particle is now pulled down to a point B which is 0.4 m vertically below E and released from rest.

(a) Prove that, while the string is taut, P moves with simple harmonic motion about E with period $\frac{2\pi}{7}$ s.

(5)

(b) Find the greatest magnitude of the acceleration of P while the string is taut.

(1)

(c) Find the speed of P when the string first becomes slack.

(3)

(d) Find, to 3 significant figures, the time taken, from release, for P to return to B for the first time.

(7)
(Total 16 marks)

7. A particle P moves on the x-axis with simple harmonic motion about the origin O as centre. When P is a distance 0.04 m from O, its speed is 0.2 m s^{-1} and the magnitude of its acceleration is 1 m s^{-2}.

(a) Find the period of the motion.

(3)

The amplitude of the motion is a metres.

Find

(b) the value of a,

(3)

(c) the total time, within one complete oscillation, for which the distance OP is greater than $\frac{1}{2}a$ metres.

(5)
(Total 11 marks)

8. A particle P of mass 0.25 kg is attached to one end of a light elastic string. The string has natural length 0.8 m and modulus of elasticity λ N. The other end of the string is attached to a fixed point A. In its equilibrium position, P is 0.85 m vertically below A.

(a) Show that $\lambda = 39.2$.

(2)

The particle is now displaced to a point *B*, 0.95 m vertically below *A*, and released from rest.

(b) Prove that, while the string remains stretched, *P* moves with simple harmonic motion of period $\frac{\pi}{7}$ s.

(6)

(c) Calculate the speed of *P* at the instant when the string first becomes slack.

(3)

The particle first comes to instantaneous rest at the point *C*.

(d) Find, to 3 significant figures, the time taken for *P* to move from *B* to *C*.

(5)
(Total 16 marks)

9. A particle *P* of mass 0.2 kg oscillates with simple harmonic motion between the points *A* and *B*, coming to rest at both points. The distance *AB* is 0.2 m, and *P* completes 5 oscillations every second.

(a) Find, to 3 significant figures, the maximum resultant force exerted on *P*.

(6)

When the particle is at *A*, it is struck a blow in the direction *BA*. The particle now oscillates with simple harmonic motion with the same frequency as previously but twice the amplitude.

(b) Find, to 3 significant figures, the speed of the particle immediately after it has been struck.

(5)
(Total 11 marks)

10. The rise and fall of the water level in a harbour is modelled as simple harmonic motion. On a particular day the maximum and minimum depths of water in the harbour are 10 m and 4 m and these occur at 1100 hours and 1700 hours respectively.

(a) Find the speed, in m h^{-1}, at which the water level in the harbour is falling at 1600 hours on this particular day.

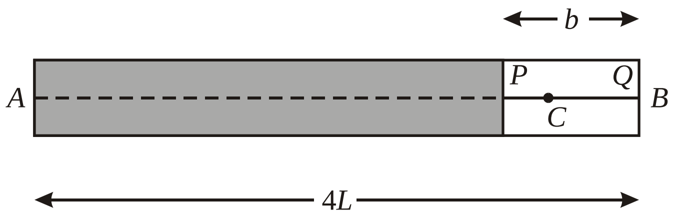
(9)

(b) Find the total time, between 1100 hours and 2300 hours on this particular day, for which the depth of water in the harbour is less than 5.5 m.

(5)
(Total 14 marks)

11.

Figure 1



In a game at a fair, a small target C moves horizontally with simple harmonic motion between the points A and B, where $AB = 4L$. The target moves inside a box and takes 3 s to travel from A to B. A player has to shoot at C, but C is only visible to the player when it passes a window PQ, where $PQ = b$. The window is initially placed with Q at the point B as shown in Figure 1. The target C takes 0.75 s to pass from Q to P.

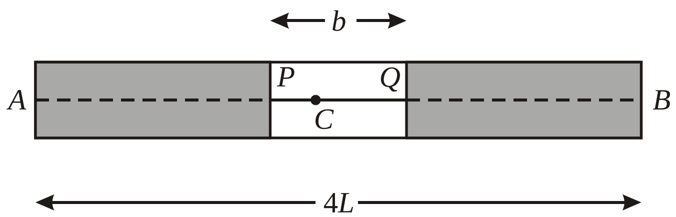
(a) Show that $b = (2 - \sqrt{2})L$.

(5)

(b) Find the speed of C as it passes P.

(2)

Figure 2



For advanced players, the window PQ is moved to the centre of AB so that $AP = QB$, as shown in Figure 2.

(c) Find the time, in seconds to 2 decimal places, taken for C to pass from Q to P in this new position.

(3)
(Total 10 marks)

12. A light spring of natural length L has one end attached to a fixed point A. A particle P of mass m is attached to the other end of the spring. The particle is moving vertically. As it passes through the point B below A, where $AB = L$, its speed is $\sqrt{(2gL)}$. The particle comes to instantaneous rest at a point C, $4L$ below A.

(a) Show that the modulus of elasticity of the spring is $\frac{8mg}{9}$.

(4)

At the point D the tension in the spring is mg.

(b) Show that P performs simple harmonic motion with centre D.

(5)

(c) Find, in terms of L and g,

(i) the period of the simple harmonic motion,

(ii) the maximum speed of P.

(5)
(Total 14 marks)

13. A particle P of mass 0.3 kg is attached to one end of a light elastic spring. The other end of the spring is attached to a fixed point O on a smooth horizontal table. The spring has natural length 2 m and modulus of elasticity 21.6 N. The particle P is placed on the table at the point A, where OA = 2 m. The particle P is now pulled away from O to the point B, where OAB is a straight line with OB = 3.5 m. It is then released from rest.

(a) Prove that P moves with simple harmonic motion of period $\frac{\pi}{3}$ s.

(4)

(b) Find the speed of P when it reaches A.

(2)

The point C is the mid-point of AB.

(c) Find, in terms of π, the time taken for P to reach C for the first time.

(3)

Later in the motion, P collides with a particle Q of mass 0.2 kg which is at rest at A.

