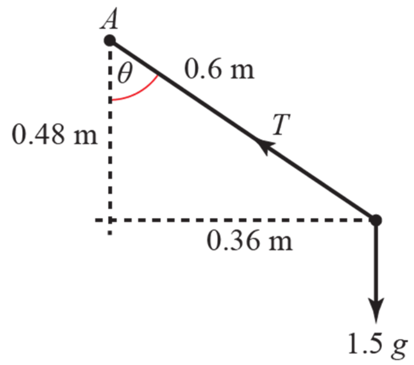
Exercise 4C

1



Let the tension in the string be T, and the angular speed be ω. The angle between the string and the vertical is θ. Since the triangle is right angled, the third side will have length 0.48 m (3, 4, 5 triangle).

$R(\updownarrow): T\cos\theta = 1.5g$

$$\Rightarrow \frac{4}{5}T = \frac{3g}{2},\ T = \frac{15g}{8} \approx 18.4\,\text{N}$$

$R(\leftrightarrow): T\sin\theta = mr\omega^2$

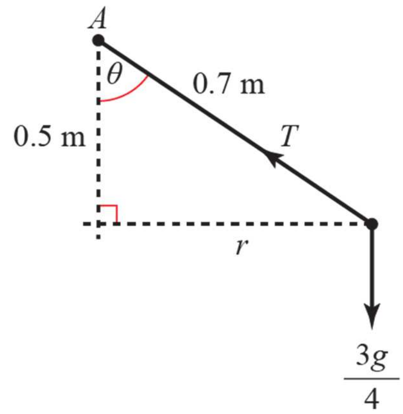
$$\Rightarrow \frac{3}{5}T = \frac{3}{2} \times 0.36 \times \omega^2,\ T = 0.9\omega^2$$

Equating the two expressions for T:

$$\frac{15g}{8} = 0.9\omega^2$$

$$\omega^2 = \frac{15g}{0.9 \times 8} \approx 20.41,\ \omega \approx 4.52\,\text{rad s}^{-1}$$

2



Let the tension in the string be T, and the angular speed be ω. The angle between the string and the vertical is θ, and the radius of the circle is r.

$R(\updownarrow): T\cos\theta = \frac{3g}{4}$

$$\Rightarrow \frac{5}{7}T = \frac{3g}{4},\ T = \frac{21g}{20} \approx 10.3\,\text{N}$$

$R(\leftrightarrow): T\sin\theta = mr\omega^2$

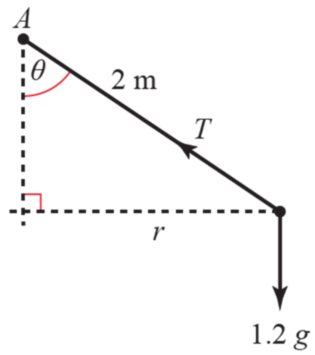
$$\Rightarrow \frac{r}{0.7}T = \frac{3}{4} \times r \times \omega^2,\ T = \frac{3}{4} \times 0.7\omega^2$$

Equating the two expressions for T:

$$\frac{21g}{20} = \frac{3}{4} \times 0.7\omega^2$$

$$\omega^2 = \frac{7g}{5 \times 0.7} \approx 19.6,\ \omega \approx 4.43\,\text{rad s}^{-1}$$

3



Let the tension in the string be T, and the angular speed be ω. The angle between the string and the vertical is θ, and the radius of the circle is r. 2 seconds to complete 2π radians $\Rightarrow$ angular speed is π rad s^{-1}

