Exercise 4B

1 $a = r\omega^2$: $a = 0.16 \times 25 = 4\,\text{m s}^{-2}$

2 $a = \dfrac{v^2}{r}$: $a = \dfrac{2.5^2}{0.3} \approx 20.8\,\text{m s}^{-2}$

3 **a** $a = r\omega^2$: $75 = 3\omega^2$, $\omega^2 = 25$, $\omega = 5\,\text{rad s}^{-1}$

b $a = \dfrac{v^2}{r}$: $75 = \dfrac{v^2}{3}$, $v^2 = 3 \times 75 = 225$, $v = 15\,\text{m s}^{-1}$

4 **a** $a = r\omega^2$: $100 = 0.6\omega^2$, $\omega^2 \approx 166.7$, $\omega \approx 12.9\,\text{rad s}^{-1}$

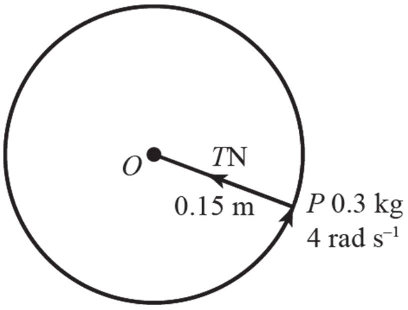
b $a = \dfrac{v^2}{r}$: $100 = \dfrac{v^2}{0.6}$, $v^2 = 100 \times 0.6 = 60$, $v = 7.75\,\text{m s}^{-1}$

5 $50\,\text{km h}^{-1} = \dfrac{50 \times 1000}{3600} \approx 13.89\,\text{m s}^{-1}$

$a = \dfrac{v^2}{r}$: $a = \dfrac{13.89^2}{90} \approx 2.14\,\text{m s}^{-2}$

6 $a = r\omega^2$: $6 = 75\omega^2$, $\omega^2 = 0.08$, $\omega \approx 0.283\,\text{rad s}^{-1}$

7



Suppose that the tension in the string is T.
Using $F = ma$

$T = 0.3 \times 0.15 \times 4^2 = 0.72\,\text{N}$

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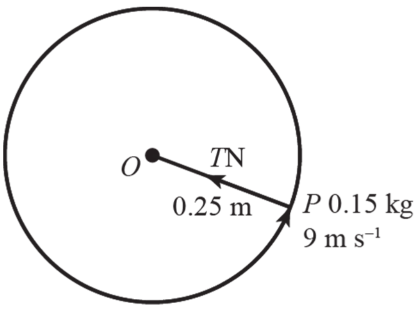
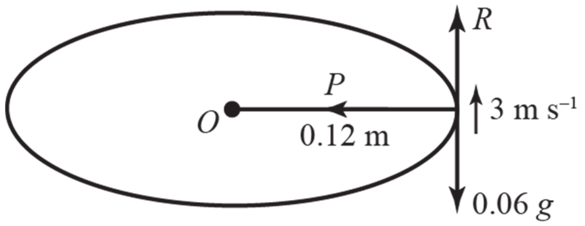


Suppose that the tension in the string is T.
Using $F = ma$

$T = \dfrac{0.15 \times 9^2}{0.25} = 48.6\,\text{N}$

9



a Suppose the vertical component of the force is R:
$R(\updownarrow): R = 0.06g = 0.588\,\text{N}$

b Suppose the horizontal component of the force is F:
Using $F = ma$,

$$F = \frac{0.06 \times 3^2}{0.12} = 4.5\,\text{N}$$

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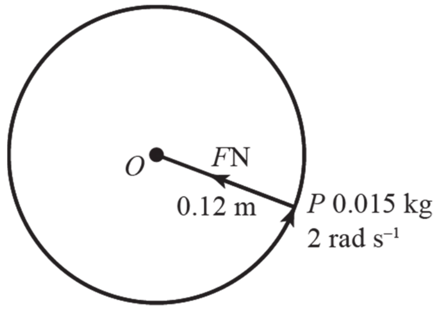

a $v = r\omega$: $v = 0.12 \times 2 = 0.24\,\text{m s}^{-1}$

b Using $F = ma$

$F = 0.015 \times 0.12 \times 2^2 = 0.0072\,\text{N}$

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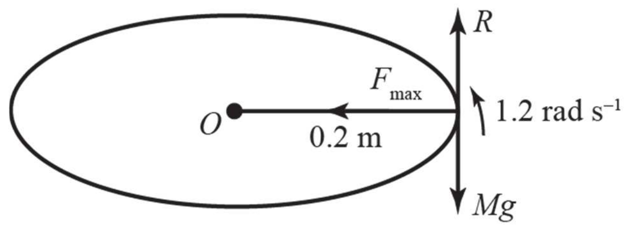

Let R be the normal reaction between the particle and the disc, F the frictional force, M the mass of the particle, and μ be the coefficient of friction between the particle and the disc.
$R(\updownarrow): R = Mg$
The particle is about to slip, so $F = F_{max} = \mu R = \mu Mg$.
Using $F = ma$, $\mu Mg = M \times 0.2 \times 1.2^2 = M \times 0.288$,

$$\mu = \frac{0.288}{g} \approx 0.0294$$

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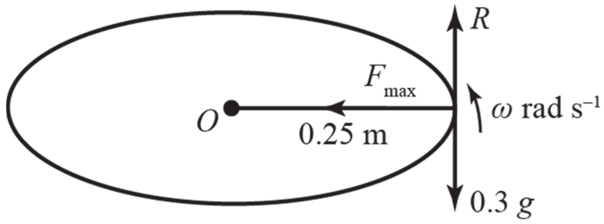


Let R be the normal reaction between the particle and the disc, F the frictional force.