After the impact, P and Q coalesce to form a single particle R.

(d) Show that R also moves with simple harmonic motion and find the amplitude of this motion.

(7)
(Total 16 marks)

14. A piston in a machine is modelled as a particle of mass 0.2 kg attached to one end A of a light elastic spring, of natural length 0.6 m and modulus of elasticity 48 N. The other end B of the spring is fixed and the piston is free to move in a horizontal tube which is assumed to be smooth. The piston is released from rest when AB = 0.9 m.

(a) Prove that the motion of the piston is simple harmonic with period $\frac{\pi}{10}$ s.

(5)

(b) Find the maximum speed of the piston.

(2)

(c) Find, in terms of π, the length of time during each oscillation for which the length of the spring is less than 0.75 m.

(5)
(Total 12 marks)

15. A particle P of mass 0.8 kg is attached to one end A of a light elastic spring OA, of natural length 60 cm and modulus of elasticity 12 N. The spring is placed on a smooth horizontal table and the end O is fixed. The particle P is pulled away from O to a point B, where OB = 85 cm, and is released from rest.

(a) Prove that the motion of P is simple harmonic with period $\dfrac{2\pi}{5}$ s.

(5)

(b) Find the greatest magnitude of the acceleration of P during the motion.

(2)

Two seconds after being released from rest, P passes through the point C.

(c) Find, to 2 significant figures, the speed of P as it passes through C.

(5)

(d) State the direction in which P is moving 2 s after being released.

(1)

(Total 13 marks)

16. A piston P in a machine moves in a straight line with simple harmonic motion about a point O, which is the centre of the oscillations. The period of the oscillations is π s. When P is 0.5 m from O, its speed is 2.4 m s^{-1}. Find

(a) the amplitude of the motion,

(4)

(b) the maximum speed of P during the motion,

(1)

(c) the maximum magnitude of the acceleration of P during the motion,

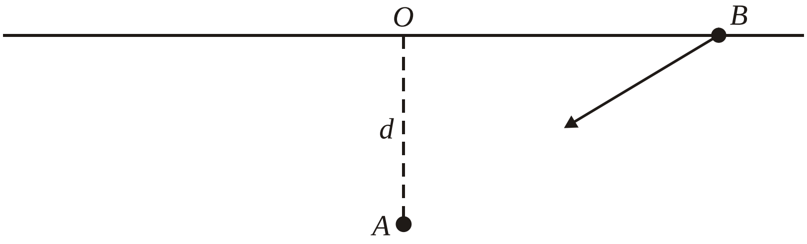
(1)

(d) the total time, in s to 2 decimal places, in each complete oscillation for which the speed of P is greater than 2.4 m s^{-1}.

(5)

(Total 11 marks)

17.



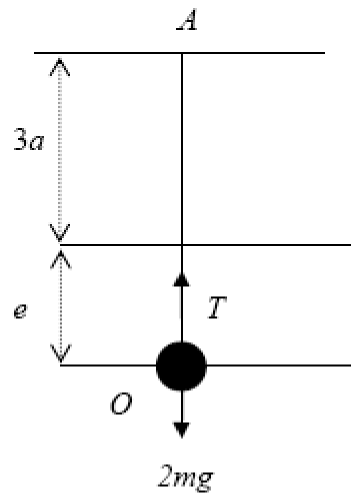
A small smooth bead B of mass 0.2 kg is threaded on a smooth horizontal wire. The point A is on the same horizontal level as the wire and at a perpendicular distance d from the wire. The point O is the point on the wire nearest to A, as shown in the diagram above. The bead experiences a force of magnitude $5(AB)$ newtons in the direction BA towards A. Initially B is at rest with $OB = 2$ m.

(a) Prove that B moves with simple harmonic motion about O, with period $\frac{2\pi}{5}$ s.
(5)

(b) Find the greatest speed of B in the motion.
(2)

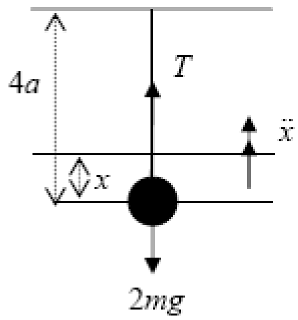
(c) Find the time when B has first moved a distance 3 m from its initial position.
(4)
(Total 11 marks)

1. (a)



	R(↑) $T = 2mg$	B1	
	Hooke's law: $T = \dfrac{6mge}{3a}$		
	$2mg = \dfrac{6mge}{3a}$	M1	
	$e = a$		
	$AO = 4a$	A1	3

(b)



H.L.	$T = \dfrac{6mg\left(a-x\right)}{3a} = \dfrac{2mg\left(a-x\right)}{a}$	B1ft	
Eqn. of motion	$-2mg + T = 2m\ddot{x}$	M1	
	$-2mg + \dfrac{2mg\left(a-x\right)}{a} = 2m\ddot{x}$	M1	
	$-\dfrac{2mgx}{a} = 2m\ddot{x}$		
	$\ddot{x} = -\dfrac{g}{a}x$	A1	
	period $2\pi\sqrt{\dfrac{a}{g}}$ *	A1	5

(c) $v^2 = \omega^2\left(a^2 - x^2\right)$

$v_{\max}{}^2 = \dfrac{g}{a}\left(\left(\dfrac{a}{4}\right)^2 - 0\right)$ M1 A1

$v_{\max} = \dfrac{1}{4}\sqrt{(ga)}$ A1 3

(d) $x = -\dfrac{a}{8}$ $v^2 = \dfrac{g}{a}\left(\dfrac{a^2}{16} - \dfrac{a^2}{64}\right)$ M1

$= \dfrac{3ag}{64}$

$v^2 = u^2 + 2as$ M1

$0 = \dfrac{3ag}{64} - 2gh$ A1

$h = \dfrac{3a}{128}$

Total height above $O = \dfrac{a}{8} + \dfrac{3a}{128} = \dfrac{19a}{128}$ A1 4

[15]

