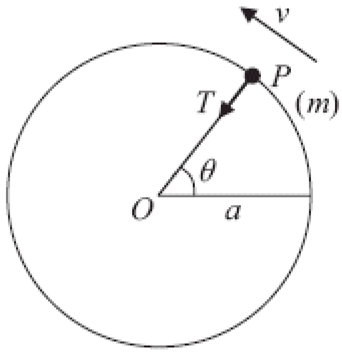
1.



A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is fixed at the point O. The particle is initially held with OP horizontal and the string taut. It is then projected vertically upwards with speed u, where $u^2 = 5ag$. When OP has turned through an angle θ the speed of P is v and the tension in the string is T, as shown in the diagram above.

(a) Find, in terms of a, g and θ, an expression for v^2.

(3)

(b) Find, in terms of m, g and θ, an expression for T.

(4)

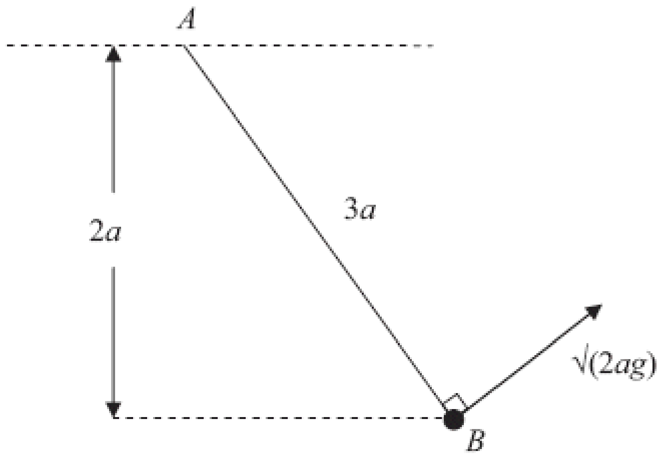
(c) Prove that P moves in a complete circle.

(3)

(d) Find the maximum speed of P.

(2)
(Total 12 marks)

2.



One end A of a light inextensible string of length $3a$ is attached to a fixed point. A particle of mass m is attached to the other end B of the string. The particle is held in equilibrium at a distance 2a below the horizontal through A, with the string taut. The particle is then projected with speed $\sqrt{(2ag)}$, in the direction perpendicular to AB, in the vertical plane containing A and B, as shown in the diagram above. In the subsequent motion the string remains taut. When AB is at an angle θ below the horizontal, the speed of the particle is v and the tension in the string is T.

(a) Show that $v^2 = 2ag(3 \sin \theta - 1)$.

(5)

(b) Find the range of values of T.

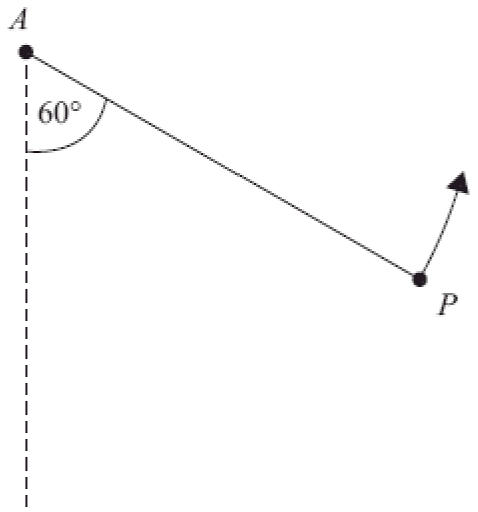
(6)
(Total 11 marks)

3. One end of a light inextensible string of length l is attached to a fixed point A. The other end is attached to a particle P of mass m, which is held at a point B with the string taut and AP making an angle arccos $\frac{1}{4}$ with the downward vertical. The particle is released from rest. When AP makes an angle θ with the downward vertical, the string is taut and the tension in the string is T.

(a) Show that

$$T = 3mg \cos \theta - \frac{mg}{2}.$$

(6)

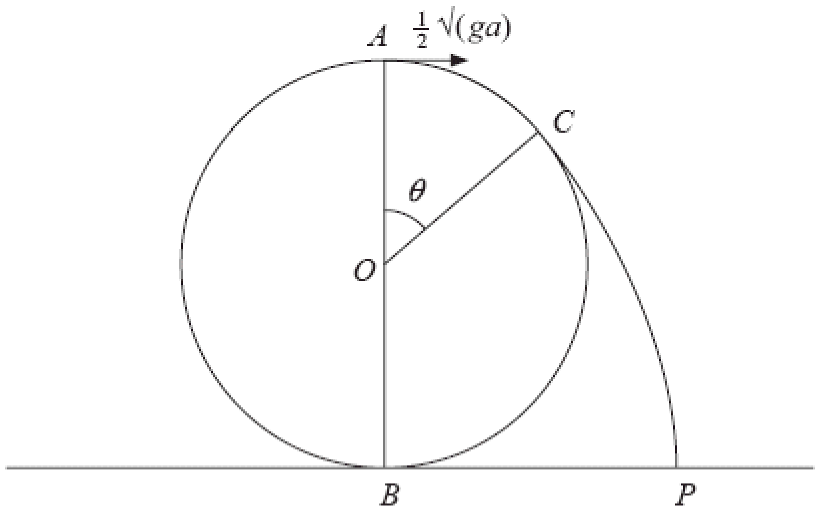


At an instant when AP makes an angle of 60° to the downward vertical, P is moving upwards, as shown in the diagram. At this instant the string breaks. At the highest point reached in the subsequent motion, P is at a distance d below the horizontal through A.

(b) Find d in terms of l.

(5)
(Total 11 marks)

4.



A particle is projected from the highest point A on the outer surface of a fixed smooth sphere of radius a and centre O. The lowest point B of the sphere is fixed to a horizontal plane. The particle is projected horizontally from A with speed $\frac{1}{2}\sqrt{(ga)}$. The particle leaves the surface of the sphere at the point C, where $\angle AOC = \theta$, and strikes the plane at the point P, as shown in the diagram above.

(a) Show that $\cos\theta = \dfrac{3}{4}$.

(7)

(b) Find the angle that the velocity of the particle makes with the horizontal as it reaches P.

(8)

(Total 15 marks)

5. A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is released from rest with the string taut and OP horizontal.

(a) Find the tension in the string when OP makes an angle of 60° with the downward vertical.

(6)

A particle Q of mass $3m$ is at rest at a distance a vertically below O. When P strikes Q the particles join together and the combined particle of mass $4m$ starts to move in a vertical circle with initial speed u.

(b) Show that $u = \sqrt{\left(\dfrac{ga}{8}\right)}$.

(3)

The combined particle comes to instantaneous rest at A.

(c) Find

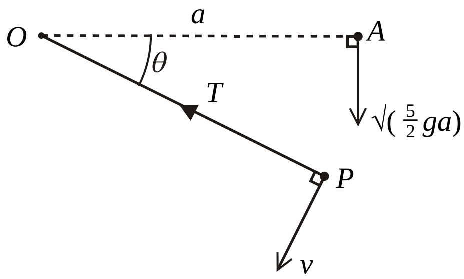
(i) the angle that the string makes with the downward vertical when the combined particle is at A,

(ii) the tension in the string when the combined particle is at A.

(6)

(Total 15 marks)

6.



A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. At time $t = 0$, P is projected vertically downwards with speed $\sqrt{\frac{5}{2}ga}$ from a point A which is at the same level as O and a distance a from O. When the string has turned through an angle θ and the string is still taut, the speed of P is v and the tension in the string is T, as shown in the diagram above.

(a) Show that $v^2 = \frac{ga}{2}(5 + 4\sin\theta)$

(3)

(b) Find T in terms of m, g and θ.

(3)

The string becomes slack when $\theta = \alpha$.

(c) Find the value of α.

(3)

The particle is projected again from A with the same velocity as before. When P is at the same level as O for the first time after leaving A, the string meets a small smooth peg B which has been fixed at a distance $\frac{1}{2}a$ from O. The particle now moves on an arc of a circle centre B. Given that the particle reaches the point C, which is $\frac{1}{2}a$ vertically above the point B, without the string going slack,

(d) find the tension in the string when P is at the point C.

