1. The rudder on a ship is modelled as a uniform plane lamina having the same shape as the region R which is enclosed between the curve with equation $y = 2x - x^2$ and the x-axis.

(a) Show that the area of R is $\frac{4}{3}$.

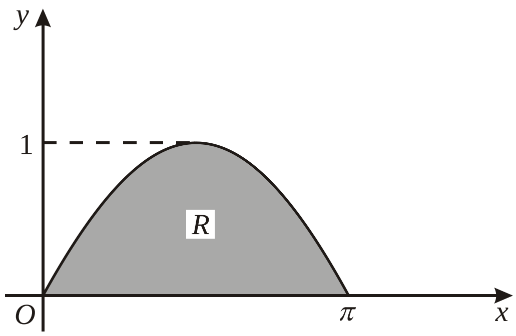
(4)

(b) Find the coordinates of the centre of mass of the lamina.

(5)
(Total 9 marks)

2.

Figure 1



A uniform lamina occupies the region R bounded by the x-axis and the curve

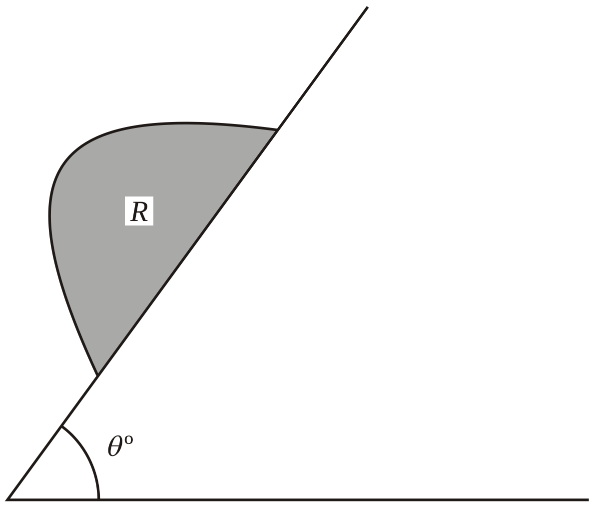
$$y = \sin x, \qquad 0 \leq x \leq \pi,$$

as shown in Figure 1.

(a) Show, by integration, that the y-coordinate of the centre of mass of the lamina is $\frac{\pi}{8}$.

(6)

Figure 2



A uniform prism S has cross-section R. The prism is placed with its rectangular face on a table which is inclined at an angle θ to the horizontal. The cross-section R lies in a vertical plane as shown in Figure 2. The table is sufficiently rough to prevent S sliding. Given that S does not topple,

(b) find the largest possible value of θ.

(3)
(Total 9 marks)

1. (a) $A = \int_0^2 (2x - x^2)\,dx$ M1A1

$= \left[x^2 - \frac{x^3}{3}\right]_{...}^{...}$ A1

$A = \left[x^2 - \frac{x^3}{3}\right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$ * cso A1 4

(b) $\bar{x} = 1$ (by symmetry) B1

$\frac{4}{3}\bar{y} = \frac{1}{2}\int y^2\,dx = \frac{1}{2}\int (2x - x^2)^2\,dx$ M1

$= \frac{1}{2}\int (4x^2 - 4x^3 + x^4)\,dx$ A1

$= \frac{1}{2}\left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5}\right]$ A1

$\frac{4}{3}\bar{y} = \frac{1}{2}\left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5}\right]_0^2 = \frac{8}{15}$

$\bar{y} = \frac{8}{15} \times \frac{3}{4} = \frac{2}{5}$ accept exact equivalents A1 5

[9]

2. (a) $\int_0^{\pi} \frac{1}{2} y^2\,dx = \int_0^{\pi} \frac{1}{2}\sin^2 x\,dx$ M1

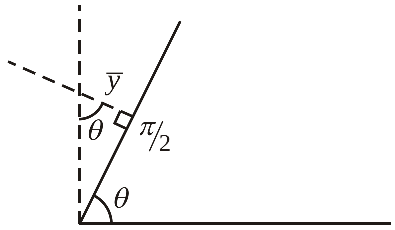
$= \frac{1}{4}\int_0^{\pi} (1 - \cos 2x)\,dx$ M1

$= \frac{1}{4}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi}$ A1

$= \frac{\pi}{4}$ A1

$\bar{y} = \frac{\pi/4}{\int_0^{\pi} \sin x\,dx} = \frac{\frac{\pi}{4}}{2} = \frac{\pi}{8}$ A1 6

(b)



$\tan\theta = \dfrac{\pi/2}{\bar{y}}$ M1

$= 4$ A1ft

$\underline{\theta = 75.96°, 76°}$ A1 3

$\underline{75.9°, 76.0°}$

[9]

1. This question was a straightforward introduction to the paper. In part (a), nearly all recognised the use of integration and found the appropriate limits. In part (b), the majority used integration to find the *x*-coordinate instead of using the symmetry of the figure. This did waste time and was not always correctly done. Those who could remember a correct formula for the *y*-coordinate usually completed the question correctly.

2. The formula required in part (a) was not always well-known and even those that did quote it correctly were not always able to cope with the resulting integral. The second part was totally independent and was generally well-answered.