2. (a) $\dfrac{2\pi}{\omega} = 2.4 \Rightarrow \omega = \dfrac{5\pi}{6}\left(\approx 2.62\right)$ M1 A1

$x = 0, t = 0 \quad \Rightarrow \quad x = a \sin \omega t$

when $t = 0.4$, $x = a \sin\left(\dfrac{5\pi}{6} \times 0.4\right)$ $\left(= \dfrac{\sqrt{3}}{2} a\right)$ M1

$v^2 = \omega^2\left(a^2 - x^2\right) \Rightarrow 16 = \dfrac{25\pi^2}{36}\left(a^2 - \dfrac{3a^2}{4}\right) \Rightarrow a = \dfrac{48}{5\pi}\left(\approx 3.06\right)$ M1 A1

$v_{\max} = a\omega = 8$ (or awrt 8.0 if decimals used earlier) cao M1 A1 7

<u>Alternative</u>

$\frac{2\pi}{\omega} = 2.4 \Rightarrow \omega = \frac{5\pi}{6}$ M1 A1

$x = 0, t = 0 \;\Rightarrow\; x = a \sin \omega t$

$\dot{x} = a\omega \cos \omega t$ M1

$4 = a\omega \cos\left(\frac{5\pi}{6} \times 0.4\right)$ M1

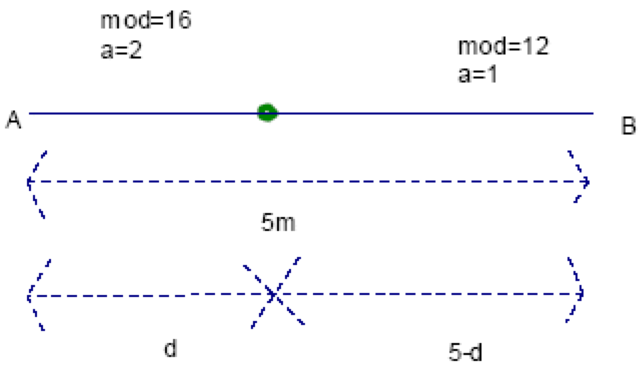
$a = \frac{48}{5\pi} (\approx 3.06)$ or $a\omega = 8$ A1

$v_{max} = a\omega = 8$ M1 A1

(b) $\ddot{x}_{max} = a\omega^2 = \frac{20\pi}{3}$ awrt 21 M1 A1 2

[9]

3. (a)



Hooke's law: Equilibrium $\Rightarrow \frac{16(d-2)}{2} = \frac{12(4-d)}{1}$ M1A1A1

$\Rightarrow\; d = 3.2$ A1

so extensions are 1.2m and 0.8m. A1

(b) If the particle is displaced distance x towards ***B*** then M1A1ft

$-m\ddot{x} = \frac{16(1.2+x)}{2} - \frac{12(0.8-x)}{1} (= 20x)$ A1ft

$\Rightarrow \ddot{x} = -40x$ or $\ddot{x} = -\frac{20}{m} (\Rightarrow \text{SHM})$ A1

(c) $T = \dfrac{2\pi}{\sqrt{40}}$ B1ft

$a = \dfrac{\sqrt{10}}{\text{their}\,\omega}$ B1ft

$x = a \sin \omega t$ their a, their ω M1

$\dfrac{1}{4} = \dfrac{1}{2}\sin\sqrt{40}t$ A1

$\sqrt{40}t = \dfrac{\pi}{6} \left(\Rightarrow t = \dfrac{\pi}{6\sqrt{40}}\right)$ M1

Proportion $\dfrac{4t}{T} = \dfrac{4\pi}{6\sqrt{40}} \times \dfrac{\sqrt{40}}{2\pi} = \dfrac{1}{3}$ M1A1

[16]

4. (a) $a = 8$ B1

$T = \dfrac{25}{2} = \dfrac{2\pi}{\omega} \Rightarrow \omega = \dfrac{4\pi}{25} (\approx 0.5.2....)$ M1 A1

$v^2 = \omega^2 (a^2 - x^2) \Rightarrow v^2 = \left(\dfrac{4\pi}{25}\right)^2 (8^2 - 3^2)$ ft their a, ω M1 A1ft

$v = \dfrac{4\pi}{25}\sqrt{55} \approx 3.7 \ (\text{mh}^{-1})$ awrt 3.7 M1 A1 7

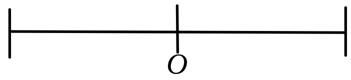
(b) $x = a\cos\omega t \Rightarrow 3 = 8\cos\left(\dfrac{4\pi}{25}t\right)$ ft their a, ω M1 A1ft

$t \approx 2.3602 \ldots$ M1

time is 12 22 A1 4

[11]

5. (a)



A *B*

$T = 3 = \frac{2\pi}{\omega} \therefore \omega = \frac{2\pi}{3}$ M1A1

$u^2 = \omega^2 (a^2 - x^2)$; $a = 0.12$, $u^2 = a^2 \omega^2$, $u = 0.12 \times \omega$ M1

$= 0.251$ m s^{-1} (0.25 m s^{-1}) A1 4

(b) Time from $O \to A \to O = 1.5$ s $\therefore t = 0.5$ B1

$x = a \sin \omega t \Rightarrow OP = 0.12 \sin\left(\frac{\pi}{3}\right)$ M1A1

Distance from *B* is $0.12 - OP = 0.12 - 0.104\ldots = 0.016$m M1A1 5

(c) $v^2 = \omega^2(a^2 - x^2)$ M1

$v = \frac{2\pi}{3}\sqrt{0.12^2 - 0.104...^2} = \frac{2\pi}{3} \times 0.0598 = 0.13$ ms^{-1} A1 2

[11]

6. (a) (Measuring *x* from *E*) $2\ddot{x} = 2g - 98(x + 0.2)$, and so $\ddot{x} = -49x$ M1 A1, A1

SHM period with $\omega^2 = 49$ so $T = \frac{2\pi}{7}$ dM1 A1cso 5

DM1 requires the minus sign.
Special case
$2\ddot{x} = 2g - 98x$ is M1A1A0M0A0 $2\ddot{x} = -98x$ is M0A0A0M0A0
No use of $\ddot{x}$, just *a* is M1 A0,A0 then M1 A0 if otherwise correct.
Quoted results are not acceptable.