(6)
(Total 15 marks)

7. A particle P is free to move on the smooth inner surface of a fixed thin hollow sphere of internal radius a and centre O. The particle passes through the lowest point of the spherical surface with speed U. The particle loses contact with the surface when OP is inclined at an angle α to the upward vertical.

(a) Show that $U^2 = ag(2 + 3\cos\alpha)$.

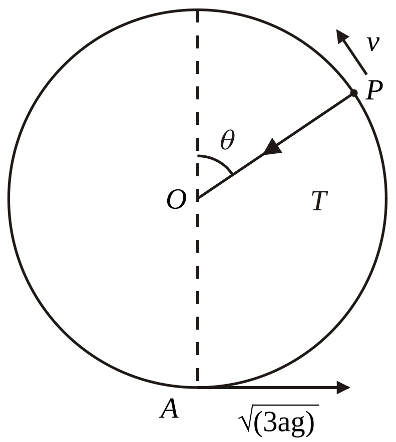
(7)

The particle has speed W as it passes through the level of O. Given that $\cos\alpha = \frac{1}{\sqrt{3}}$,

(b) show that $W^2 = ag\sqrt{3}$.

(5)
(Total 12 marks)

8.



A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a point O. The point A is vertically below O, and $OA = a$. The particle is projected horizontally from A with speed $\sqrt{(3ag)}$. When OP makes an angle θ with the upward vertical through O and the string is still taut, the tension in the string is T and the speed of P is v, as shown in the diagram above.

(a) Find, in terms of a, g and θ, an expression for v^2.

(3)

(b) Show that $T = (1 - 3\cos\theta)mg$.

(3)

The string becomes slack when P is at the point B.

(c) Find, in terms of a, the vertical height of B above A.

(2)

After the string becomes slack, the highest point reached by P is C.

(d) Find, in terms of a, the vertical height of C above B.

(5)
(Total 13 marks)

9. One end of a light inextensible string of length l is attached to a particle P of mass m. The other end is attached to a fixed point A. The particle is hanging freely at rest with the string vertical when it is projected horizontally with speed $\sqrt{\frac{5gl}{2}}$.

(a) Find the speed of P when the string is horizontal.

(4)

When the string is horizontal it comes into contact with a small smooth fixed peg which is at the point B, where AB is horizontal, and $AB < l$. Given that the particle then describes a complete semicircle with centre B,

(b) find the least possible value of the length AB.

(9)
(Total 13 marks)

10. One end of a light inextensible string of length l is attached to a fixed point A. The other end is attached to a particle P of mass m which is hanging freely at rest at a point B. The particle P is projected horizontally from B with speed $\sqrt{(3gl)}$. When AP makes an angle θ with the downward vertical and the string remains taut, the tension in the string is T.

(a) Show that $T = \text{mg}(1 + 3\cos\theta)$.

(6)

(b) Find the speed of P at the instant when the string becomes slack.

(3)

(c) Find the maximum height above the level of B reached by P.

(5)
(Total 14 marks)

11. A smooth solid sphere, with centre O and radius a, is fixed to the upper surface of a horizontal table. A particle P is placed on the surface of the sphere at a point A, where OA makes an angle α with the upward vertical, and $0 < \alpha < \frac{\pi}{2}$. The particle is released from rest. When OP makes an angle θ with the upward vertical, and P is still on the surface of the sphere, the speed of P is v.

(a) Show that $v^2 = 2ga(\cos\alpha - \cos\theta)$.

(4)

Given that $\cos\alpha = \frac{3}{4}$, find

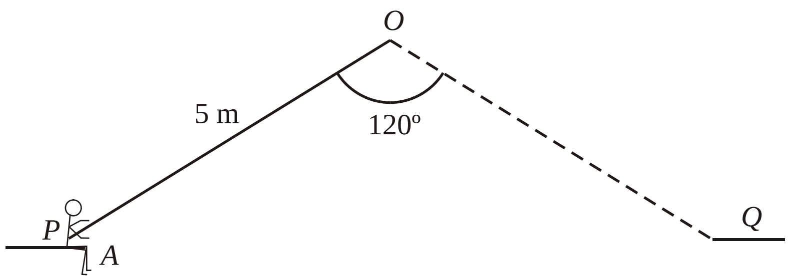
(b) the value of θ when P loses contact with the sphere,

(5)

(c) the speed of P as it hits the table.

(4)
(Total 13 marks)

12.



A trapeze artiste of mass 60 kg is attached to the end A of a light inextensible rope OA of length 5 m. The artiste must swing in an arc of a vertical circle, centre O, from a platform P to another platform Q, where PQ is horizontal. The other end of the rope is attached to the fixed point O which lies in the vertical plane containing PQ, with $\angle POQ = 120°$ and $OP = OQ = 5$ m, as shown in the diagram above.

As part of her act, the artiste projects herself from P with speed $\sqrt{15}$ m s^{-1} in a direction perpendicular to the rope OA and in the plane POQ. She moves in a circular arc towards Q. At the lowest point of her path she catches a ball of mass m kg which is travelling towards her with speed 3 m s^{-1} and parallel to QP. After catching the ball, she comes to rest at the point Q.

By modelling the artiste and the ball as particles and ignoring her air resistance, find

(a) the speed of the artiste immediately before she catches the ball,

(4)

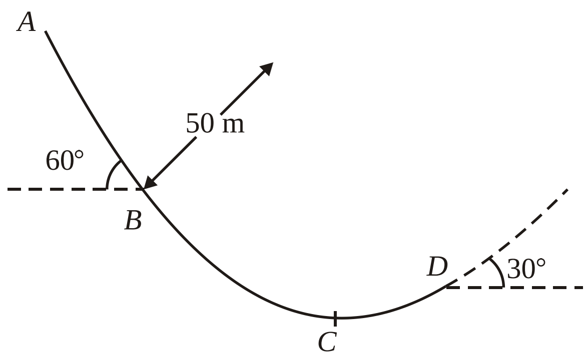
(b) the value of m,

(7)

(c) the tension in the rope immediately after she catches the ball.

(3)
(Total 14 marks)

13.



The diagram above represents the path of a skier of mass 70 kg moving on a ski-slope $ABCD$. The path lies in a vertical plane. From A to B, the path is modelled as a straight line inclined at 60° to the horizontal. From B to D, the path is modelled as an arc of a vertical circle of radius 50 m. The lowest point of the arc BD is C.

At B, the skier is moving downwards with speed 20 m s^{-1}. At D, the path is inclined at 30° to the horizontal and the skier is moving upwards. By modelling the slope as smooth and the skier as a particle, find

(a) the speed of the skier at C,

(3)

(b) the normal reaction of the slope on the skier at C,

(3)

(c) the speed of the skier at D,

(3)

(d) the change in the normal reaction of the slope on the skier as she passes B.

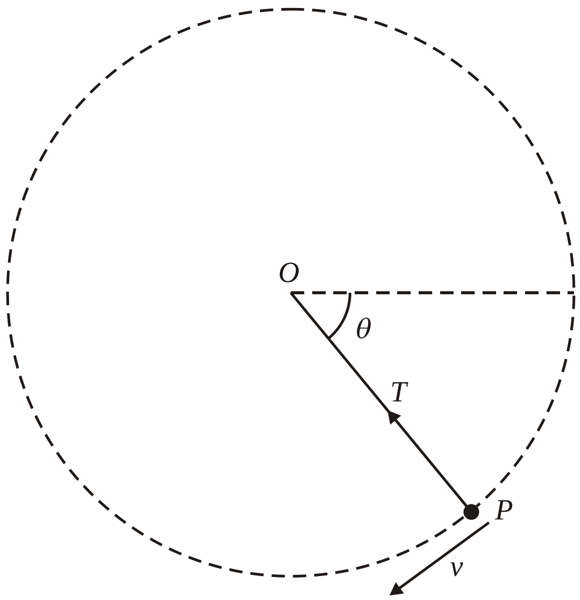
(4)

The model is refined to allow for the influence of friction on the motion of the skier.

(e) State briefly, with a reason, how the answer to part (b) would be affected by using such a model. (No further calculations are expected.)

(2)
(Total 15 marks)

14.



