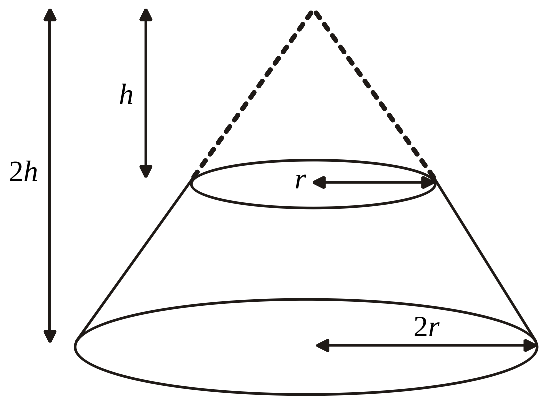
**1.**




A uniform solid $S$ is formed by taking a uniform solid right circular cone, of base radius $2r$ and height $2h$, and removing the cone, with base radius $r$ and height $h$, which has the same vertex as the original cone, as shown in the diagram above.

(a) Show that the distance of the centre of mass of $S$ from its larger plane face is $\frac{11}{28}h$ .

**(5)**

The solid $S$ lies with its larger plane face on a rough table which is inclined at an angle $\theta°$ to the horizontal. The table is sufficiently rough to prevent $S$ from slipping.
Given that $h = 2r$,

(b) find the greatest value of $\theta$ for which $S$ does not topple.

**(3)**
**(Total 8 marks)**

**1.** (a)

| | Large cone | small cone | $S$ | | |
|---|---|---|---|---|---|
| Vol. | $\frac{1}{3}\pi(2r)^2(2h)$ | $\frac{1}{3}\pi r^3 h$ | $\frac{7}{3}\pi r^2 h$ | (accept ratios 8 : 1 : 7) B1 | |
| C of M | $\frac{1}{2}h$ | $\frac{5}{4}h$ | $\bar{x}$ | (or equivalent) B1, B1 | |

$\frac{8}{3}\pi r^2 h.\frac{1}{2}h - \frac{1}{3}\pi r^2 h.\frac{5}{4}h = \frac{7}{3}\pi r^2 h.\bar{x}$ or equivalent M1

$\rightarrow \bar{x} = \frac{11}{28}h*$ A1 5

(b) $\tan\theta = \frac{2r}{\bar{x}} = \frac{2r}{\frac{11}{28}h}, = \frac{2r}{\frac{11}{14}r} = \frac{28}{11}$ M1, A1

$\theta \approx 68.6°$ or 1.20 radians A1 3

(Special case – obtains complement by using $\tan\theta = \frac{2r}{\bar{x}}$

giving 21.4° or .374 radians M1A0A0)

Centres of mass may be measured from another point (e.g. centre of small circle, or vertex) The Method mark will then require a complete method (Moments and subtraction) to give required value for $\bar{x}$). However B marks can be awarded for correct values if the candidate makes the working clear.

**[8]**

**1.** Part (a) was well done, with the aid of the given answer. There was some uncertainty concerning the formula for the volume of a cone but most had the correct dimensions and ratios, and were not penalised. The position of the centroid of a cone was well known by most, though a common error was to use h/4 instead of 5h/4 for the smaller cone. Difficulties did arise from using a different point and then attempting to deduce the correct answer. Moments about a point O were taken without any attempt to say where it was in many cases, though a few candidates used a calculus method from first principles and were successful.

In part (b) candidates mainly used $\tan\theta = \frac{2r}{\bar{x}}$,and obtained the correct answer. The common mistakes involved using r instead of 2r and finding the complementary angle. It was noticeable that candidates who failed to prove the required result in (a) generally made no attempt at (b), even though the information needed had been supplied.