(b) Max. acceleration = 49 × max. $x = 49 \times 0.4 = 19.6$ m s^{-2} B1 1

Answer must be positive and evaluated for B1

(c) String slack when $x = -0.2$: $v^2 = 49(0.4^2 - 0.2^2)$ M1 A1

$\Rightarrow v \approx 2.42 \text{ m s}^{-1} = \dfrac{7\sqrt{3}}{5}$ A1 3

M1 – Use correct formula with their ω, a and x but **not** $x = 0$.
A1 Correct values but allow $x = +0.2$
Alternative
It is possible to use energy instead to do this part

$\frac{1}{2}mv^2 + mg \times 0.6 = \frac{\lambda \times 0.6^2}{2l}$ M1 A1

(d) Uses $x = a \cos \omega t$ or use $x = a \sin \omega t$ but not with $x = 0$ or $\pm a$ M1
Attempt complete method for finding time when string goes

slack $-0.2 = 0.4 \cos 7t \Rightarrow \cos 7t = -\dfrac{1}{2}$ dM1 A1

$t = \dfrac{2\pi}{21} \approx 0.299$ s A1

Time when string is slack = $\dfrac{(2)\times 2.42}{g} = \dfrac{2\sqrt{3}}{7} \approx 0.495$ s

(2 needed for A) M1 A1ft
Total time = $2 \times 0.299 + 0.495 \approx 1.09$ s A1 7

If they use $x = a \sin \omega t$ with $x = \pm 0.2$ and add $\dfrac{\pi}{7}$ or $\dfrac{\pi}{14}$ this is dM1,
A1 if done correctly
If they use $x = a \cos \omega t$ with $x = -0.2$ this is dM1, then A1
(as in scheme)
If they use $x = \text{a} \cos \omega t$ with $x = +0.2$ this needs *their* $\dfrac{\pi}{7}$ minus
answer to reach dM1, then A1

[16]

7. (a) $\ddot{x} = -\omega^2 x \Rightarrow 1 = \omega^2 \times 0.04 (\Rightarrow \omega = 5)$ M1A1

$T = \dfrac{2\pi}{5}$ awrt 1.3 A1 3

(b) $v^2 = \omega^2(a^2 - x^2) \Rightarrow 0.2^2 = 5^2(a^2 - 0.04^2)$ ft their ω M1A1ft

$a = \dfrac{\sqrt{2}}{25}$ accept exact equivalents or awrt 0.057 A1 3

(c) Using $x = a \cos \omega t$

$\frac{1}{2}a = a \cos \omega t$ — ft their ω — M1A1ft

$5t = \frac{\pi}{3}$

$t = \frac{\pi}{15}$ — A1

$T' = 4t = \frac{4\pi}{15}$ — awrt 0.84 — M1A1 — 5

Alternative

Using $x = a \sin \omega t$

$\frac{1}{2}a = a \sin \omega t$ — ft their ω — M1A1ft

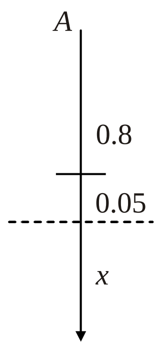
$5t = \frac{\pi}{6}$

$t = \frac{\pi}{30}$ — A1

$T' = T - 4t = \frac{4\pi}{15}$ — awrt 0.84 — M1A1 — 5

[11]

8. (a)



$T = \frac{\lambda}{0.8}(0.05) = 0.25g$ — M1

$\lambda = \frac{(0.8)(0.25g)}{0.05} = 39.2$ (*) — A1 — 2

(b)	$T = \dfrac{39.2}{0.8}(x + 0.05)$		M1	
	$mg - T = ma$	(3 term equn)	M1	
	$0.25g - \dfrac{39.2}{0.8}(x + 0.05) = 0.25\ \ddot{x}$ (or equivalent)		A1	
	$\ddot{x} = -196x$		A1	
	SHM with period $\dfrac{2\pi}{\omega} = \dfrac{2\pi}{14} = \dfrac{\pi}{7}$ s (*)		M1 A1 cso	6

1st M1 must have extn as $x + k$ with $k \neq 0$ (but allow M1 if e.g. $x + 0.15$), or must justify later

For last four marks, *must* be using $\ddot{x}$ (not a)

(c)	$v = 14\sqrt{\{(0.1)^2 - (0.05)^2\}}$	M1 A1ft	
	$= 1.21(24\ldots) \approx$ <u>1.21 m s^{-1}</u> (3 s.f.) Accept $7\sqrt{3}/10$	A1	3

Using $x = 0$ is M0

(d)	Time T under gravity = $\dfrac{1.21..}{g}$ (= 0.1237 s)	B1ft	
	Complete method for time T' from B to slack.		
	[↑ e.g. $\dfrac{\pi}{28} + t$, where $0.05 = 0.1\sin 14t$	M1 A1	
	OR T', where $-0.05 = 0.1\cos 14T'$]		
	$T'' = 0.1496$s	A1	
	Total time = $T + T'$ = <u>0.273 s</u>	A1	5

M1 – must be using distance for when string goes slack.
Using $x = -0.1$ (i.e. assumed end of the oscillation) is M0

[16]

9.	(a)	$a = 0.1$	B1	
		$\dfrac{2\pi}{\omega} = \dfrac{1}{5} \Rightarrow \omega = 10\pi$	M1 A1	
		$F_{max} = ma\omega 2$	M1	
		$= 0.2 \times 0.1 \times (l0\pi)^2$	M1	
		≈ 19.7 (N)	cao Al	6

(b) $a' = 0.2$, $\omega' = 10\pi$ B1ft, B1ft

$v^2 = \omega^2(a^2 - x^2) = 100\pi^2(0.2^2 - 0.1^2)$ $(= 3\pi^2 \approx 29.6\ldots)$ M1 A1

$v \approx 5.44$ (ms^{-1}) cao Al 5

If answers are given to more than 3 significant figures a maximum of one A mark is lost in the question.

