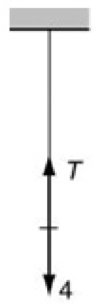
Exercise 2A

1



$(\uparrow)\ T = 4$ ← Hooke's law

$T = \frac{\lambda x}{3}$

So, $\frac{\lambda x}{3} = 4$

$\Rightarrow x = \frac{12}{\lambda}$

$\therefore$ Total length of string, $L = 3 + \frac{12}{\lambda}$

a $\lambda = 30:\ L = 3 + \frac{12}{30}$

$= 3.4$ m

b $\lambda = 12:\ L = 3 + \frac{12}{12}$

$= 4$ m

c $\lambda = 16:\ L = 3 + \frac{12}{16}$

$= 3.75$ m

2 By Hooke's law,

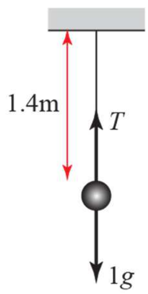
$20 = \frac{25(l - 0.8)}{l}$

$4l = 5l - 4$

$4 = l$

Natural length is 4 m

3



Let natural length be l

$(\uparrow)\ T = g = 9.8$

$T = \frac{20(1.4 - l)}{l}$

$9.8 = 20\frac{(1.4 - l)}{l}$

$9.8l = 28 - 20l$

$29.8l = 28 \Rightarrow l = \frac{28}{29.8} = \frac{140}{149}$

Let the new extension be x

$0.8g = \frac{20x}{\left(\frac{140}{149}\right)}$

$0.8g = \frac{\cancel{20}x \times 149}{\cancel{140}^{7}}$

$\frac{5.6g}{149} = x$

$x \approx 0.3683\ldots$

Total length of string is $0.3683 + \frac{140}{149}$

$= 1.31$ m (3 s.f.)

4 Let the initial extension be x_1

$(\uparrow)\ T = Mg$

$Mg = \frac{\lambda x_1}{a} \Rightarrow x_1 = \frac{Mga}{\lambda}$

When the mass m is added to the scale pan, extension is x_2

$(\uparrow)\ T = (M + m)g$

$(M + m)g = \frac{\lambda x_2}{a} \Rightarrow x_2 = \frac{(M + m)ga}{\lambda}$

$\therefore\ x_2 - x_1 = \frac{ga}{\lambda}(M + m - M) = \frac{mga}{\lambda}$

New equilibrium is $\frac{mga}{\lambda}$ below the old one.

5 $m_1 g = \dfrac{\lambda(a_1 - l)}{l}$ **(1)**

$m_2 g = \dfrac{\lambda(a_2 - l)}{l}$ **(2)**

Dividing **(1)** by **(2)**:

$$\frac{m_1}{m_2} = \frac{a_1 - l}{a_2 - l}$$

$$m_1(a_2 - l) = m_2(a_1 - l)$$

$$m_1 a_2 - m_1 l = m_2 a_1 - m_2 l$$

$$m_1 a_2 - m_2 a_1 = l(m_1 - m_2)$$

$$l = \frac{m_1 a_2 - m_2 a_1}{m_1 - m_2}$$

The natural length $l = \dfrac{m_1 a_2 - m_2 a_1}{m_1 - m_2}$

Subtracting **(2)** from **(1)**:

$$m_1 g - m_2 g = \frac{\lambda a_1}{l} - \lambda - \left(\frac{\lambda a_2}{l} - \lambda\right)$$

$$lg(m_1 - m_2) = \lambda(a_1 - a_2)$$

$$\lambda = gl\frac{(m_1 - m_2)}{(a_1 - a_2)}$$

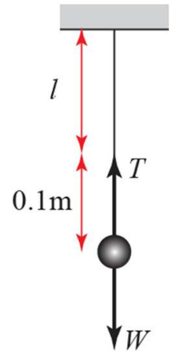
Substituting for l:

$$\lambda = g\frac{(m_1 - m_2)}{(a_1 - a_2)}\frac{(m_1 a_2 - m_2 a_1)}{(m_1 - m_2)}$$

