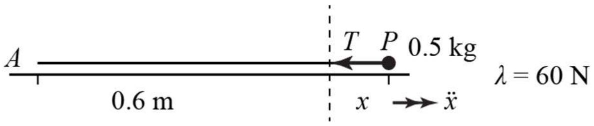
Exercise 3D

1



a $F = ma$

$-T = 0.5\ddot{x}$

Hooke's law: $T = \dfrac{\lambda x}{l}$

$$T = \frac{60}{0.6} = 100x$$

$$-100x = 0.5\ddot{x}$$

$$\ddot{x} = -\frac{100}{0.5}x$$

$$\ddot{x} = -200x$$

$\therefore$ S.H.M.

The equation of motion must reduce to the form $\ddot{x} = -\omega^2 x$.

b $\omega^2 = 200 \quad \omega = \sqrt{200} = 10\sqrt{2}$

$$\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{10\sqrt{2}} = \frac{\pi}{10}\sqrt{2}$$

$\therefore$ period is $\dfrac{\pi}{10}\sqrt{2}$ s (or 0.444 s (3 s.f.))

amplitude $= 0.9 - 0.6 = 0.3$

$\therefore$ amplitude is 0.3 m

The amplitude is the same as the initial extension.

c $v^2 = \omega^2(a^2 - x^2)$

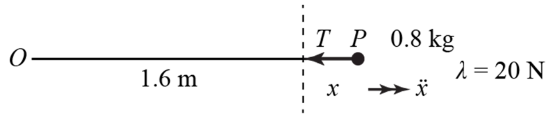
$v_{max} = \omega a = 10\sqrt{2} \times 0.3$

$= 3\sqrt{2}$

Use $x = 0$ for the maximum speed.

The maximum speed is $3\sqrt{2}\,\text{m s}^{-1}$ or 4.24 m s^{-1} (3 s.f.)

2



a $F = ma$

$-T = 0.8\ddot{x}$

Hooke's Law: $T = \dfrac{\lambda x}{l}$

$$T = \frac{20}{1.6}x$$

$$-\frac{20}{1.6}x = 0.8\ddot{x}$$

$$\ddot{x} = -\frac{20x}{1.6 \times 0.8} = -\frac{10x}{0.8^2}$$

$\therefore$ S.H.M.

b

$$\omega = \frac{\sqrt{10}}{0.8}$$

$$\therefore \text{period} = \frac{2\pi}{\omega} = 2\pi \times \frac{0.8}{\sqrt{10}} = \frac{1.6\pi}{\sqrt{10}}$$

amplitude $= 2.6 - 1.6 = 1\,\text{m}$

$$v^2 = \omega^2(a^2 - x^2)$$

$$v_{max} = \omega a = 1 \times \frac{\sqrt{10}}{0.8}$$

The oscillation is split into 2 parts which are twice the natural length apart

total distance at this speed $= 4 \times 1.6$

$= 6.4\,\text{m}$

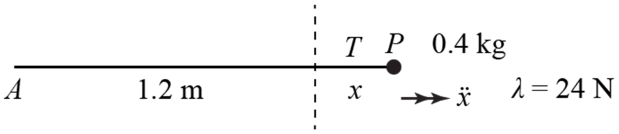
$$\text{time} = 6.4 \times \frac{0.8}{\sqrt{10}}$$

For the middle section the particle moves at a constant speed (= the maximum speed of the S.H.M.)

$$\therefore \text{total time} = 6.4 \times \frac{0.8}{\sqrt{10}} + \frac{1.6\pi}{\sqrt{10}} = 3.208\ldots$$

The total time is 3.21 s (3 s.f.)

3



a $F = ma$

$-T = 0.4\ddot{x}$

Hooke's Law: $T = \dfrac{\lambda x}{l}$

$$T = \frac{\lambda x}{l}$$

$$T = \frac{24x}{1.2} = 20x$$

$\therefore -20x = 0.4\ddot{x}$

$$\ddot{x} = -\frac{20}{0.4}x$$

$$\ddot{x} = -50x$$

$\therefore$ S.H.M.

b For the impact $I = mv - mu$

$$1.8 = 0.4v$$

$$v = \frac{1.8}{0.4} = 4.5$$

This is the speed of P while the string is slack. It is also the maximum speed for the S.H.M.

$$\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{5\sqrt{2}}$$

The required time includes half a period.

$\therefore$ time for half an oscillation $= \dfrac{\pi}{5\sqrt{2}}$ s

time at constant speed

$$= \frac{0.2}{4.5} = \frac{2}{45}\text{ s}$$

P travels 0.2 m before the string becomes taut.

$$\text{total time} = \frac{\pi}{5\sqrt{2}} + \frac{2}{45} = 0.4887...$$

time is 0.489 s (3 s.f.)

c $v^2 = \omega^2(a^2 - x^2)$

ω and the maximum speed are known so the amplitude can be found.