[11]

10. (a) $a = 3$, $T = 12$ (or $\frac{1}{2}T = 6$) B1, B1

$T = \frac{2\pi}{\omega} = 12 \Rightarrow \omega = \frac{\pi}{6}$ (□ 0.52) M1 A1

In the scheme below, when a and/or ω appear in a line, accept the symbols or the candidate's values of a and/or ω for the marks in that line.

(Taking $x = a$ when $t = 0$) $x = a\cos\omega t$ M1

$\dot{x} = -a\omega\sin\omega t$ M1 A1

When $t = 5$ $\dot{x} = -3 \times \frac{\pi}{6}\sin\frac{5\pi}{6}$ M1

$|\dot{x}| = \frac{\pi}{4}$ (m h^{-1}) A1 9

awrt 0.79

Alternative

When $t = 5$ $x = 3\cos\frac{5\pi}{6} = -\frac{3\sqrt{3}}{2}$ (□ –2.60) M1

$v^2 = \omega^2(a^2 - x^2)$ M1

$= \frac{\pi^2}{6^2}\left(9 - \frac{9\times 3}{4}\right)\left(= \frac{\pi^2}{16}\right)$ M1 A1

$|v| = \frac{\pi}{4}$ (m h^{-1}) A1

awrt 0.79

(b)	Depth of 5.5 m $\Rightarrow x = -1.5$		
	$-1.5 = a\cos\omega t$	M1	
	$\cos\omega t = -\frac{1}{2}$	A1ft	
	$\frac{\pi}{6}t = \frac{2\pi}{3}, \left(\frac{4\pi}{3}\right)$	M1	
	$t = 4, 8$	A1	
	Required time is $t_2 - t_1 = 8 - 4 = 4$ (h)	A1	5
	The following should be accepted		
	$1.5 = a\cos\omega t$	M1	
	$\cos\omega t = \frac{1}{2}$	A1ft	
	$\frac{\pi}{6}t = \frac{\pi}{3}$	M1	
	$t = 2$	A1	
	Required time is $2t = 4$ (h)	A1	5

[14]

Alternatives measuring x from the centre of oscillation

(a)	(Using 1400 as $t = 0$)		
	The first 4 marks are as above	B1 B1	
	$x = a\sin\omega t$	M1 A1	
	$\dot{x} = a\omega\cos\omega t$	M1	
	When $t = 2$ $\quad \dot{x} = 3 \times \frac{\pi}{6}\cos\frac{2\pi}{6}$	M1 A1	
	$t = 2$ oe is essential for this M	M1	
	$= \frac{\pi}{4}$ (m h^{-1})	A1	9
(b)	$1.5 = 3\sin\omega t$	M1	
	$\sin\omega t = \frac{1}{2}$	A1ft	
	$\frac{\pi}{6}t = \frac{\pi}{6}, \left(\frac{5\pi}{6}\right)$	M1	
	$t = 1, 5$	A1	
	Required time is $t_2 - t_1 = 5 - 1 = 4$ (h)	A1	5

[14]

11.	(a)	$6 = \dfrac{2\pi}{w} \Rightarrow w = \dfrac{\pi}{3}$	M1	
		$a = 2L$	B1	
		$x = 2L\cos wt$	M1	
		$2L - b = 2L\cos\left(\dfrac{\pi}{3}.\dfrac{3}{4}\right)$	M1 A1 ft	
		$\underline{b = L(2 - \sqrt{2}}$ (*)	A1c.s.o.	5
	(b)	$\dot{x} = -2Lw \sin wt$	M1	
		$= -2L\dfrac{\pi}{3}\sin\dfrac{\pi}{4}$		
		Speed = $\dfrac{\sqrt{2}L\pi}{3}$	A1	2
	(c)	$\dfrac{1}{2}(2-\sqrt{2})L = 2L \sin wt$	M1 A1	
		$t = (0.1469 \times \dfrac{3}{4})$		
		$\therefore$ Total time = 2 × 0.14		
		= <u>0.28 (2dp)</u>	A1	3

[10]

12.	(a)	KE loss + PE loss = EPE Gain		
		$\dfrac{1}{2}.m2gL + mg3L = \dfrac{\lambda(3L)^2}{2L}$	M1 A2(–1 e.e.)	
		$(*)\dfrac{8mg}{9} = \lambda$	A1	4
	(b)	$mg - T = m\ddot{x}$	M1 A1	
		$mg - \dfrac{8mg}{9L}(x+e) = m\ddot{x}$	M1 A1	
		$-\dfrac{8g}{9L}x = \ddot{x}$		
		Hence SHM about D	A1 c.s.o.	5

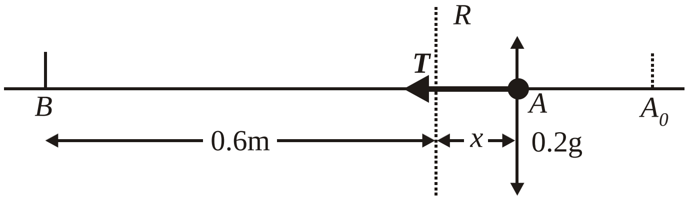
(c)	(i)	Period = $\frac{2\pi}{w} = 2\pi\sqrt{\frac{9L}{8g}} = 3\pi\sqrt{\frac{L}{2g}}$	M1 A1ft	
	(ii)	mg = $\frac{8mg}{9L}e \Rightarrow e = \frac{9L}{8}$	B1	
		$9 = 3L - \frac{9L}{8} = \frac{15L}{8}$		
		$v_{max} = 9w = \frac{15L}{8}\sqrt{\frac{8g}{9L}}$	M1	
		$= \frac{5}{4}\sqrt{2gL}$	A1	5

[14]