Given $\mu = 0.25$ and $F = F_{max} = 0.25R$.

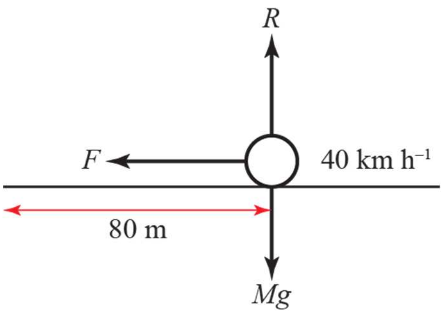
$R(\updownarrow): R = 0.3g$, so $F_{max} = 0.25 \times 0.3g$.

Using $F = ma$, $0.25 \times 0.3g = 0.3 \times 0.25 \times \omega^2$,

$g = \omega^2$

$\omega \approx 3.13 \text{ rad s}^{-1}$

13



$40 \text{ km h}^{-1} = \dfrac{40 \times 1000}{3600} \approx 11.11 \text{ m s}^{-1}$

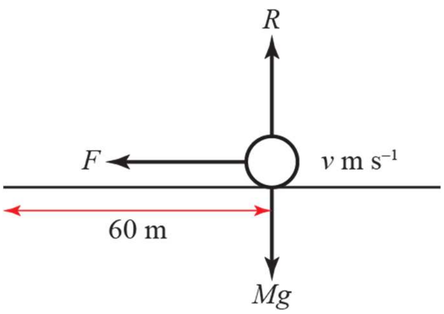
Let the mass of the car be M. Let F be the force due to friction between the car tyres and the road, μ the coefficient of friction, and R the normal reaction between the car and the road.

At maximum speed the car is about to slip, so $F = F_{max}$

$R(\updownarrow): R = Mg$, so $F = F_{max} = \mu R = \mu Mg$

$R(\leftrightarrow)$: Using $F = ma$, $\mu Mg = \dfrac{M \times 11.11^2}{80}$, $\mu = \dfrac{11.11^2}{80 \times 9.8} \approx 0.157$

14



Let the mass of the car be M.

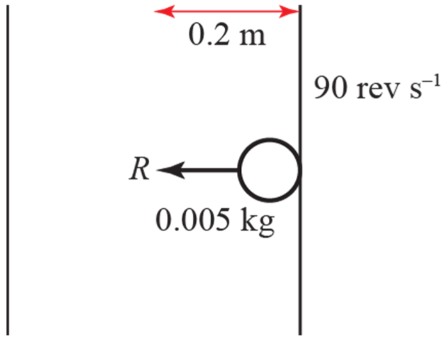
Let F be the force due to friction between the car tyres and the road, and R the normal reaction between the car and the road.

Max speed $\Rightarrow F = F_{max}$

$R(\updownarrow): R = ma$, so $F = F_{max} = \mu R = \dfrac{1}{3}Mg$

$R(\leftrightarrow)$: Using $F = ma$, $\dfrac{1}{3}Mg = M \times 60 \times \omega^2$, $\omega^2 = \dfrac{g}{180} \approx 0.0544$, $\omega \approx 0.233 \text{ rad s}^{-1}$

15 a



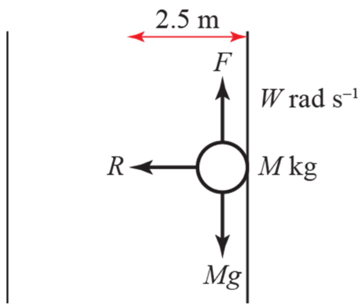
$90 \text{ rev s}^{-1} = 90 \times 2\pi \text{ rad s}^{-1}$
$= 180\pi \text{ rad s}^{-1}$

Let the normal reaction between the particle and the cylinder be R.

$R(\leftrightarrow)$: Using $F = ma$, $R = 0.005 \times 0.2 \times (180\pi)^2 = 319.775... \approx 320$ N (2 s.f.)

b $F = mg$
$= 0.005 \times 9.8$
$R = 319.775$
$\mu = \frac{F}{R}$
$= 1.5 \times 10^{-4}$
$= 0.00015$ (2 s.f.)

16 a



Suppose that the person has mass M. Let the normal reaction between the person and the cylinder be R. F is the frictional force between the person and the wall of the cylinder.