A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is fixed at a point O. The particle is held with the string taut and OP horizontal. It is then projected vertically downwards with speed u, where $u^2 = \frac{3}{2}ga$. When OP has turned through an angle θ and the string is still taut, the speed of P is v and the tension in the string is T, as shown in the diagram above.

(a) Find an expression for v^2 in terms of a, g and θ.

(2)

(b) Find an expression for T in terms of m, g and θ.

(3)

(c) Prove that the string becomes slack when $\theta = 210°$.

(2)

(d) State, with a reason, whether P would complete a vertical circle if the string were replaced by a light rod.

(2)

After the string becomes slack, P moves freely under gravity and is at the same level as O when it is at the point A.

(e) Explain briefly why the speed of P at A is $\sqrt{\left(\frac{3}{2}ga\right)}$.

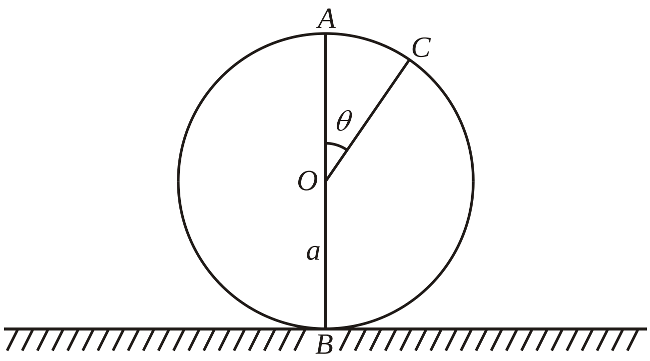
(1)

The direction of motion of P at A makes an angle φ with the horizontal.

(f) Find φ.

(4)
(Total 14 marks)

15.



A particle is at the highest point A on the outer surface of a fixed smooth sphere of radius a and centre O. The lowest point B of the sphere is fixed to a horizontal plane. The particle is projected horizontally from A with speed u, where $u < \sqrt{(ag)}$. The particle leaves the sphere at the point C, where OC makes an angle θ with the upward vertical, as shown in the diagram above.

(a) Find an expression for $\cos \theta$ in terms of u, g and a.

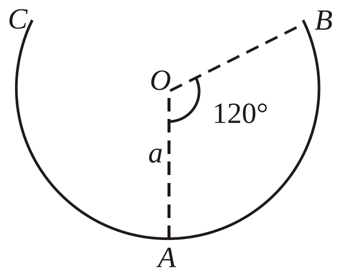
(7)

The particle strikes the plane with speed $\sqrt{\left(\dfrac{9ag}{2}\right)}$.

(b) Find, to the nearest degree, the value of θ.

(7)
(Total 14 marks)

16.



Part of a hollow spherical shell, centre O and radius a, is removed to form a bowl with a plane circular rim. The bowl is fixed with the circular rim uppermost and horizontal. The point A is the lowest point of the bowl. The point B is on the rim of the bowl and $\angle AOB = 120°$, as shown in the diagram above. A smooth small marble of mass m is placed inside the bowl at A and given an initial horizontal speed u. The direction of motion of the marble lies in the vertical plane AOB. The marble stays in contact with the bowl until it reaches B. When the marble reaches B, its speed is v.

(a) Find an expression for v^2.

(3)

(b) For the case when $u^2 = 6ga$, find the normal reaction of the bowl on the marble as the marble reaches B.

(3)

(c) Find the least possible value of u for the marble to reach B.

(3)

The point C is the other point on the rim of the bowl lying in the vertical plane OAB.

(d) Find the value of u which will enable the marble to leave the bowl at B and meet it again at the point C.

(7)
(Total 16 marks)

17. A smooth solid hemisphere is fixed with its plane face on a horizontal table and its curved surface uppermost. The plane face of the hemisphere has centre O and radius a. The point A is the highest point on the hemisphere. A particle P is placed on the hemisphere at A. It is then given an initial horizontal speed u, where $u^2 = \frac{1}{2}(ag)$. When OP makes an angle θ with OA, and while P remains on the hemisphere, the speed of P is v.

(a) Find an expression for v^2.

(3)

(b) Show that, when $\theta = \arccos 0.9$, P is still on the hemisphere.

(5)

(c) Find the value of $\cos\theta$ when P leaves the hemisphere.

(2)

(d) Find the value of v when P leaves the hemisphere.

(2)

After leaving the hemisphere P strikes the table at B.

(e) Find the speed of P at B.

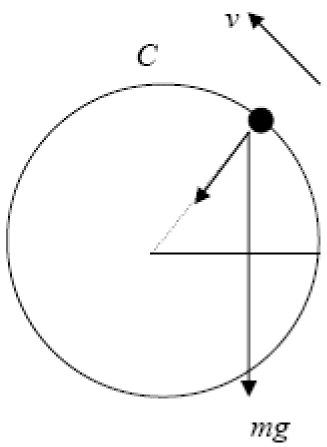
(2)

(f) Find the angle at which P strikes the table.

(3)

(Total 17 marks)

1. (a)



Energy: $mga\sin\theta = \frac{1}{2}m \times 5ag - \frac{1}{2}mv^2$ M1 A1

$v^2 = 5ag - 2ag\sin\theta$ A1 3

(b) Eqn of motion along radius:

$T + mg\sin\theta = \frac{mv^2}{a}$ M1 A1

$T = \frac{m}{a}\left(5ag - 2ag\sin\theta\right) - mg\sin\theta$ M1

$T = mg\left(5 - 3\sin\theta\right)$ A1 4

(c) At C, $\theta = 90°$

$T = mg\left(5-3\right) = 2mg$ M1 A1

$T > 0 \quad \therefore P$ reaches C A1 3

(d) Max speed at lowest point

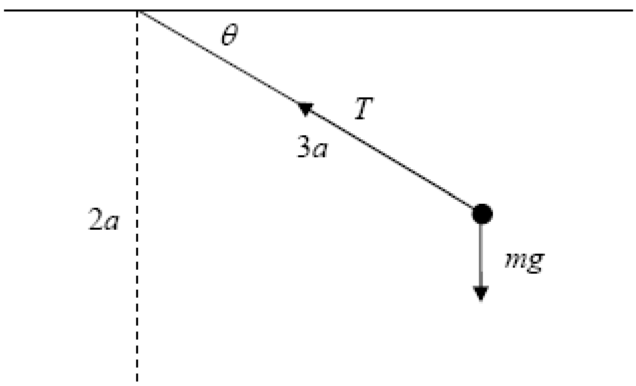
($\theta = 270°$; $v^2 = 5ag - 2ag\sin 270$) M1

$v^2 = 5ag + 2ag$

$v = \sqrt{(7ag)}$ A1 2

[12]

2. (a)



$\frac{1}{2}m \times 2ag - \frac{1}{2}mv^2 = mg(2a - 3a\sin\theta)$ M1 A1=A1

leading to $v^2 = 2ga(3\sin\theta - 1)$ * cso M1 A1 5

(b) minimum value of T is when $v = 0 \Rightarrow \sin\theta = \frac{1}{3}$ B1

$T = mg\sin\theta = \frac{mg}{3}$ M1 A1

maximum value of T is when $\theta = \frac{\pi}{2}$ $(v^2 = 4ag)$

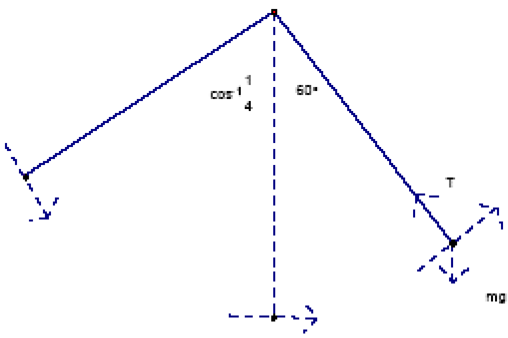
$\uparrow \quad T = \frac{mv}{3a} + mg$ M1 A1

$= \frac{7mg}{3}$ A1 6

$\left(\frac{mg}{3} \le T \le \frac{7mg}{3}\right)$

[11]

3. (a)



Energy:

$\left(\frac{1}{2}mu^2 + \right)mgl\left(\cos\theta - \frac{1}{4}\right) = \frac{1}{2}mv^2$ M1A1

Resolving:

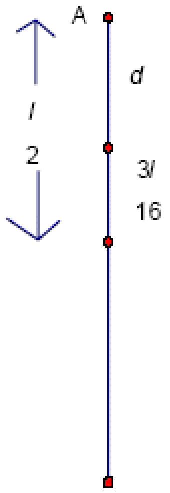
$$T - mg\cos\theta = \frac{mv^2}{l}$$ M1A1

Eliminate v^2:

$$T = mg\cos\theta + \frac{1}{l}(2mgl(\cos\theta - \frac{1}{l}))$$ M1

$$T = 3mg\cos\theta - \frac{mg}{2} \quad *$$ A1

(b)



$$\theta = 60° \Rightarrow mv^2 = 2mgl(\frac{1}{2} - \frac{1}{4})$$ M1

$$\Rightarrow v^2 = \frac{gl}{2}$$

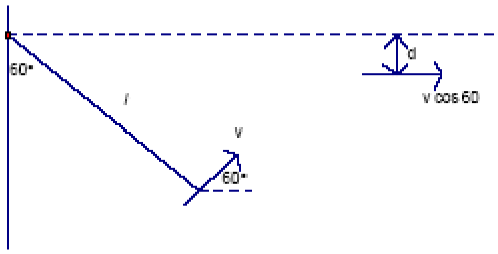
vertical motion under gravity:

$\updownarrow$ $0 = (v\cos 30°)^2 - 2gs$ M1

$$0 = \frac{gl}{2} \times \frac{3}{2} - 2gs \Rightarrow s = \frac{3l}{16}$$ A1

Distance below A = $\frac{l}{2} - \frac{3l}{16} = \frac{5l}{16}$ M1A1

<u>Alternative</u>



$$\frac{1}{2}mv^2 - mgl\cos 60 = \frac{1}{2}m(v\cos 60)^2 - mgd$$ M1A1

$\frac{gl}{4} - \frac{gl}{2} = \frac{gl}{4} \times \frac{1}{4} - gd$ M1

$d = \frac{1-4+8}{16} l = \frac{5l}{16}$ A1

[11]

4. (a) Let speed at C be u

CE $\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mga\,(1-\cos\theta)$ M1 A1

$u^2 = \frac{9ga}{4} - 2ga\cos\theta$

$mg\cos\theta\,(+R) = \frac{mu^2}{a}$ M1 A1

$mg\cos\theta = \frac{9mg}{4} - 2mg\cos\theta$ eliminating u M1

Leading to $\cos\theta = \frac{3}{4}$* M1 A1 7

(b) At C $u^2 = \frac{9ga}{4} - 2ga \times \frac{3}{4} = \frac{3}{4}ga$ B1

$(\rightarrow)$ $u_x = u\cos\theta = \sqrt{\left(\frac{3ga}{4}\right)} \times \frac{3}{4} = \sqrt{\left(\frac{27ga}{64}\right)} = 2.033\sqrt{a}$ M1 A1ft

$(\downarrow)$ $u_y = u\cos\theta = \sqrt{\left(\frac{3ga}{4}\right)} \times \frac{\sqrt{7}}{4} = \sqrt{\left(\frac{21ga}{64}\right)} = 1.792\sqrt{a}$ M1

$v_y^2 = u_y^2 + 2gh \;\Rightarrow\; v_y^2 = \frac{21}{64}ga + 2g \times \frac{7}{4}a = \frac{245}{64}ga$ M1 A1

$\tan\psi = \frac{v_y}{u_x} = \sqrt{\left(\frac{245}{27}\right)} \approx 3.012...$ M1

$\psi \approx 72°$ awrt 72° A1 8

Or 1.3° (1.2502°) awrt 1.3°

Alternative for the last five marks

Let speed at P be v.

CE $\quad \frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mg \times 2a$ — or equivalent — M1

$v^2 = \frac{17mga}{4}$ — M1 A1

$\cos\psi = \frac{u_x}{v} = \sqrt{\left(\frac{27}{64}\times\frac{4}{17}\right)} = \sqrt{\left(\frac{27}{272}\right)} \approx 0.315$ — M1

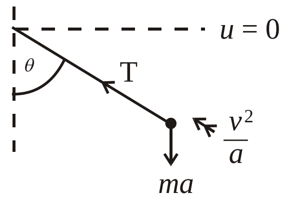
$\psi \approx 72°$ — awrt 72° — A1

<u>Note</u>

The time of flight from C to P is $\frac{\sqrt{235}-\sqrt{21}}{8}\sqrt{\left(\frac{a}{g}\right)} \approx 1.38373\sqrt{\left(\frac{a}{g}\right)}$

[15]

5. (a)



Energy $\frac{1}{2}mv^2 = mga\cos\theta$ — M1A1

$v^2 = 2ga\cos\theta$

$F = ma$ ↖ $T - mg\cos\theta = \frac{mv^2}{a}$ — M1A1

Sub for $\frac{v^2}{a}$: $T = mg\cos\theta + 2mg\cos\theta$: $\theta = 60$ $\therefore$ $T = \frac{1}{3}mg$ — M1A1 — 6

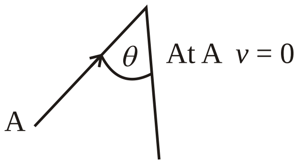
(b) Speed of P before impact = $\sqrt{2ga}$ — B1

$\rightarrow \sqrt{2ga} \rightarrow 0 \rightarrow u$

PCLM: • • • $\therefore u = \frac{\sqrt{2ga}}{4} = \sqrt{\frac{ga}{8}}$ * — M1A1cso — 3

m 3m 4m

(c) (i)



At A $v = 0$ so conservation of energy gives:

$\frac{1}{2}\cancel{4m}u^2 = \cancel{4m}ga(1-\cos\theta)$ M1A1

$\frac{ga}{16} = ga(1-\cos\theta)$ M1

$\cos\theta = \frac{15}{16}, \theta = 20°$ A1

(ii) $\nearrow$ At A $T = 4mg\cos\theta = \frac{15mg}{4}$ (accept 3.75mg) M1A1 6

[15]

6. (a) Energy equation with two terms on RHS,

$\frac{1}{2}mv^2 = \frac{1}{2}m.\frac{5ga}{2} + mga\sin\theta$ M1, A1

$\Rightarrow v^2 = \frac{ga}{2}(5 + 4\sin\theta)$ * A1 cso 3

Use of $v^2 = u^2 + 2gh$ is M0

(b) R(\\ string) $T - mg\sin\theta = \frac{mv^2}{a}$ (3 terms) M1A1

$\Rightarrow T = \frac{mg}{2}(5 + 6\sin\theta)$ o.e. A1 3

(c) $T = 0 \Rightarrow \sin\theta, = -\frac{5}{6}$ M1, A1

Has a solution, so string slack when $\alpha \approx 236(.4)°$ or 4.13 radians A1 3

(d) At top of small circle, $\frac{1}{2}mv^2 = \frac{1}{2}m.\frac{5ga}{2} - \frac{mga}{2}$ M1A1A1

(M1 for energy equation with 3 terms)

$\Rightarrow v^2 = \frac{3}{2}ga = 14.7a$

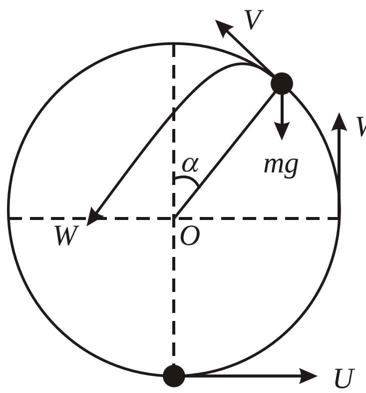
Resolving and using Force = $\frac{mv^2}{r}$, $T + mg = m.\frac{\frac{3}{2}ga}{\frac{1}{2}a}$ M1A1

(M1 needs three terms, but any v)

$\Rightarrow T = 2mg$ A1 6

[15]

7.