13.	(a)	$(-)\ \frac{21.6x}{2} = 0.3\ddot{x}$	M1 A1	
		$-36x = \ddot{x}$	M1	
		S.H.M., period = $\frac{2\pi}{\sqrt{36}} = \frac{\pi}{3}$ (*)	A1 c.s.o.	4
	(b)	At A: $v = aw = 1.5 \times 6 = 9$ ms^{-1}	M1 A1	2
	(c)	$x = a\cos\omega t$		
		$0.75 = 1.5\cos 6t$	M1	
		$\frac{\pi}{3} = 6t \Rightarrow t = \frac{\pi}{18}$ (no decimals)	dep. M1 A1	3
	(d)	$(-)\ \frac{21.6x}{2} = 0.5\ddot{x}$	M1 A1	
		$-21.6x = \ddot{x} \Rightarrow$ S.H.M., $\omega = \sqrt{21.6}$	A1	
		At collision: CLM: $0.3 \times 9 = 0.5v \Rightarrow v = 5.4$	M1 A1 ft	
		$a \times \sqrt{21.6} = 5.4$	M1	
		$a = 1.16$ m (3SF)	A1	7

[16]

13. (a)



$\lambda = 48$N

$BA_0 = 0.9$ m

Applying Hooke's Law correctly : e.g. $T = \frac{48x}{0.6}$ (= $80x$) M1

[Note: x may be other forms e.g. ("x" – 0.6) or "(0.3 – "x")

Equation of motion: (–) T = 0.2 $\ddot{x}$ (or a) M1

Correct equation of motion: e.g. $-\frac{48x}{0.6} = -0.2\ddot{x}$, 0.2"$\ddot{x}$" = $\frac{48x}{0.6}$(0.3 – "x") A1

Period (= $\frac{2\pi}{\omega}$) = $\frac{2\pi}{20} = \frac{\pi}{10}$ * (no incorrect working seen) A1 5

(b) max $v = aw$; = 0.3 × 20 = 6 ms^{-1} M1A1 2

(c) Using $x = a\cos 20t$ or $x = a\sin 20T$ or $\cos\alpha = \frac{x}{a}$ M1

Using $x = 0.15$ to give either $\cos 20t = ½$ or $\sin 20T = ½$ M1

Either $t = \frac{\pi}{60}$, ($\frac{5\pi}{60}$) or $T = \frac{\pi}{120}$ or $\alpha = \frac{\pi}{3}$ A1

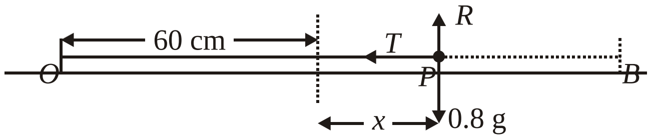
Complete method for time:

$t_2 - t_1$, or $\frac{\pi}{10} - 2t_1$, or $\frac{\pi}{20} + 2T$ or $\frac{2\pi - 2\alpha}{2\pi} = \frac{t}{(\pi/10)}$ M1

Time = $\frac{\pi}{15}$ s (must be in terms of π) A1 5

[12]

15. (a)



λ = 12N

OB = 85 cm

λ = 12N
OB = 85 cm

Hooke's Law: $T = \dfrac{12x}{0.6}$ [= 20x]	M1	
Equation of motion: (–)$T = 0.8\ddot{x}$	M1	
$-\dfrac{12x}{0.6} = 0.8\ddot{x}$ ($\ddot{x} = -25x$)	A1	
Finding ω from derived equation of form $\ddot{x} = -\omega^2 x$	M1	
Period = $\dfrac{2\pi}{\omega} = \dfrac{2\pi}{5}$ (*) no incorrect working seen	A1	5

[*Alternative for first 3 marks*

Energy equation: e.g. $\left(\dfrac{1}{2}\right)\dfrac{12}{0.6}\{0.25^2 - x^2\} = \left(\dfrac{1}{2}\right)0.8\dot{x}^2$	M1	
Differentiation wrt *time*; to give $\ddot{x}(\dot{x}) = -25x(\dot{x})$	M1; A1]	

(b)

Substituting (candidate's) ω and a **in** $\boldsymbol{\omega^2 a}$; = 25 × 0.25 = 6.25 (ms^{-2})	M1; A1	2
[Or finding $T_{max} = 0.8a \Rightarrow a = 5/0.8 = 6.25$]		

(c)

Complete method for x; x = 0.25 cos 10° (–0.2098)	M1 A1	
Using $v^2 = \omega^2(a^2 - x^2) \Rightarrow v = (\pm)5\sqrt{[(0.25)^2 - (0.25\cos 10^2)]}$ *[ft on wrong amplitude]*	M1 A1 ft	
v = (±) 0.68 (m s^{-1})	A1	5

(d)

Direction $\overrightarrow{OB}$ or equivalent	B1	1

[13]

16. (a)

$\dfrac{2\pi}{\omega} = \pi \Rightarrow = 2$	B1	
$2.4^2 = 4(a^2 - 0.5^2)$	M1 A1 ft	
a = 1.3 m	A1	4

(b)

$v_{max} = a\omega = 2.6$ m s^{-1}	B1	1

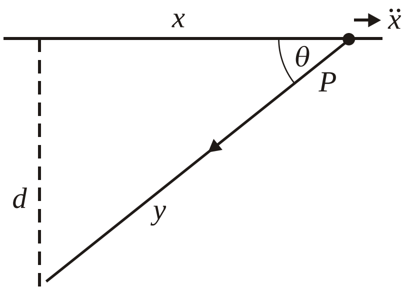
(c)

$\text{arct}_{max} = a\omega^2 = 5.2$ m s^{-2}	B1 ft	1

(d)	$0.5 = 1.3 \sin 2t$	M1	
	$t = \frac{1}{2} \sin^{-1}\left(\frac{0.5}{1.3}\right)$	M1 A1	
	$\therefore$ Total time $= 4t = 0.79$ (2 dp)	M1 A1	5

[11]

17. (a)