$R(\updownarrow): T\cos\theta = 1.2g$

$R(\leftrightarrow): T\sin\theta = mr\omega^2$

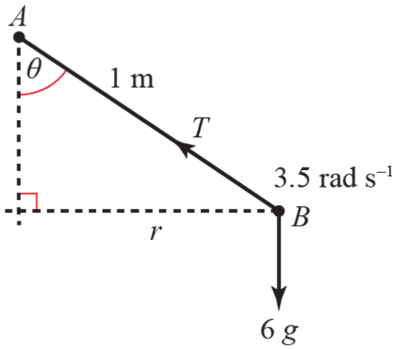
$$\Rightarrow T \times \frac{r}{2} = 1.2 \times r \times \pi^2, T = 2.4\pi^2$$

$$= 23.7\ \text{N}$$

and using this value in the first equation gives

$$\theta = \cos^{-1}\left(\frac{1.2g}{T}\right) \approx \cos^{-1} 0.496 \approx 60^\circ$$

4



Let the tension in the string be T. The angle between the string and the vertical is θ, and the radius of the circle is r.

$\mathrm{R}(\updownarrow): T\cos\theta = 6g$

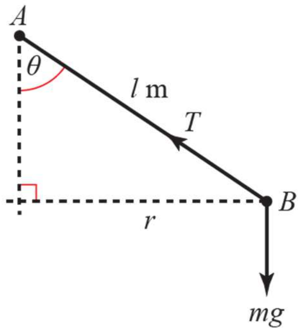
$\mathrm{R}(\leftrightarrow): T\sin\theta = 6\times r\times 3.5^2$

$$T\times\frac{r}{1} = 73.5r, T = 73.5\,\mathrm{N}$$

and using this value in the first equation gives

$73.5\cos\theta = 6g$, $\cos\theta = 0.8$ radius $= \sin\theta = 0.6\,\mathrm{m}$

5



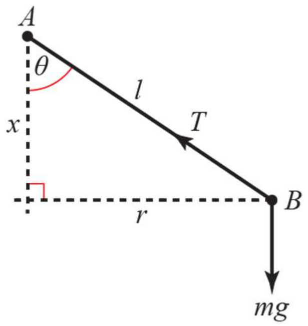
Let the tension in the string be T. The angle between the string and the vertical is θ, and the radius of the circle is r.

$\mathrm{R}(\leftrightarrow): T\sin\theta = m\times r\times\omega^2$

$$T\frac{r}{l} = m\times r\times\omega^2$$

$$T = ml\omega^2$$

6



Let the tension in the string be T. The angle between the string and the vertical is θ, and the radius of the circle is r.

$\mathrm{R}(\leftrightarrow): T\sin\theta = m\times r\times\omega^2$

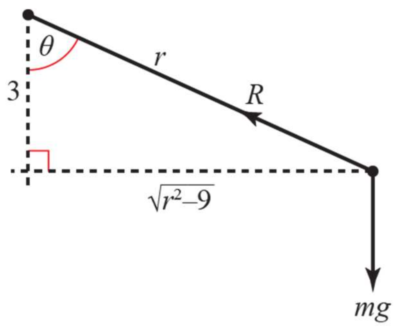
$\mathrm{R}(\updownarrow): T\cos\theta = mg$

Dividing the first equation by the second

$$\Rightarrow \tan\theta = \frac{mr\omega^2}{mg}$$

$$\frac{r}{x} = \frac{r\omega^2}{g},\ \omega^2 x = g$$

7



R is the normal reaction at the marble.

Using geometry, we know that the radius at the marble is perpendicular to the tangent at that point, so R acts along this radius.

θ is the angle between the radius and the vertical.

Using Pythagoras' theorem we know that the radius of the circle is $\sqrt{r^2-9}$.