(a) Energy $\frac{1}{2}m(U^2 - v^2) = mga(1 + \cos\alpha)$ M1A1=A1

(T+) mg cos $\alpha = \frac{mv^2}{a}$ M1A1

Leaves circle when $T = 0$

$g \cos\alpha = \frac{U^2 - 2ga - 2ga\cos\alpha}{a}$ Eliminating v M1

Leading to $U^2 = ag(2 + 3\cos\alpha)$ * cso A1 7

(b) Using conservation of energy from the lowest point of the surface

$\frac{1}{2}m(U^2 - W^2) = mga$ M1A1=A1

$W^2 = U^2 - 2ag$

Using $\cos\alpha = \frac{1}{\sqrt{3}}$, $W^2 = ag\left(2 + \frac{3}{\sqrt{3}}\right) - 2ag$ M1

$= ag\sqrt{3}$ * cso A1 5

Alternative using conservation of energy from the point where P loses contact with surface.

$\left(V^2 = ag\cos\alpha = \frac{ga}{\sqrt{3}}\right)$

Energy $\frac{1}{2}m(W^2 - V^2) = mga\cos\alpha$ M1A1

$\frac{1}{2}m\left(W^2 - \frac{1}{\sqrt{3}}ag\right) = mga \times \frac{1}{\sqrt{3}}$ A1

Leading to $W^2 = ag\sqrt{3}$ * cso M1A1

Alternative using projectile motion from the point where P loses contact with surface.

$V^2 = ag\cos\alpha = \frac{ga}{\sqrt{3}}$

$\downarrow W_y^2 = V^2\sin^2\alpha + 2ga\cos\alpha$

$= \frac{1}{\sqrt{3}}ag\left(1 - \frac{1}{3}\right) + 2ga \times \frac{1}{\sqrt{3}} = \frac{8\sqrt{3}}{9}ag$ M1A1

$\leftarrow V_x = V\cos\alpha$ A1

$W^2 = W_\gamma^2 + V_x^2 = \frac{8\sqrt{3}}{9}ag + \frac{1}{3}ag\sqrt{3} \times \frac{1}{3} = ag\sqrt{3}$ * cso M1A1

[12]

8. (a) Energy: $\frac{1}{2}m.3ag - \frac{1}{2}mv^2 = mga(1+\cos\theta)$ M1 A1

$\underline{v^2 = ag(1 - 2\cos\theta)}$ **(o.e.)** A1 3

(b) $T + \text{mg}\cos\theta = m\frac{v^2}{a}$ M1 A1

Hence $\underline{T = (1 - 3\cos\theta)mg}$ (*) A1 cso 3

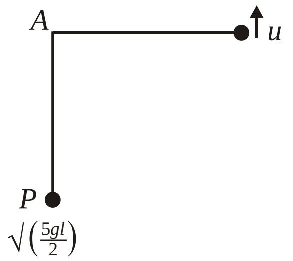
(c) Using $T = 0$ to find $\cos\theta$ M1

Hence height above $A = \underline{\frac{4}{3}a}$ Accept $1.33a$ (but must have 3+ s.f.) A1 2

(d)	$v^2 = \frac{1}{3}ag$ (**o.e.**)	f.t. using $\cos\theta = \frac{1}{3}$ in v^2	B1ft	
	consider vert motion: $(v\sin\theta)^2 = 2gh$	(with v resolved)	M1A1	
	$\sin^2\theta = \frac{8}{9}$ (or $\theta = 70.53$, $\sin\theta = 0.943$) and solve for h (as ka)		M1	
	$\underline{h = \frac{4}{27}a}$ or $0.148a$ (awrt)		A1	5
OR	consider energy: $\frac{1}{2}m(v\cos\theta)^2 + mgh = \frac{1}{2}mv^2$ (3 non-zero terms)		M1A1	
	Sub for v, θ and solve for h		M1	
	$\underline{h = \frac{4}{27}a}$ or $0.148a$ (awrt)		A1	

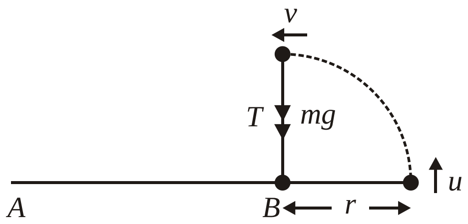
[13]

9. (a)



Conservation of Energy		
$\frac{1}{2}m\left(\frac{5gl}{2} - u^2\right) = mgl$	M1 A1 = A1	
Leading to $u = \sqrt{\left(\frac{gl}{2}\right)}$	A1	4

(b)



Conservation of Energy

$\frac{1}{2}m(u^2 - v^2) = mgr$ M1 A1

$v^2 = u^2 - 2gr$

$R(\downarrow)\ T + mg = \frac{mv^2}{r}$ M1 A1

$T = \frac{m}{r}(u^2 - 2gr) - mg$ M1

$= \frac{mu^2}{r} - 3mg$ A1

$= \frac{mgl}{2r} - 3mg$ M1

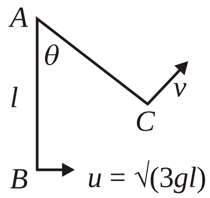
$T \geq 0 \Rightarrow \frac{mgl}{2r} \geq 3mg$ M1

$\Rightarrow \frac{1}{6} \geq r$

$AB_{MIN} = \frac{5l}{6}$ A1 9

[13]

10. (a)



Energy $\frac{1}{2}m(u^2 - v^2) = mgl\,(1 - \cos\theta)$ M1 A1

$[v^2 = gl + 2gl\cos\theta]$

N2L $T - mg\cos\theta = \frac{mv^2}{l}$ M1 A1

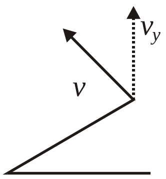
$\frac{mg\cancel{l}(1 + 2\cos\theta)}{\cancel{l}}$ M1

$T = mg\,(1 + 3\cos\theta)$ * cso A1 6

(b) $T = 0 \Rightarrow \cos\theta = -\frac{1}{3}$ B1

$v^2 = gl - \frac{2}{3}gl \Rightarrow v = \left(\frac{gl}{3}\right)^{1/2}$ M1 A1 3

(c) i $v_y = \left(\frac{gl}{3}\right)^{1/2}\sin\theta \quad \left[\left(\frac{gl}{3}\right)^{1/2}\frac{2\sqrt{2}}{3}\right]$ M1

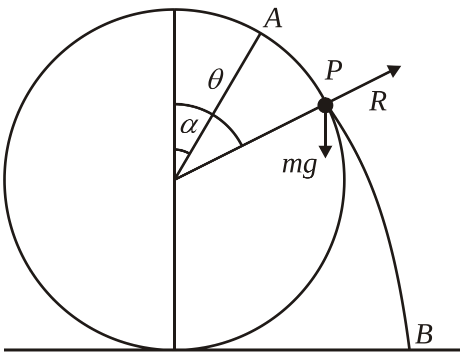


$v^2 = u^2 - 2gh \Rightarrow 2gh = \frac{gl}{3}.\frac{8}{9} \Rightarrow h = \frac{4l}{27}$ M1 A1

$H = l\,(1 - \cos\theta) + \frac{4l}{27} = \frac{40l}{27}$ M1 A1 5

[14]

11.



(a)	$\frac{1}{2}mv^2 = mg(\underline{\alpha\cos\alpha - a\cos\theta})$	M1 A1 $\underline{\text{A1}}$	
	$v^2 = 2ga(\cos\alpha - \cos\theta)$ (*)	cso A1	4
(b)	$[mg\cos\theta\,(-R) = \frac{mv^2}{a}$ $\quad(R = 0)$	M1 A1=A1	
	$g\cos\theta = 2g\left(\frac{3}{4} - \cos\theta\right)$	M1	
	$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$	A1	5
	accept 60°		
(c)	From A to B $\quad\frac{1}{2}mw^2 = mg(\underline{a + a\cos\alpha})$	M1 A1 $\underline{\text{A1}}$	
	$w^2 = 2ga\left(1 + \frac{3}{4}\right) \Rightarrow w = \left(\frac{7ga}{2}\right)^{\frac{1}{2}}$	A1	4
	Alternative		
	From P to C		
	$v_P^2 = 2ga\left(\frac{3}{4} - \frac{1}{2}\right) = \frac{ga}{2}$		
	$\frac{1}{2}mw^2 - \frac{1}{2}m\left(\frac{ga}{2}\right) = mg(\underline{a + a\cos\theta})$	M1 A1 $\underline{\text{A1}}$	
	$w^2 - \frac{ga}{2} = 2mga\left(1 + \frac{1}{2}\right) \Rightarrow w = \left(\frac{7ga}{2}\right)^{\frac{1}{2}}$	A1	4