Minimum $W \Rightarrow$ the person is about to slip $\Rightarrow F = F_{max} = \frac{2}{3}R$

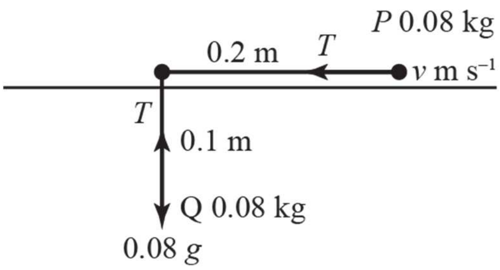
Also $R(\updownarrow) \Rightarrow F_{max} = Mg$, so $Mg = \frac{2}{3}R$, $R = \frac{3Mg}{2}$

$R(\leftrightarrow)$: Using $F = ma$,

$R = \frac{3Mg}{2} = M \times 2.5 \times W^2$, $W^2 = \frac{3g}{5}$, $W = \sqrt{\frac{3g}{5}} \approx 2.42 \text{ rad s}^{-1}$

b No, because it is the minimum possible value for W. If the speed or the coefficient of friction reduced at all, the people would slip down the cylinder.

17



Let the tension in the string be TN, and the speed of P be v m s^{-1}
Q is in equilibrium, so
$R(\updownarrow)$ at $Q \Rightarrow T = 0.08g$

For P, Using $F = ma$,

$T = 0.08g = \frac{0.08v^2}{0.2}$, $v^2 = 0.2 \times g \approx 1.96$, $v = 1.4 \text{ ms}^{-1}$

18 a Let the frictional force between the car tyre and the road be F, and the coefficient of friction be μ. The normal reaction between the car and the road is R.

$$\text{R}(\uparrow): R = Mg$$

$$\text{R}(\leftarrow): F = \frac{mv^2}{r}$$

The car does not slip at this speed,

$$F > \mu R$$

$$\mu > \frac{Mv^2}{MgR}$$

$$\mu > \frac{v^2}{gR}$$

b Model assumes that the tyres all experience the same friction.

19 If the extension in the spring is x m, then the radius of the circle is $(0.3 + x)$.

The tension in the string is given by $T = \frac{\lambda x}{a} = \frac{10x}{0.3}$

Using $F = ma$,

$$\frac{10x}{0.3} = 0.25 \times (0.3 + x) \times 3^2$$

$$10x = \frac{9}{4} \times \frac{3}{10}(0.3 + x)$$

$$400x = 8.1 + 27x$$

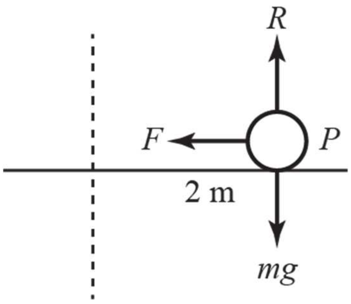
$$373x = 8.1$$

$$x = 0.02171...$$

$$\text{Radius} = 0.3 + 0.0217$$

$$= 0.322 \text{ m (3 s.f.)}$$

20



F is force due to friction,

R is the nornal reaction.

$\text{R}(\updownarrow): R = mg$

$\text{R}(\leftrightarrow): F = mr\omega^2$

If P is not to slip then

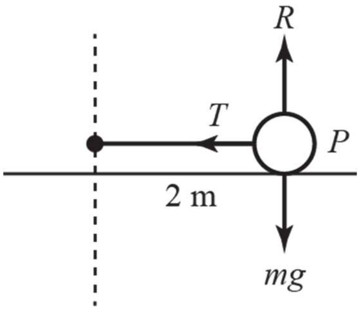
$0.3 \times 4\text{g} \geqslant 4 \times 2 \times \omega^2$

$\therefore \omega^2 \leqslant \frac{147}{100}$

T is the tension in the elastic string:

$T = \frac{\lambda x}{l} = \frac{12 \times (2 - 1.5)}{1.5}$

$= 4 \text{ N}$



The limits for ω^2 depends on whether the friction is acting with the tension or against it:

$\text{R}(\leftrightarrow): 0.3 \times 4 \times g + 4 \geqslant 4 \times 2 \times \omega^2$

$\omega^2 = \frac{197}{100}$

$\text{R}(\leftrightarrow): -0.3 \times 4 \times g + 4 \leqslant 4 \times 2 \times \omega^2$

$\omega^2 \geqslant \frac{97}{100}$

$\frac{\sqrt{97}}{10} \leqslant \omega \leqslant \frac{\sqrt{197}}{10}$

Challenge

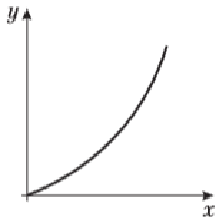
a

$x = pt$

$t = \frac{x}{p}$

$y = q\left(\frac{x}{p}\right)^2$

$= \frac{q}{p^2}x^2$



b $\frac{dx}{dt} = p; \ \frac{dy}{dt} = 2qt$

Acceleration is $2q$ in the positive y-direction. Speed at the origin is p.

c Solve for y,

$x^2 + (y - R)^2 = R^2$

$(y - R)^2 = R^2 - x^2$

$y = R \pm \sqrt{R^2 - x^2}$

For lower half of a circle

$y = R - \sqrt{R^2 - x^2}$

d $R = \frac{p^2}{2q}$

e $2q$

f The acceleration of P and Q are equal.