$\mathrm{R}(\leftrightarrow): R\sin\theta = m\sqrt{r^2-9}\,\omega^2$

$$R\frac{\sqrt{r^2-9}}{r} = m\sqrt{r^2-9}\,\omega^2$$

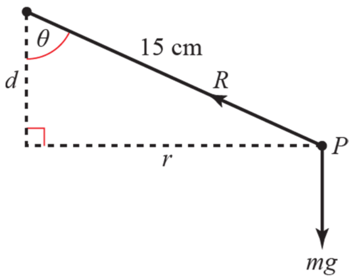
$$\omega^2 = \frac{R}{mr}$$

$\mathrm{R}(\updownarrow): R\cos\theta = mg = R\times\frac{3}{r}, R = \frac{mgr}{3}$

Substituting this expression for R in the first equation:

$$\omega^2 = \frac{mgr}{3mr} = \frac{g}{3},\ \omega = \sqrt{\frac{g}{3}}\ \mathrm{rad\ s^{-1}}$$

8



R is the normal reaction at the marble. Using geometry, we know that the radius at P is perpendicular to the tangent at that point, so R acts along this radius.

θ is the angle between the radius of the bowl and the vertical.

The particle moves on a circle of radius r m, depth d m below the rim of the bowl.

$\text{R}(\updownarrow): R\cos\theta = mg \quad \textbf{(1)}$

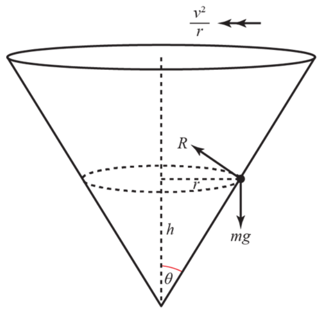
$\text{R}(\leftrightarrow): R\sin\theta = mr\omega^2 \quad \textbf{(2)}$

Dividing $\textbf{(2)} \div \textbf{(1)}$ eliminate R,

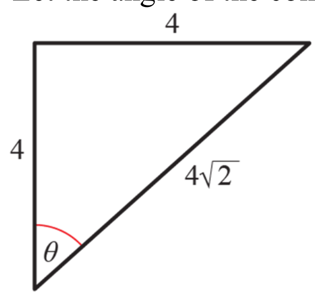
$$\tan\theta = \frac{r\omega^2}{g} = \frac{r}{d}$$

$$\Rightarrow d = \frac{g}{\omega^2} \approx \frac{9.8}{196} = 0.05\text{m} = 5\,\text{cm}$$

9



Let the angle of the cone be 2θ and the radius of the circle the particle is moving r.



$$\text{R}(\leftarrow):\; R\cos 45^\circ = m\frac{v^2}{r}$$

$$\text{R}(\leftarrow):\; R\sin 45^\circ = mg$$

$$\text{Dividing: } \cot 45^\circ = \frac{v^2}{rg}$$

$$= \frac{h}{r} = 3$$

$$v^2 = 3g$$

$$v = \sqrt{3g}$$

$$= 5.4249...$$

$$= 5.4\text{ m s}^{-1} \text{ (2 s.f.)}$$

$$\omega = \frac{v}{r}$$

$$= \frac{5.4249...}{3}$$

$$= 1.8083...$$

$$= 1.8\text{ rad s}^{-1} \text{ (2 s.f.)}$$

10 $R(\uparrow): R\sin\theta - F\cos\theta = mg$

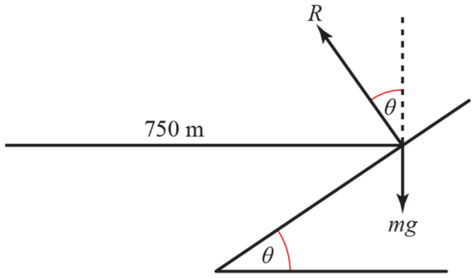
$R(\rightarrow): R\cos\theta + F\sin\theta = m\times 0.1\times 14^2\times 5$

Using $F = \mu R = m\times 0.1\times 14^2\times 5 = m\times 0.3\times \omega^2$,

we have

$\omega = 14\sqrt{\frac{5}{3}} = 18.1\text{ rad s}^{-1}(3\text{ s.f.})$

11



No friction, so just the normal reaction, R, between the car and the road with a horizontal component.