Alternatives using projectile motion from P

$v_P = \left(\frac{ga}{2}\right)^{\frac{1}{2}}$, as above

$\downarrow u_y = \left(\frac{ga}{2}\right)^{\frac{1}{2}} \sin 60° = \left(\frac{3ga}{8}\right)^{\frac{1}{2}}$

$\downarrow v_y^2 = u_y^2 + 2g \times \frac{3a}{2}, = \frac{27ga}{8}$ M1, A1

$\rightarrow u_x = \left(\frac{ga}{2}\right)^{\frac{1}{2}} \cos 60° = \left(\frac{ga}{8}\right)^{\frac{1}{2}}$ A1

$w^2 = u_x^2 + v_y^2 = \frac{ga}{8} + \frac{27ga}{8} = \frac{7ga}{2} \Rightarrow w = \left(\frac{7ga}{2}\right)^{\frac{1}{2}}$ A1 4

There are also longer projectile methods using time of flight

In outline, solving $\frac{3a}{2} = \left(\frac{3ga}{8}\right)^{\frac{1}{2}} t + \frac{1}{2} gt^2$ gives $t = \left(\frac{3a}{2g}\right)^{\frac{1}{2}}$,

then, using $v = u + at$ gives $v_y = \left(\frac{3ga}{8}\right)^{\frac{1}{2}} + g\left(\frac{3a}{2g}\right)^{\frac{1}{2}} = \left(\frac{27ga}{8}\right)^{\frac{1}{2}}$,

then as before. M1 A1

[13]

12. (a) $\frac{1}{2} m(v^2 - 15) = mg5(1 - \cos 60°)$ M1 A1 A1

$\underline{v = 8 \text{ ms}^{-1}}$ A1 4

(b) $\frac{1}{2} Mw^2 = Mg5(1 - \cos 60°)$ M1 A1

<u>CLM</u>: $60 \times 8 - 3m = (60 + m)w$ M1 A1ft A1

$480 - 3m = 420 + 7m$

$60 = 10m$ M1 (solving form)

$\underline{6 = m}$ A1 7

	(c)	$T - 66g = \dfrac{66 \times 7^2}{5}$	M1 A1ft	
		$T = 132g$		
		$= \dfrac{1290 \text{ N}}{1300 \text{ N}}$	A1	3

[14]

13.		*All M marks require correct number of terms with appropriate terms resolved*		
	(a)	B to C: $\frac{1}{2}mv^2 - \frac{1}{2}m20^2 = mg \times 50(1 - \sin 30°)$	M1 A1	
		$v = 30 \text{ ms}^{-1}$ (29.8)	A1	3
	(b)	$(\uparrow)$ at C, $R - mg = m\dfrac{890}{50}$	M1 A1 ft	
		$R = 1900$ N (1930 N)	A1	3
	(c)	C to D: $\frac{1}{2}\ m\ 890 - \frac{1}{2}mw^2 = mg \times 50\ (1 - \cos 30°)$	M1 A1 ft	
		$w = 28 \text{ ms}^{-1}$ (27.5)	A1	3
	(d)	Before: $R = mg \cos\theta$	B1	
		After: $R = mg\cos\theta + m\ \dfrac{20^2}{50}$	M1 A1	
		Change $= 70 \times \dfrac{20^2}{50} = 560$ N	A1 c.s.o	4
	(e)	Lower speed at $C \Rightarrow$ R reduced	M1 A1	2

[15]

31.	(a)	Energy: $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga\sin\theta$	M1	
		$v^2 = \dfrac{3}{2}ga + 2ga\sin\theta$	A1	2
	(b)	Radial equation: $T - mg\sin\theta = m\dfrac{v^2}{a}$	M1A1	
		$T = \dfrac{3mg}{2}(1 + 2\sin\theta)$ any form	A1	3

(c)	Setting $T = 0$ and solving trig. equation; $(\sin\theta = -\frac{1}{2}) \Rightarrow \theta = 210°$ *	M1;A1	2
(d)	Setting $\boldsymbol{v} = \mathbf{0}$ in (i) and solving for θ	M1	
	$\sin\theta = -\frac{3}{4}$ so not complete circle	A1	2
	or (ii) looking at energy at top (mga) and at start ($\frac{3}{4}mga$) so not possible		
	or substituting $\theta = 270°$ in (a); $v^2 < 0$ so not possible to complete		
(e)	No change in PE $\Rightarrow$ no change in KE (Cof E) so $v = u$	B1	1
(f)	When string becomes slack, $V^2 = \frac{1}{2}ga$ [$\sin\theta = -\frac{1}{2}$ in (a)]	B1ft	
	Working horizontally: $\sqrt{\frac{ga}{2}}\cos 60° = \sqrt{\frac{3ga}{2}}\cos\phi$	M1A1	
	or vertically: $\frac{3}{2}ag\sin^2\phi = \frac{1}{2}ag.\sin^2 60° + ag(= \frac{11ag}{8})$		
	or finding V_V **and** V_H and using to find $\tan\phi$		
	ϕ = **73°** or **73.2°**	A1	4

[14]

14.	(a)	Energy: $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga(1 - \cos\theta)$	M1 A1 A1	
		Radial: $(\pm R) + mg\cos\theta = \dfrac{mv^2}{a}$	M1 A1	
		Eliminating v and finding $\cos\theta$ =.....; $\cos\theta\ \dfrac{u^2+2ga}{3ga}$	M1, A1	7

(b) Energy (C and ground): $\frac{1}{2}m\left(\frac{9ag}{2}\right) - \frac{1}{2}mv^2 = (m)ga(1 + \cos\theta)$ M1 A1

Substituting $v^2 = ga\cos\theta$ M1 A1

Finding $\cos\theta$ $\cos\theta = \frac{5}{6}$ M1 A1 ft

[ft on cos θ only if 0 < cos θ < 1]

$\theta = 34°$ A1

cao

[Or energy (A and ground): $\frac{1}{2}m\left(\frac{9ag}{2}\right) - \frac{1}{2}mu^2 = 2mga$ M1 A1

Substituting u^2 from (a)

Using with (a) to find $\cos\theta = \frac{5}{6}$ M1 A1 ft; $\theta = 34°$ A1 7

Projectile approach: $V_x = v\cos\theta$; $V_y^2 = (v\sin\theta)^2 + 2ga(1 + \cos\theta)$

$\left(\frac{9ag}{2}\right) = V_x^2 + V_y^2 \Rightarrow \left(\frac{9ag}{2}\right) - v^2 = 2ga(1 + \cos\theta)$ – M1 A1, then as scheme]

[14]

15. (a) $\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mga(1 + \cos 60°)$ M1 A1

$v^2 = u^2 - 3ga$ A1 3

(b) $R + mg\cos 60° = \frac{mv^2}{a}$ M1 A1

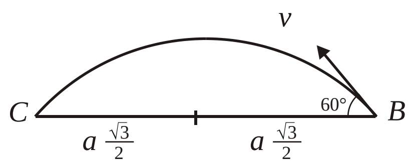
$R = \frac{m}{a}(6ga - 3ga) - \frac{mg}{2}$

$= \frac{5mg}{2}$ A1 3

(c) $R = 0$ at $B \Rightarrow \frac{mg}{2} = \frac{mv^2}{a} \Rightarrow v^2 = \frac{1}{2}ag$ M1

$\Rightarrow u^2 = \frac{7ga}{2} \Rightarrow u = \sqrt{\frac{7ga}{2}}$ M1 A1 3

(d)



($\rightarrow$) B to C: $v \cos 60° \times t = a\sqrt{3}$ M1 A1

$t = \dfrac{2a\sqrt{3}}{v}$

($\uparrow$) B to C: $0 = v \sin 60t - \frac{1}{2}gt^2$ M1 A1

$\Rightarrow t = \dfrac{2v\sin 60°}{g} = \dfrac{v\sqrt{3}}{g}$

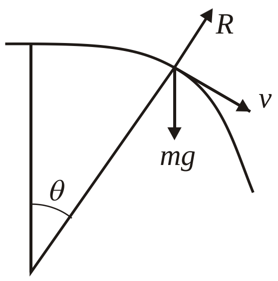
$\therefore \dfrac{2a\sqrt{3}}{v} = \dfrac{v\sqrt{3}}{g} \Rightarrow v^2 = 2ga$ M1 A1