$$= g\frac{(m_1 a_2 - m_2 a_1)}{(a_1 - a_2)}$$

The modulus of elasticity is $g\dfrac{(m_1 a_2 - m_2 a_1)}{(a_1 - a_2)}$

6

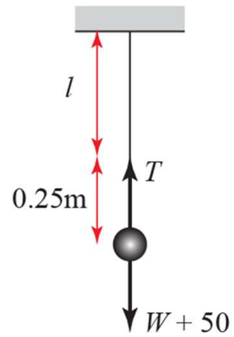


$(\uparrow)\ T = W$

$$T = \frac{\lambda \times 0.1}{l}$$

$$W = \frac{\lambda \times 0.1}{l}$$

So $\lambda = 10Wl$



$(\uparrow)\ T = W + 50$

$$T = \frac{\lambda \times 0.25}{l}$$

$$W + 50 = \frac{\lambda \times 0.25}{l}$$

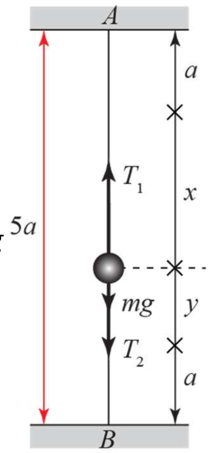
$$W + 50 = \frac{10Wl \times 0.25}{l}$$

$$W + 50 = \frac{10W}{4}$$

$$50 = \frac{3W}{2}$$

So $W = \dfrac{100}{3}$ N

7 a



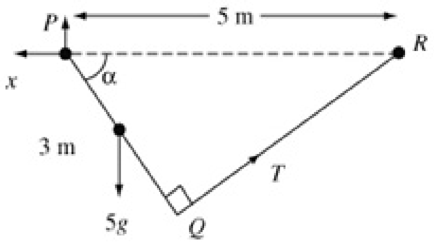
$$a + x + y + a = 5a$$
$$y = 3a - x$$
$$\lambda = 2mg$$
$$(\uparrow)\ T_1 = T_2 + mg$$
$$\frac{2mgx}{a} = \frac{2mg(3a-x)}{a} + mg$$
$$2x = 2(3a - x) + a$$
$$2x = 6a - 2x + a$$
$$4x = 7a$$
$$x = \frac{7a}{4}$$

Distance of particle from ceiling is

$$a + x = a + \frac{7a}{4} = \frac{11a}{4}$$

b If the spring is not light, then in effect the mass would increase, the extension would increase and hence the distance of the particle below the ceiling would increase.

8



$$PQR = 90° \Rightarrow QR = 4\text{m}$$
$$\cos\alpha = \frac{3}{5};\ \sin\alpha = \frac{4}{5}$$

a Taking moments about P:

$$5g \times \frac{3}{2}\cos\alpha = 3T$$
$$5g \times \frac{3}{2} \times \frac{3}{5} = 3T$$
$$T = \frac{3g}{2} = 14.7$$

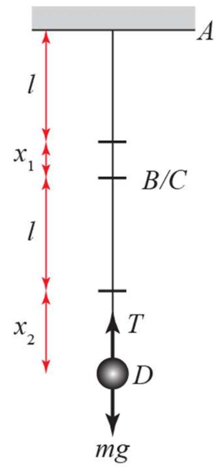
Tension is 14.7 N

8 b Using Hooke's law:

$$14.7 = \frac{30(4-l)}{l}$$
$$14.7l = 120 - 30l$$
$$44.7l = 120$$
$$l = 2.68\ldots$$

Natural length is 2.7 m (2 s.f.)

9 $(\uparrow)$ $T = mg$ throughout the length.



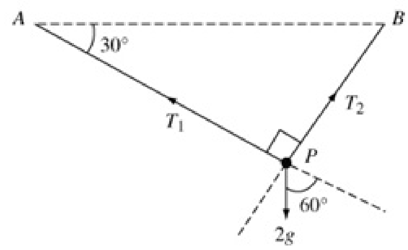
So, $mg = \frac{2mgx_1}{l} \Rightarrow x_1 = \frac{1}{2}l$

and $mg = \frac{4mgx_2}{l} \Rightarrow x_2 = \frac{1}{4}l$

$$\therefore \quad AD = 2l + x_1 + x_2$$
$$= \frac{11l}{4}$$

The length AD is $\frac{11l}{4}$

10



a $(\nwarrow)$ (along PA)

$$T_1 = 2g\cos 60° = g = 9.8\text{N}$$

so $\frac{9.8x_1}{0.5} = 9.8$

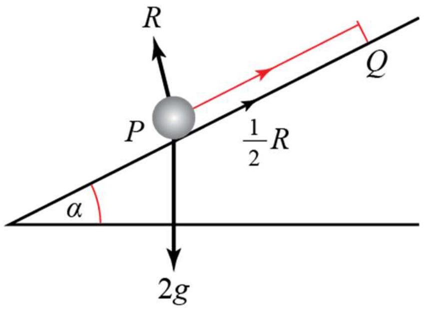
$$x_1 = 0.5$$
$$\therefore \quad PA = 0.5 + 0.5$$
$$= 1\text{ m}$$

b $\frac{PB}{PA} = \tan 30°$

$$PB = PA\tan 30° = \tan 30° = \frac{1}{\sqrt{3}}$$
$$PB \approx 0.577\text{ m}$$
$$= 0.58\text{ m (2 s.f.)}$$

10 c (↗) (along PB)
$$T_2 = 2g\cos 30°$$
$$= 2g\frac{\sqrt{3}}{2}$$
$$= g\sqrt{3}\text{ N}$$
$$\approx 17\text{ N (2 s.f.)}$$
The tension in PB is 17 N (2 s.f.)

11 a



$\tan\alpha = \frac{3}{4}$ so $\cos\alpha = \frac{4}{5}$ and $\sin\alpha = \frac{3}{5}$

(↖) $R = 2g\cos\alpha = \frac{8g}{5}$

$\therefore\quad F = \mu R = \frac{1}{2}\times\frac{8g}{5} = \frac{4g}{5}$

(↗) $T + F = 2g\sin\alpha$

$$T = 2g\sin\alpha - F$$
$$= \left(2g\times\frac{3}{5}\right) - \frac{4g}{5} = \frac{2g}{5}$$
$$= 3.92$$

The tension in the string is 3.9 N (2 s.f.)

b $T = \frac{\lambda x}{l}$

$$3.92 = \frac{20x}{0.8}$$
$$x = 0.1568 = 0.16\text{ (2 s.f.)}$$

Length of the string $= 0.16 + 0.8$
$= 0.96$ m (2 s.f.)