$$126\,\text{km h}^{-1} = \frac{126\times 1000}{3600} = 35\,\text{m s}^{-1}$$

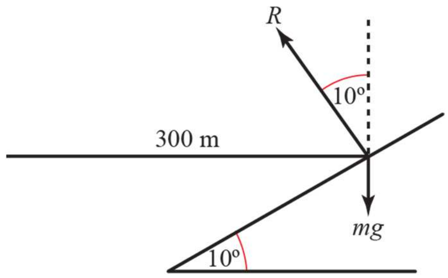
$R(\updownarrow): R\cos\theta = mg$ **(1)**

$R(\leftrightarrow): R\sin\theta = m\frac{v^2}{r} = m\times\frac{35^2}{750}$ **(2)**

Dividing **(2)** ÷ **(1)** to eliminate R and m

$$\Rightarrow \tan\theta = \frac{35^2}{750g} \approx 0.167,\ \theta \approx 9.5^\circ$$

12



No friction, so just the normal reaction, R, between the car and the road with a horizontal component.

$R(\updownarrow): R\cos 10^\circ = mg$

$R(\leftrightarrow): R\sin 10^\circ = \frac{mv^2}{r} = \frac{mv^2}{300}$

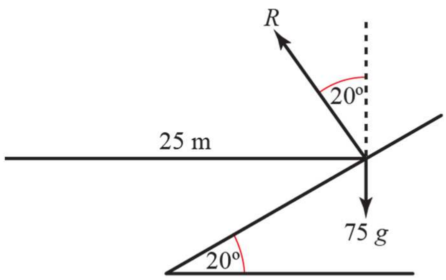
Dividing to eliminate R

$$\Rightarrow \tan 10^\circ = \frac{mv^2}{300mg}$$

$$v^2 = 300g\tan 10^\circ = 518.4\ldots$$

$$v \approx 22.8\,\text{m s}^{-1}$$

13



No friction, so just the normal reaction, R, between the cycle and the road with a horizontal component.

$R(\updownarrow): R\cos 20^\circ = 75g$

$R(\leftrightarrow): R\sin 20^\circ = \frac{75\times v^2}{25}$

Dividing to eliminate R

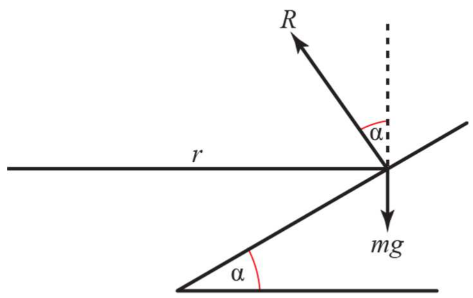
$$\Rightarrow \tan 20^\circ = \frac{75v^2}{25\times 75g} = \frac{v^2}{25g}$$

$$v^2 = 25g\tan 20^\circ = 89.172\ldots$$

$$v \approx 9.44\,\text{m s}^{-1}$$

It was not necessary to know the value of the mass because it cancels out at the stage when the two equations are combined to find $\tan 20^\circ$.

14 a



No friction, so just the normal reaction, R, between the vehicle and the road with a horizontal component.

$R(\updownarrow): R\cos\alpha = mg$

$R(\leftrightarrow): R\sin\alpha = \frac{mv^2}{r}$

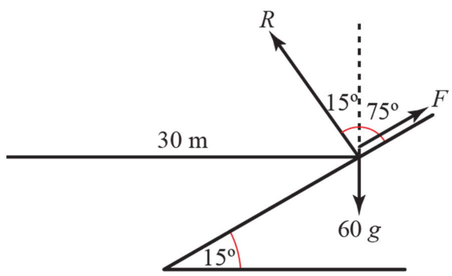
Dividing to eliminate R

$\Rightarrow \tan\alpha = \frac{mv^2}{rmg} = \frac{v^2}{rg}$

$v^2 = rg\tan\alpha, v = \sqrt{rg\tan\alpha}$

b This model assumes there is no friction between the tyres and the road.