$v_{\text{max}} = 4.5\,\text{m}\,\text{s}^{-1}$

$\therefore 4.5 = a\omega$

$$a = \frac{4.5}{5\sqrt{2}}$$

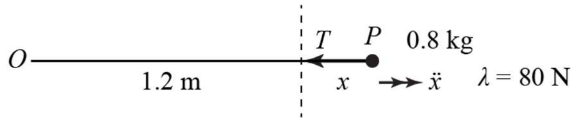
$$AB = 1.2 + \frac{4.5}{5\sqrt{2}}$$

$$= 1.836$$

AB is the natural length of the string plus the amplitude of the S.H.M.

Distance AB is 1.84 m (3 s.f.)

4



a $F = ma$

$-T = 0.8\ddot{x}$

Hooke's Law: $T = \dfrac{\lambda x}{l}$

$$T = \frac{80x}{1.2}$$

$$0.8\ddot{x} = -\frac{80}{1.2}x$$

$$\ddot{x} = -\frac{100}{1.2}x$$

$\therefore$ SHM

b $\omega = \sqrt{\dfrac{100}{1.2}} = \dfrac{10}{\sqrt{1.2}}$

$$\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{10}\sqrt{1.2}$$

$$= 0.6882...$$

period is 0.688 s (3 s.f.)

amplitude = 1.2 – 0.6 = 0.6 m

c $v^2 = \omega^2(a^2 - x^2)$

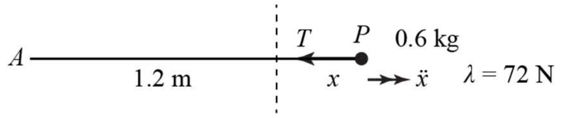
$$v_{\max} = \omega a$$

$$= \frac{10}{\sqrt{1.2}} \times 0.6$$

$$= 5.477...$$

The max speed is 5.48 m s^{-1} (3 s.f.)

5



a $F = ma$

$-T = 0.6\ddot{x}$

Hooke's Law: $T = \dfrac{\lambda x}{l}$

$T = \dfrac{72x}{1.2} = 60x$

$\therefore -60x = 0.6\ddot{x}$

$\ddot{x} = -\dfrac{60}{0.6}x$

$\ddot{x} = -100x$

$\therefore \omega^2 = 100, \omega = 10$

Use $F = ma$ and Hooke's Law to obtain the value of ω.

For the impact: $I = mv - mu$

$3 = 0.6v - 0$

$v = \dfrac{3}{0.6} = 5$

Use impulse = change of momentum to obtain the maximum speed.

$\therefore$ maximum speed is 5 m s^{-1}

$v^2 = \omega^2(a^2 - x^2)$

Now the amplitude can be obtained.

$v_{\max} = \omega a$

$5 = 10a$

$a = \dfrac{5}{10} = 0.5$

$x = a\sin\omega t$

P is at the centre of the oscillation when $t = 0$.

$\therefore x = 0.5\sin 10t$

b $\ddot{x} = -100x$

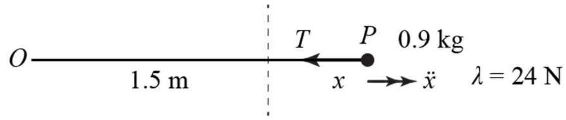
$|\ddot{x}| = 100|x|$

The amplitude gives the maximum value of $|x|$.

$|\ddot{x}|_{\max} = 100 \times 0.5 = 50$

The maximum magnitude of the acceleration is 50 m s^{-2}.

6



a amplitude $= (2 - 1.5)\text{ m} = 0.5\text{ m}$

b energy; K.E. gained $= \frac{1}{2}mv^2 = \frac{1}{2}\times 0.9v^2$

$$\text{E.P.E. lost} = \frac{\lambda x^2}{2l} = \frac{24\times 0.5^2}{2\times 1.5}$$

$$\frac{1}{2}\times 0.9v^2 = 24\times\frac{0.5^2}{2\times 1.5}$$

$$v^2 = \frac{2\times 24\times 0.5^2}{0.9\times 2\times 1.5}$$

$$v = 2.108...$$

The speed is 2.11 m s^{-1} (3 s.f.)

b can be solved by using conservation of energy or by S.H.M. methods, finding the maximum speed for the oscillation.

c **i** Impact with the wall:

Newton's law of impact : $eu = v$

$$\therefore v = \frac{3}{5}\times 2.108...$$
$$= 1.264...$$

$\therefore$ maximum speed for the new oscillation is 1.264 m s^{-1}

$F = ma$

$-T = 0.9\ddot{x}$

S.H.M. methods essential for this part.