$R(\rightarrow)$: $0.2\ddot{x} = -5y\cos\theta$	M1 A1	
$P \cos\theta = \frac{x}{y}$	M1	
$\Rightarrow \ddot{x} = -25x$	A1	
$\Rightarrow$ SHM period $= \frac{2\pi}{5}$	A1	5

(b)	$d = 2$; max speed = '$d\omega$' $= 2 \times 5 = 10$ m s^{-1}	M1 A1	2
	$x = 2\cos 5t$	M1	
	Distance 3 m from start $\Rightarrow$ $x = -1$	B1	
	$\cos 5t = -\frac{1}{2}$		
	$\Rightarrow$ $5t = \frac{2\pi}{3}$, $t = \frac{2\pi}{15}$ s	M1 A1	4

[11]

1. The majority gained full marks for part (a) but some used m instead of $2m$ for the mass and others forgot to complete the question by adding $3a$ to their extension. However, part (b) was a different story. This was a standard question and should have been routine for most but was very poorly done. Some candidates measured the extension from the natural length, which can produce a correct result provided the appropriate substitution is employed. Probably they were not fully aware of what they were doing and so stopped work when they did not reach the required equation. Many had inconsistent masses, with $2mg$ and $m\ddot{x}$ appearing in the same line of working, although some realised and backtracked successfully. The omission of $2mg$ in the equation of motion led to the correct answer when the extension was incorrectly measured from the natural length so candidates assumed that they had answered correctly. Poor notation was often seen; "acc." or "a" was used for acceleration (as well as its length application) and e was used for the variable extension along with an acceleration $\ddot{x}$. There were many sign errors seen in the equations and although some candidates realised they had made errors and either corrected their work or fiddled the result, others seemed to think that the equation $\ddot{x} = \omega^2 x$ proved S.H.M. Good attempts were presented for parts (c) and (d) although some thought that the amplitude was 3a/4; $\omega = \sqrt{\frac{a}{g}}$ was another common error. Some forgot to complete their work in part (d) by adding $\frac{a}{8}$ and $\frac{3a}{128}$ to obtain the final answer.

2. This was a relatively routine SHM question and was answered well by the majority of candidates with many completely correct solutions seen. It was rare to see a script where the first two marks of part (a) were not earned. Most candidates realised that as the particle was at the centre of the oscillation when $t = 0$ they should use $x = a \sin\omega t$ rather than $x = a \cos\omega t$. Use of the latter equation led to loss of accuracy marks but method marks were still available. Some candidates did not realise that to find the amplitude they needed to find the value of x when t = 0.4 and use this in $v^2 = \omega^2 (a^2 – x^2)$. Almost all those who obtained a value for a (correct or otherwise) were able to gain at least the method mark in part (b).

3. Part (a) was usually handled confidently with Hooke's Law in the equilibrium position applied to each spring and the resulting tensions equated. There were some unfortunate careless errors such as "total extension = 5 – 2 – 1 = 3" which appeared far too often. Mistakes in the extensions led to difficulties in the later parts of the question but some able candidates didn't think to check their extensions when their confidently applied method in (b) failed to work as expected. The work seen in part (b) suggested that many candidates are not aware that to prove SHM they need an equation of motion that reduces to the form $\ddot{x} = –\omega^2 x$; some even tried to give a written explanation of the motion, with no equations provided at all. Even if the question had not told them that the equilibrium position was the centre of the oscillation, candidates should have known this and measured their displacement from this point; many chose some other point instead. There were also numerous sign errors in the equation of motion, often "corrected" but not always in a valid manner.

Lack of success in part (b) meant many candidates had no suitable value for ω to use in part (c). Some were content to invent a number and continue, thereby making the method and follow through marks available, others gave up, though this may have been due to lack of time. Most who worked on this part realised that the amplitude could be obtained using max $v_{max} = a\omega$ and hence the time required using $x = a \sin\omega t$. Solutions using $x = a\cos\omega t$ were seen occasionally but the extra work needed to obtain the necessary time was often omitted. The instruction to find the *proportion* of the time was usually ignored completely resulting in the loss of three marks as the period was not calculated since the necessity for it was not apparent.

4. The methods and S.H.M. formulae were well known and many completely correct solutions were seen. However, it was disappointing to see how many candidates could not cope with the time aspect of this question; 12 hr 30 min is not 12.3 hr. Even those who converted correctly at the start sometimes forgot to reverse the process at the end, giving their final answer as 12.36.

Some candidates preferred to work with seconds, but on the whole they remembered to change their final answers appropriately. x was not always measured from the centre of the oscillation, resulting in the use of $x = 5$ rather than $x = 3$ in both parts of this question. In (b), $x = –3$ was seen quite often, almost certainly because the point in question was below the centre of the oscillation. The symmetry of S.H.M. would allow a correct result to be deduced from this and also from use of $x = \sin\omega t$, but the appropriate deduction was rarely seen.

5. As the question stated that the particle was moving with SHM most candidates could make some attempt at a solution. Where SHM was well understood these solutions were concise and correct. $T = 1.5$ was seen too often and a few candidates worked with an amplitude of 2.4. In part (b) most candidates used $t = 2$ and the majority of those who used $x = a \sin \omega t$ obtained the correct answer but some thought they had finished when x had been found. Those who used $x = a \cos \omega t$ generally failed to appreciate that $t = 1.25$; they did not understand the question and rarely obtained any marks. Identifying the correct x to use in part (c) caused some candidates problems as they used their final answer from (b) instead.

6. Relatively few candidates proved SHM successfully. The need for x double dot and a minus sign has been stressed many times before in reports. Mistakes included: all terms numerical with no x, arguing with a general ‘a’ term and both special cases mentioned in the mark scheme. Many recovered and scored full marks in (b) though sometimes $x = 0.2$ and 0.6 were used as the amplitude. Part(c) was usually successfully tackled by using $v^2 = \omega^2(a^2 - x^2)$ but there were some candidates who thought that the string was slack when $x = 0$. The candidates using $x = a \cos wt$ and differentiating to find v, had to calculate t when the string became slack as an intermediate step and usually went wrong A third group used energy to solve this part and frequently obtained the correct answer.