15



R is the normal reaction between the cycle and the track. F is the force due to friction. At minimum speed the force due to friction is acting up the slope to stop the cycle from sliding down. (At maximum speed the friction will act down the slope to prevent sliding up the slope.)

As slipping is about to occur, $F = \mu R$.

$R(\updownarrow): R\cos 15^\circ + F\cos 75^\circ = 60g$

$R\left(\cos 15^\circ + \frac{\cos 75^\circ}{4}\right) = 60g$

$R(\leftrightarrow): R\cos 75^\circ - F\cos 15^\circ = 60 \times \frac{v^2}{30}$

$R\left(\cos 75^\circ - \frac{\cos 15^\circ}{4}\right) = 2v^2$

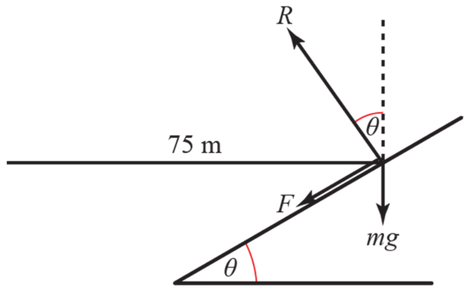
Dividing to eliminate R

$\Rightarrow \frac{\cos 75^\circ - \frac{\cos 15^\circ}{4}}{\cos 15^\circ + \frac{\cos 75^\circ}{4}} = \frac{2v^2}{60g}$

$v^2 = \frac{\cos 75^\circ - 0.25 \times \cos 15^\circ}{\cos 15^\circ + 0.25 \times \cos 75^\circ} \times 30g$

$v^2 = 4.94\ldots, \quad v \approx 2.22\,\text{m s}^{-1}$

16



R is the normal reaction between the van and the track. F is the force due to friction. At maximum speed the force due to friction is acting down the slope to stop the van from sliding up.
μ is the coefficient of friction between the tyres and the road.

As slipping is about to occur, $F = \mu R$.

$$90\,\text{km h}^{-1} = \frac{90 \times 1000}{3600} = 25\,\text{m s}^{-1}$$

$$\text{R}(\updownarrow): F\sin\theta + mg = R\cos\theta$$

$$\text{R}(\leftrightarrow): F\sin\theta + R\sin\theta = m \times \frac{25^2}{75}$$

Substituting $F = \mu R$

$$\Rightarrow mg = R(\cos\theta - \mu\sin\theta)$$

$$\frac{25m}{3} = R(\mu\cos\theta + \sin\theta)$$

Dividing to eliminate m

$$\Rightarrow \frac{25}{3g} = \frac{\mu\cos\theta + \sin\theta}{\cos\theta - \mu\sin\theta} = \frac{\mu + \tan\theta}{1 - \mu\tan\theta}$$

(on dividing top and bottom by $\cos\theta$)

$$= \frac{\mu + \frac{1}{3}}{1 - \frac{\mu}{3}} \left(\text{using } \tan\theta = \frac{1}{3}\right)$$

$$= \frac{3\mu + 1}{3 - \mu}$$

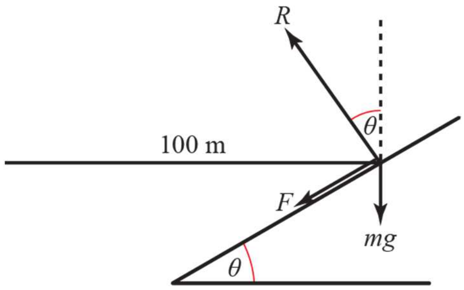
$$\Rightarrow 25(3 - \mu) = 3g(3 + 1)$$

Rearranging this equation gives

$$\mu(9g + 25) = 75 - 3g$$

$$\mu = \frac{75 - 3g}{9g + 25} \approx 0.40$$

17



R is the normal reaction between the car and the track. F is the force due to friction. At maximum speed the force due to friction is acting down the slope to stop the car from sliding up. The track is banked at θ to the horizontal.
As slipping is about to occur, $F = \mu R$.