Hooke's Law: $T = \frac{\lambda x}{l}$

$$T = \frac{24}{1.5}x = 16x$$

$$\therefore -16x = 0.9\ddot{x}$$

$$\ddot{x} = -\frac{16}{0.9}x$$

$$\therefore \omega = \frac{4}{\sqrt{0.9}}$$

$$\text{period} = \frac{2\pi}{\omega} = 2\pi\frac{\sqrt{0.9}}{4} = 1.490...$$

The period is 1.49s (3 s.f.).

ii $v^2 = \omega^2(a^2 - x^2)$

$v_{max} = \omega a$

Now ω is known you can find the amplitude using $v^2 = \omega^2(a^2 - x^2)$ with the maximum speed.

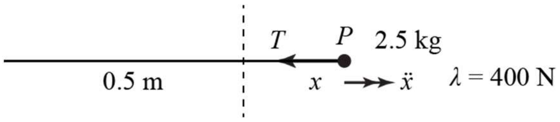
$$1.264 = \frac{4}{\sqrt{0.9}}a$$

$$a = 1.264\times\frac{\sqrt{0.9}}{4}$$

$$a = 0.2997$$

The amplitude is 0.300 m (3 s.f.)

7



a $F = ma$

$-T = 2.5\ddot{x}$

Hooke's Law: $T = \dfrac{\lambda x}{l}$

$$T = \frac{400x}{0.5} = 800x$$

$$-800x = 2.5\ddot{x}$$

$$\ddot{x} = -\frac{800}{2.5}x$$

$$\ddot{x} = -320x$$

$$\omega = \sqrt{320}$$

$$\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{320}} = 0.3512...$$

The period is 0.351 s (3 s.f.).

b $\text{amplitude} = (50 - 42)\,\text{cm}$

$$= 0.08\,\text{m}$$

$$v^2 = \omega^2(a^2 - x^2)$$

$$v_{\max} = \omega a$$

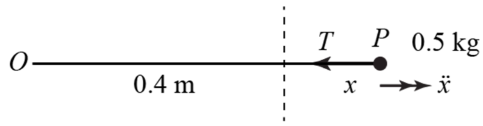
$$= \sqrt{320 \times 0.08}$$

$$\text{maximum K.E} = \frac{1}{2} \times 2.5 \times \left(\sqrt{320 \times 0.08}\right)^2$$

$$= 2.56$$

The maximum K.E. is 2.56 J.

8



a $F = ma$

$-T = 0.5\ddot{x}$

Hooke's Law: $T = \dfrac{\lambda x}{l}$

$T = \dfrac{30}{0.4}x = 75x$

$\therefore 0.5\ddot{x} = -75x$

$\ddot{x} = -\dfrac{75}{0.5}x$

$\ddot{x} = -150x$

$\therefore \omega = \sqrt{150}$

amplitude $= 0.6 - 0.4 = 0.2$ m

$v^2 = \omega^2(a^2 - x^2)$

$v_{\max} = a\omega$

$= \sqrt{150} \times 0.2$

$= 2.449...$

When the string becomes slack P's speed is 2.45 m s^{-1} (3 s.f.).

a can be done by conservation of energy but the period of the oscillation is needed for **b**.

b period $= \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\sqrt{150}}$

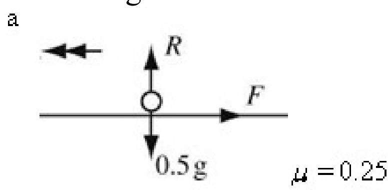
The first part of the motion is $\dfrac{1}{4}$ of an oscillation.

On the smooth floor:

time $= \dfrac{0.1}{2.449}$

For the first 0.2 m the string is taut.

On the rough floor:



$-F = 0.5a$

$F = \mu R = 0.25 \times 0.5g$

$\therefore 0.5a = -0.25 \times 0.5g$

$a = -0.25g$

Find the acceleration.

$v = u + at$

$0 = 2.449 - 0.25gt$

$t = \dfrac{2.449}{0.25 \times 9.8}$

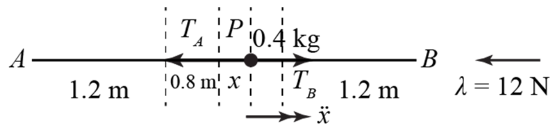
Use $v = u + at$ with $u = 2.449$ and $a = -0.25g$ to find the time taken to come to rest.

total time $= \dfrac{1}{4} \times \dfrac{2\pi}{\sqrt{150}} + \dfrac{0.1}{2.449} + \dfrac{2.449}{0.25 \times 9.8}$

$= 1.168...$

$\therefore T = 1.17$ N(3 s.f.)