$\Rightarrow u^2 = 5ga$

$\Rightarrow u = \sqrt{5ga}$ A1 7

[16]

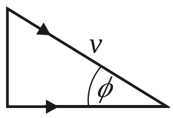
16. (a)



Energy $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(1 - \cos\theta)$ M1 A1

$v^2 = \frac{1}{2}ga + 2ga(1 - \cos\theta)$

$= \frac{1}{2}ga(5 - 4\cos\theta)$ A1 3

(b)	$R(\nearrow)$: $mg\cos\theta - R = \dfrac{mv^2}{a}$	M1 A1	
	so $R = mg(3\cos\theta - \frac{5}{2})$	A1	
	$\cos\theta = 0.9 \Rightarrow R = mg(2.7 - 2.5)$	M1	
	$= 0.2\,mg > 0 \Rightarrow P$ still on hemisphere		
	P leaves hemisphere when $R = 0 \Rightarrow 3\cos\theta - \frac{5}{2} = 0 \Rightarrow \cos\theta = \frac{5}{6}$	A1	5
	$\cos\theta = \frac{5}{6} \Rightarrow v^2 = \frac{1}{2}ga(5 - 4 \times \frac{5}{6})$	M1 A1	2
	$= \dfrac{5ga}{6}, \quad v = \sqrt{\dfrac{5ga}{6}}$	A1	2
	At B, speed v is given by $v^2 = u^2 + 2ga = \frac{5}{2}ga, \; v = \sqrt{\dfrac{5ga}{2}}$	A1	2
	A fter leaving hemisphere, horizontal component of velocity remains		
	constant $= \sqrt{\dfrac{5ga}{6}}\dfrac{5}{6}$	B1	
	$\cos\phi = \dfrac{\frac{5}{6}\sqrt{\frac{5ga}{6}}}{\sqrt{\frac{5ga}{2}}} = \dfrac{5}{6\sqrt{3}}$ 	M1	
	$\Rightarrow \phi = 61.2°$ or $61°$ to horizontal	A1	3

[17]

1. Most candidates were able to make a good start on this question and a considerable number scored highly. Part (a) was done well by the vast majority of students. A few used the bottom of the circle as their reference point for GPE, which did not necessarily produce an incorrect solution, but did require more work. The rare errors were sign errors, using cosine or failing to eliminate the half from the KE correctly. Part (b) was also straightforward for the majority, although some failed to include the component of the weight in their equation of motion and others made sign errors once again. The majority of students realized the use of the result from (b) was required to prove that the particle moved in complete circles and produced a convincing argument, though a few started from scratch. A lot of students thought that having v^2 greater than zero at the top was the required condition and many added this to an already correct solution; perhaps some thought that both $T > 0$ and $v^2 > 0$ were needed. The majority of students realized that $\theta = 90°$ was required, but some students produced convincing arguments based on the range of values for sin θ. Part (d) was done well by most students Again most used their result from (a), with a few solutions worked out from scratch using energy. The common error was to maximize v by simply stating $\theta = 90°$ or sin $\theta = 0$. This result not only ignores the mechanics but also shows a lack of imagination.

2. Part (a) was a straightforward application of the principle of conservation of energy and was done well by many candidates, although some needed several attempts before they arrived at the given answer. Those who tried to quote a single potential energy term involving only the general position, without any reference to the starting point, were misled into thinking they had obtained a correct result. Finding the height difference between the general position and the starting level proved to be much more challenging than it should have been and some candidates were thrown by having to deal with an angle with the horizontal rather than the more usual angle with the vertical.

Obtaining a complete solution to part (b) proved impossible for many candidates. Usually only the maximum value for T was found correctly, by using sinθ = 1; many thought that the minimum value was 0. Some candidates found the tension at the top of the "circle", even though their calculations showed that v^2 was negative at this point! It was rare to find a candidate who appreciated that the minimum tension would occur when $v = 0$.

3. Part (a) of this question was a fairly standard vertical circular motion question and many candidates could produce two equations by considering change in kinetic and potential energies and resolving along the radius. Some tried to resolve vertically with little success. The need to find a difference in potential energy between two points created problems for some candidates who seemed to think the particle either started at the top of the circle or from the horizontal level of A. The presence of a printed answer did enable some candidates to retrace their steps and correct the signs in their working. This was not a problem as long as the result was consistently correct. Part (b) produced a different set of problems. Some candidates thought that the string broke at the point where $T = 0$ and so found an incorrect value for the initial velocity of the projectile. The projectile problem could be solved by using vertical motion under gravity or by energy considerations. Many who opted for energy forgot that the particle was still moving (horizontally) at the highest point of its path; some of those who opted for vertical motion under gravity forgot that the initial velocity they had found needed to be resolved vertically before use in the appropriate equation. Most who had worked to this point in the question remembered to finish off by finding the distance below the horizontal through A.

4. Vertical circle questions usually present problems for many candidates and this one was no exception. Not all seem to be aware that an energy equation and an equation of motion along a radius (in this case at *C*) should be sufficient to make a sound start on the question. There were incorrect signs in the energy equation which were then adjusted later to arrive at the given result. Similarly, incorrect trigonometric functions became correct ones. Part (b) was attempted by most of those who had achieved success with (a) but the projectile motion defeated many. Some could not relate θ correctly to the horizontal and vertical components of the velocity at *C*. A variety of methods were seen in (b). Some used the separate components, finding the components at *P* and finishing off with the tangent of the required angle. Others used an energy approach, working from either *A* or *C* to obtain the final velocity at *P* and finished off with the horizontal component and the cosine. Some made extra work for themselves by finding the final velocity and the final vertical component, finishing off with the sine of the required angle. The usual mistakes such as using $v^2 = u^2 - 2as$ when energy was required occurred.

5. There were a lot of completely, or nearly completely, correct solutions to this question. Some lost the final A mark only for using m instead of $4m$. Most candidates who made an attempt at this question scored highly on it. They seemed well versed in the two significant principles of using Conservation of Energy and Newton's Second Law towards the centre. Unfortunately again it may have been a case of results learnt rather than mechanics principles understood. In part (a), either $v^2 = u^2 + 2ag(1 - \cos 60)$ or $v^2 = 2ag(1 - \cos 60)$ was used in the C of E equation rather than identifying a correct trigonometric expression to link the GPE at the two positions. This happened again in part (c)(i) when they used mgh as change in GPE and then had $\cos \theta = 1/16$ as their result for finding θ. Similarly in part (c)(ii) where $v = 0$ was the condition, too many thought that meant no acceleration and hence resolved vertically. In both (c)(i) and (ii) $4m$ was not used consistently.

6. Vertical circle solutions have continued to improve and candidates understood what was required. Part (a)was usually completed successfully with the printed answer providing a useful check. Part (b) caused more problems and was a good discriminator - some forgot the weight component, others resolved as mg $\cos \theta$ instead of mg $\sin \theta$. In part(c) many who had answered part (b) correctly achieved the first two marks but a significant group could not produce the correct angle.

Part (d) was often omitted. Some did not realise that the speed going up when the string was horizontal was the same as the original speed and wasted time on another energy equation. Some worked with a general position until the end which made the algebra more difficult.

Mistakes involved not using the top of the circle, having a wrong radius and having a negative mg in their Force = mass acceleration equation

7. Most candidates tackled this question well and, usually, if they were successful in part (a), they were also successful in part (b). As in Q2, however, part (b) was correctly answered more often than part (a). In part (a), among the less successful candidates, most were aware that they had to use an energy equation but a fairly common mistake was assuming the velocity to be zero at the point of leaving the surface. Some candidates used memorised formulae involving an acute angle θ measured with the downward vertical. A few of these managed successfully to justify the changes needed to produce the required result for angle α but more tried simply to manipulate their signs. In part (b), it was pleasing to see that the majority found the most efficient way of completing the question, considering the change of energy between the lowest point and the point on the horizontal through *O*. A significant minority took the approach of finding the velocity where the particle left the surface of the sphere and working from that point to the point on the horizontal through *O*. Often, though, they did not make it very clear that they were doing this and some lost track of their zero level for the gravitational potential energy. There were very few valid projectile attempts. Most candidates attempting this method forgot that they had to consider two components and, although a few managed to calculate the correct vertical component of *W*, successful attempts to combine this with a horizontal component were very rare.