$$144\,\text{km}\,\text{h}^{-1} = \frac{144 \times 1000}{3600} = 40\,\text{m}\,\text{s}^{-1}$$

$$\text{R}(\updownarrow): mg = R\cos\theta - F\sin\theta$$

$$\text{R}(\leftrightarrow): R\sin\theta + F\cos\theta = m \times \frac{40^2}{100}$$

Substituting $F = \mu R$ and dividing to eliminate m

$$\Rightarrow \frac{\sin\theta + 0.3\cos\theta}{\cos\theta - 0.3\sin\theta} = \frac{40^2}{100g}$$

Dividing top and bottom of the left-hand side by $\cos\theta$

$$\Rightarrow \frac{\tan\theta + 0.3}{1 - 0.3\tan\theta} = \frac{1600}{100g} = \frac{16}{g}$$

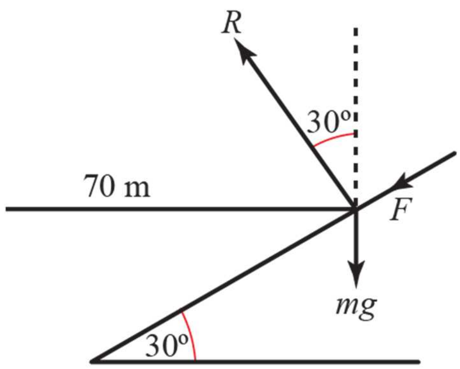
$$g(\tan\theta + 0.3) = 16(1 - 0.3\tan\theta)$$

$$\tan\theta(g + 4.8) = 16 - 0.3g$$

$$\tan\theta = \frac{16 - 0.3g}{g + 4.8} = 0.894\ldots$$

$$\theta \approx 42^\circ$$

18



R is the normal reaction between the car and the track, F is force due to friction. At maximum speed F acts down the slope and is equal to $\mu R = 0.4R$

$$\text{R}(\updownarrow): R\cos 30^\circ - F\sin 30^\circ = mg$$

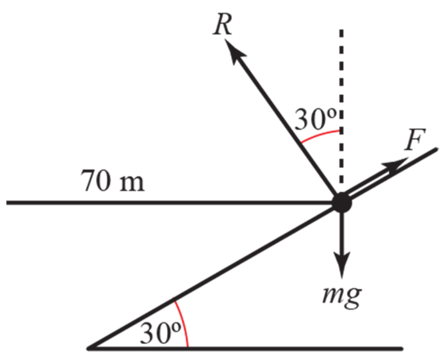
$$\text{R}(\leftrightarrow): F\cos 30^\circ + R\sin 30^\circ = \frac{mv}{70}$$

Substituting $F = 0.4R$ and dividing

$$\Rightarrow \frac{v^2}{70g} = \frac{0.4\cos 30^\circ + \sin 30^\circ}{\cos 30^\circ - 0.4\sin 30^\circ}$$

$$\Rightarrow v^2 = 871.7\ldots$$

$$\Rightarrow v \approx 29.5\,\text{m}\,\text{s}^{-1}$$



At minimum speed, F acts up the slope

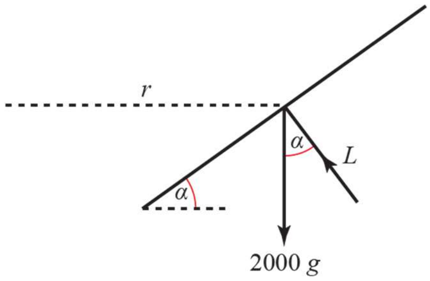
$$\text{R}(\updownarrow): R\cos 30^\circ + F\sin 30^\circ = mg$$

$$\text{R}(\leftrightarrow): R\sin 30^\circ - F\cos 30^\circ = \frac{mv^2}{70}$$

which leads to $\dfrac{v^2}{70g} = \dfrac{\sin 30^\circ - 0.4\cos 30^\circ}{\cos 30^\circ + 0.4\sin 30^\circ}$

giving $v^2 = 98.83\ldots \Rightarrow v \approx 9.94\,\text{m}\,\text{s}^{-1}$

19 a



L is the lift force, and r is the radius of the circular arc.