9



The centre of the oscillation is at the mid-point of AB.

a $F = ma$

$T_B - T_A = 0.4\ddot{x}$

Hooke's Law : $T = \dfrac{\lambda x}{l}$

AP: extension $= (0.8 + x)$

$\therefore T_A = \dfrac{12(0.8 - x)}{1.2} = 10(0.8 + x)$

BP: extension $= (0.8 - x)$

$\therefore T_B = \dfrac{12(0.8 - x)}{1.2} = 10(0.8 - x)$

$\therefore 10(0.8 - x) - 10(0.8 + x) = 0.4\ddot{x}$

$-20x = 0.4\ddot{x}$

$$\ddot{x} = -\frac{20}{0.4}x = -50x$$

$\therefore$ P moves with S.H.M.

The tensions in the two parts of the string are different.

b $\omega^2 = 50$

amplitude $= 0.6$ m

$$v^2 = \omega^2(a^2 - x^2)$$

$$v_{\max}^2 = \omega^2 a^2$$

$$= 50 \times 0.6^2$$

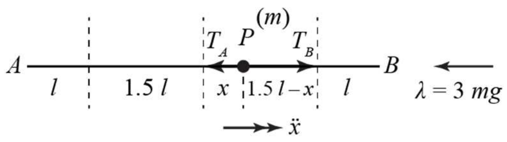
$$\text{maximum K.E.} = \frac{1}{2}mv_{\max}^2$$

$$= \frac{1}{2} \times 0.4 \times 50 \times 0.6^2$$

$$= 3.6$$

The maximum K.E. is 3.6 J.

10



The centre of the oscillation is at the mid-point of AB.

a $F = ma$

$T_B - T_A = m\ddot{x}$

Hooke's Law : $T = \dfrac{\lambda x}{l}$

extension $= 1.5l + x$

AP: $\quad T_A = \dfrac{3mg(1.5l + x)}{l}$

PB: extension $= 1.5l - x$

$T_B = \dfrac{3mg(1.5l - x)}{l}$

$$\therefore \frac{3mg(1.5l - x)}{l} - \frac{3mg(1.5l + x)}{l} = m\ddot{x}$$

$$-\frac{6mgx}{l} = m\ddot{x}$$

$$\ddot{x} = -\frac{6g}{l}x$$

$\therefore$ S.H.M.

b $\omega^2 = \dfrac{6g}{l} \quad \omega^2 = \sqrt{\dfrac{6g}{l}}$

period $= \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{l}{6g}}$

c Amplitude = $1.5l$

d $\quad v^2 = \omega^2(a^2 - x^2)$

$AP = 3l \Rightarrow x = \dfrac{1}{2}$

$$\therefore v^2 = \frac{6g}{l}\left(\left(\frac{3l}{2}\right)^2 - \left(\frac{l}{2}\right)^2\right)$$

$$v^2 = \frac{6g}{l}\left(\frac{9l^2}{4} - \frac{l^2}{4}\right)$$

$$v^2 = \frac{6g}{l} \times \frac{8l^2}{4}$$

$$v^2 = 12gl$$

When $AP = 3l, P$' s speed is $\sqrt{12gl}$ (or $2\sqrt{3gl}$).

11 a When P is in equilibrium:

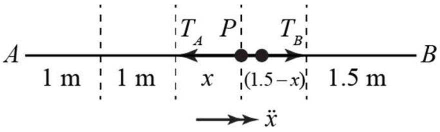
Use the ratio condition to obtain the necessary lengths for the two parts of the string.

$$AP = \frac{2}{5} \times 5 = 2\text{ m}$$

$$BP = 3\text{ m}$$

Natural lengths: $AP = 1\text{m}$

$$BP = 1.5\text{m}$$



$$F = ma$$

$$T_B - T_A = 0.5\ddot{x}$$

Hooke's Law : $T = \dfrac{\lambda x}{l}$

AP: extension $= 1 + x$

$$T_A = \frac{15(1+x)}{1}$$

BP: extension $= 1.5 - x$

$$T_B = \frac{15(1.5-x)}{1.5} = 10(1.5-x)$$

$$\therefore 10(1.5-x) - 15(1+x) = 0.5\ddot{x}$$

$$-25x = 0.5\ddot{x}$$

$$\ddot{x} = -50x$$

$\therefore$ S.H.M.

$$\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{50}} = \frac{2\pi}{5\sqrt{2}} = \frac{\pi}{5}\sqrt{2}$$

b Amplitude = (3 – 2)m = 1 m.