In part (d) candidates who used a circle and angles subtended at the centre, not often seen, invariably found the correct answer in a very simple way. Many of those using $a \sin wt$ and $a \cos wt$ did not have a clear method in mind and combined a variety of $x = 0.2$ and $x = –0.2$ along with a fraction of the period in attempting, usually unsuccessfully, to find the correct time. A few candidates thought that the time under SHM was simply the period.

The part of the motion that was under gravity was usually tackled correctly. There were, however, some excellent solutions to this question, candidates who appeared to be well drilled and competent and the question was a good discriminator.

7. The basic formulae for SHM were well known and many candidates scored well on this question but full marks were relatives uncommon; part (c) proving difficult to complete correctly. In part (a), some candidates tried to use $v = r\omega$ incorrectly and, as it happened, this gave, numerically, the correct period. It is, however, a false method and gained no credit in this part, although some follow through was allowed in the remaining parts of the question. Part (b) was generally well done and, in part (c), candidates generally knew how to find at least one time when *OP* was $\frac{1}{2}a$. However relating this to the total time for which $OP > \frac{1}{2}a$ proved demanding and $\frac{2\pi}{15}$ was commonly seen instead of the correct $\frac{4\pi}{15}$. Some very neat solutions were seen using the reference circle associated with SHM.

8. Part (a) was almost universally correct. Part (b) caused considerable problems: many assumed that they could find the acceleration as a (unspecified) '*a*', and that if they showed that this was equal to '$-196x$' they had succeeded in showing that the motion was SHM. Such candidates failed to realise that any acceleration given as an unspecified *a* needs its direction clearly specified. Hence, without the use of an expression for $\ddot{x}$ as equal to $-196x$, they could make no progress. Weaker candidates also failed to see that the equation of motion had to include the weight as well. In parts (c) and (d) a common mistake was effectively to assume that the particle came to rest at the end of an oscillation within the simple harmonic motion. Nevertheless, more able candidates were able to complete the question accurately and overall, this proved to be a good discriminating question for the final one on the paper.

9. Those who had revised their SHM formulae did well on this question. and many gained full marks very quickly. The main source of error in part (a) was taking $T = 5$, which led to $\omega = \frac{2\pi}{5}$ In part (b), a common error was to assume that, when the particle was struck, $x = 0$, whereas the conditions of the question implied that $x = 0.1$ had to be used in the equation $v^2 = \omega^2 (a^2 - x^2)$ The request for 3 significant figures was generally heeded but a few lost a mark by ignoring this request. Candidates are penalised a maximum of one mark in a question for such matters.

10. Some candidates had difficulty in interpreting this question with candidates being confused between clock time and elapsed time or between the depth of the water and SHM displacement. A few did not understand the 24 hour clock and thought there were six hundred hours between low and high water, despite the question specifying that the times were on a particular day. In part (a), the indirect method of finding *x* first and then using $v^2 = \omega^2(a^2 - x^2)$ was more popular than using the differentiation method but those who chose the latter almost always obtained the correct answer. Many thought the period of motion was six rather than twelve hours. However there were many good solutions to this questions and it was pleasing to see answers clearly explained through sketches or graphs. In part (b), most could find a value of the time when the depth of the water was 5.5m but not all could interpret this correctly and complete the question.

11. Part (a) was successfully completed by the majority but there were were errors in parts (b) and (c) where a significant number measured *x* from the end of the oscillation rather than from the centre. The method was generally known in part (b) but the final part was a good test of comprehension and those that restarted using $x = 2L\sin wt$ were usually successful.

12. Most realised that the first part required the use of energy and were able to obtain the required result. There was a disappointing response to the “standard proof” in part (b); many candidates simply ignored the weight and scored no marks. Part (c) (i) was well done but there were few correct solutions to (ii), where most were unable to find the amplitude of the oscillation.

13. This was a good source of marks, even for weaker candidates. It was unfortunate that it was the last question because some candidates ran out of time half way through. The first part was generally the weakest, with many candidates substituting $x = 1.5$ immediately so that there was no actual proof. Part (c) was often completely correct but there were some answers in degrees and others where it was assumed that because the particle was halfway between the centre and the end that the time for that part was also half of a quarter period.

14. For many candidates this was a good source of marks, although full marks were rarely gained.The proof provided five easy marks for many candidates but it is disappointing to see a significant number of candidates who seem to have little idea of what is required. It was not uncommon to see:

T (at B) = $\dfrac{48(0.3)}{0.6}$ = 24 N;

$T = ma$ so a (at B)= 120 ms^{-2};

as $a = (-)\ \omega^2 x$, $120 = (-)\ \omega^2 0.3)$ so ω = 20 and motion is SHM with period $\dfrac{2\pi}{20} = \dfrac{\pi}{10}$.

Part (b) was generally well answered, but in (c) the final two marks proved harder to gain.

As mentioned in the introduction, a good number of candidates proved the result in (a) for vertical motion. Often they then went on to work horizontally in (b) and (c), which suggests that they had only learnt the proof for the vertical case. Those candidates who worked with vertical motion throughout the question clearly made it harder for themselves and were rarely successful in part (c).

15. Part (a) was usually only answered well by the better candidates. A very common error was in not taking a general point in the motion, T often calculated at B as 5N, followed by an equation of motion of $\ddot{x} = 6$□ or –6□. Neither these results nor $\ddot{x} = 25x$ deterred many candidates from calculating the period, assuming the motion to be SHM.

Many candidates realised what was required for parts (b) and (c) although as many used 0.85cm for the amplitude as used the correct value of 0.25cm. Other errors in using the equation $x = 0.25\cos 10$ were to work in degrees, and to use sine rather than cosine.

Part (d) was not answered particularly well.

16. The first three parts of this question were very well-answered with S.H.M. formulae well known and used appropriately. It was pleasing to see most candidates working in radians in part (d), although many were unable to correctly identify the appropriate portion of the cycle with the most common incorrect answer being half of the required value.

17. No Report available for this question.