8. Parts (a) and (b) were generally well done with the relevant principles clearly well known, and part (c) caused little problem. In part (d), only the better candidates realised that the particle still had some speed at its highest point, and several simply assumed that they had to find when it came to rest (which it never does!).

9. Part (a) was very well done. The only error that was seen at all commonly was having the difference in kinetic energies the wrong way round. In part (b) the error of thinking that $v = 0$ at the highest point of the semi-circle was widespread. This lead to a very brief solution which could gain at most two of the nine marks. Those who did write down a correct equation for the tension in the string often arrived at $AB = \frac{5l}{6}$ but had often assumed that $T = 0$ and were unable to give a justification for their result being a minimum. Inequalities remain unpopular with candidates in mechanics. A considerable number of candidates appeared to rely on memorised formulae involving an angle θ for both the energy and the tension without any clear idea of what θ was in this question. The introduction of such an angle is an unnecessary complication in this question, where only the highest point needed to be considered.

10. Part (a) was done either very well or very badly. Many candidates were completely comfortable with the theory and produced clear and concise derivations of T. Nearly everyone else managed to arrive at the same formula using some kind of fudged argument.. A favourite approach was $T = \frac{mv^2}{l} + mg$ and $v^2 = 3gl$ at B, so the tension in new position is mg(1 + 3cosθ). There were many variations on this. Again, there were versions that may have been correctly reasoned but weren't expressed clearly enough for the examiners. For instance, one proof was perfect up to the point where it stated ma for circular motion $= \frac{mv^2}{l} = mg(1 + 2\cos\theta)$ and then said "Must add $mg\cos\theta$ to account for the weight". Candidates should be under no illusion that this type of argument will receive no credit whatsoever.

Part (b) followed on nicely from (a) for the majority who used a result obtained for v^2 in (a) with $\cos\theta = -\frac{1}{3}$.

Part (c) did discriminate between the top few candidates and the rest. Very few recognized the need to use v_y, either as the initial velocity in $v^2 = u^2 + 2as$, or = 0 (with v_x still present) when using energy. Too many thought that finding the height when the string went slack or to when $v = 0$ was sufficient to answer this part.

11. Part (a) was well done with the printed answer enabling many candidates to correct incorrect signs. In such questions the examiners require the clear use of the principle of conservation of energy and the constant acceleration formula $v^2 = u^2 + 2as$ is not accepted, as it stands, without some evidence that kinetic and potential energies have been used.

In part (b), a few thought that the particle left the sphere when the velocity was zero but the majority realised that the normal reaction between the sphere and the particle was zero and, apart from some confusion which was not infrequently seen between θ and α, could complete the question.

In part (c), there was a fairly even split between those use energy and those using projectile methods. The easiest solution, using conservation of energy from the initial position, was not often seen but, when it was, it was almost invariably completely correct. The projectile solutions were longer, harder and less frequently correct. Those using projectile methods often dealt only with the vertical component of velocity.

12. Part (a) was generally well done but the second part proved to be much more demanding. Some thought that energy was conserved in the impact and scored few marks and even those that realised that conservation of momentum was required were unable to complete the question due to poor algebraic skills – an early simplification gave a linear equation rather than a very complex quadratic. The method was generally known in part (c) but there were relatively few correct answers, either due to the wrong mass being used or else a failure to round off the answer to 2 sf or 3 sf because of the use of g = 9.8.

13. This section of the syllabus seems to be poorly understood by many and there was much evidence that some candidates were relying on formulae. The comments made above about the use of $mv2/r$ as an extra force (Q4) are equally applicable here. In (a) and (c) it was common to see $1/2\ mv2 - 1/2mu2 = mgr(1 - \cos\alpha)$ quoted and then used without apparent understanding of where the angle referred to as □could be found or that this version of the formula was not appropriate when the skier was travelling upwards after point C. The best solutions to these two parts, and there were a great many of them, showed a clear calculation of the height difference between the two points before consideration of the energy equation. In (b) and (d) the same reliance on formulae was apparent among weaker candidates. $R - mg\cos\alpha = mv2/\mathrm{r}$ was often quoted but not adapted for the vertical position required in (b). $R = mv2/\mathrm{r}$ was also very common. Even so, there were a lot of correct solutions, the vast majority of them losing a mark for giving the inappropriate 4 figure answer 1932N. There were very few completely correct solutions to (d), most candidates failing to realise that passage through point B caused a change in R from that required for equilibrium perpendicular to the incline to one which would allow motion in a circle. It was very common for candidates to apply $R - mg\cos\alpha = mv2/\mathrm{r}$ at two different points and then to subtract the answers. Answers to part (e) were often excellent. While a minority of candidates argued that the frictional force would have no effect as it was at all times perpendicular to the reaction (wrong, but at least showing some grasp of a basic principle), many gave correct and very lucid explanations of the relationship between friction, work, energy, speed and finally reaction.

14. There was some evidence of shortage of time with some scrappy and rushed answers towards the end of this question. It is perhaps not surprising then to report that it was very rare to see full marks gained here. The first two parts were familiar and candidates knew the principles, errors tending to be either with signs or wrong trigonometry. It is worth reminding candidates, however, that omission of a term, for example $mg\sin\theta$ with part (b), results in no marks being awarded. In part (c) many candidates translated "prove" as "verify" and lost marks.

There were some good answers to parts (d) and (e), but as with all questions where reasons and explanations are required some candidates lost the odd mark. Very few candidates successfully negotiated the last part.

15. The condition $u < \sqrt{(ag)}$, necessary for the particle to remain on the sphere initially, was clearly misunderstood by some candidates who, at some stage in the question, equated u^2 to ag. This happened less frequently in part (a) but in part (b) it was a more common error.

In part (a) the majority of candidates made some progress, usually having the energy equation, and often the radial equation of motion, substantially correct. This is a fairly common piece of bookwork but the manipulation required did catch out a significant number of candidates who either made sign errors, set $v = 0$ at C, or "lost" either g or a.

Section (b) caused major problems and only the better candidates scored well in this part. Those who considered an energy approach rather than a projectile approach usually fared better, although there was much confused thinking in either. Candidates who considered the motion from C to the ground often confused their "u" with that given in the question; the energy equation $\frac{1}{2}m\left(\frac{9ag}{2}\right) - \frac{1}{2}mu^2 = mga(1 + \cos\theta)$ with u^2 replaced by the expression in part (a) was common.

Those candidates who used a projectile approach often made a serious error and very rarely gained many marks. The commonest errors were, when working vertically, to use either $\sqrt{\frac{9ag}{2}}$, or $\sqrt{\frac{9ag}{2}}\sin\theta$ instead of $\sqrt{\frac{9ag}{2}}\sin\phi$, as the final velocity component.

Those candidates who started their solution with $\left(\frac{9ag}{2}\right) - v^2 = 2ga(1 + \cos\theta)$, without any reference to energy, did lose marks. Although it is only a factor of $\frac{1}{2}m$ away from a correct energy equation, the fact that most candidates preceded their equation with "using $v^2 = u^2 + 2as$" sheds considerable doubt on the thinking! It is also a consideration that candidates using the projectile approach needed to have done a few steps of working before reaching the same point in their solution.

16. In part (a) most candidates knew that energy methods were required but there were often sign errors or incorrect terms. Similarly, the radial resolution in the second part often contained an incorrect trig ratio or further sign errors. Many candidates assumed in part (c) that $v = 0$ at B was sufficient for the marble to reach B instead of considering the reaction. The final part was a good discriminator with only a few fully correct solutions. Most realised that the marble was now moving as a projectile but only a small number were able to write down correct equations for the horizontal and vertical motion, eliminate t and solve for u. A significant minority just assumed that the marble would follow the missing part of the sphere and obtained a correct answer but scored no marks!

17. No Report available for this question.