$$400\,\text{km h}^{-1} = \frac{400 \times 1000}{3600} = \frac{1000}{9}\,\text{m s}^{-1}$$

In 25 seconds the aircraft travels $\frac{25\,000}{9}$ m.

The direction changes by 45° or 315° depending on the direction of turning.

For $45^\circ, \frac{25\,000}{9} = \frac{1}{8} \times 2\pi r, r = 3536.7\ldots$ m

For $315^\circ, \frac{25\,000}{9} = \frac{7}{8} \times 2\pi r, r = 505.25$m

$R(\updownarrow): L\cos\alpha = 2000g$

$R(\leftrightarrow): L\sin\alpha = \frac{2000v^2}{r}$

Dividing the second equation by the first

$\Rightarrow \tan\alpha = \frac{v^2}{rg}$

When $r = 3536.7\ldots$

$\tan\alpha = \frac{1000^2}{9^2 \times 3537 \times 9.8} \approx 0.356$

$\alpha \approx 20^\circ$, and $L \approx 20\,800$N

When $r = 505.2\ldots$

$\tan\alpha = \frac{1000^2}{9^2 \times 505.3 \times 9.8} \approx 2.493$

$\alpha \approx 68^\circ$, and $L \approx 52\,700$ N

b To turn in a shorter time the aircraft will need to decrease the radius of the circular arc in which it turns. Thus the angle to the horizontal and lift force must both increase.

20 $R(\uparrow): T\sin\theta + R\sin\theta = mg$

$\Rightarrow T + R = \frac{mg}{\sin\theta}$

$R(\rightarrow): T\cos\theta - R\cos\theta = ml\cos\theta\omega^2$

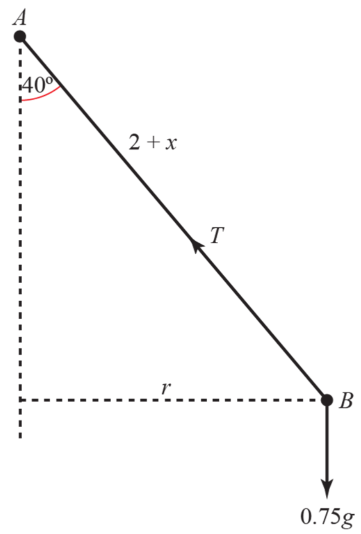
$\Rightarrow T - R = ml\omega^2$

Eliminating R:

$2T = \frac{mg}{\sin\theta} + ml\omega^2$

$T = \frac{1}{2}m\left(l\omega^2 + g\,\text{cosec}\,\theta\right)$

21



T is the tension in AB. AB is an elastic string, if the extension in the string is x then $T = \frac{\lambda x}{l} = \frac{30x}{2}$

$$R(\updownarrow): T\cos 40^\circ = 0.75g$$
$$R(\leftrightarrow): T\sin 40^\circ = 0.75 \times r \times \omega^2$$

The radius of the circle is $(2+x)\sin 40^\circ$, so substituting for r and T gives

$$15x\sin 40^\circ = 0.75 \times (2+x)\sin 40^\circ \times \omega^2$$
$$\Rightarrow \omega^2 = \frac{15x}{0.75(2+x)} = \frac{20x}{2+x}$$

From the first equation:

$$15x\cos 40^\circ = 0.75g$$
$$x = \frac{0.75g}{15\cos 40^\circ} = 0.639\ldots$$
$$\Rightarrow \omega^2 = 4.846,\ \omega \approx 2.2\,\text{rad s}^{-1}$$