GCE Examinations

Advanced Subsidiary / Advanced Level

Mechanics
Module M3

Paper E

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

Written by Shaun Armstrong & Chris Huffer

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M3 Paper E – Marking Guide

1. *(a)* $\mathbf{a} = \frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{v}) = (2\mathrm{e}^{2t}\mathbf{i} + 2\mathbf{j})\ \mathrm{cm\,s^{-2}}$ — M1 A1

(b) $2\mathrm{e}^{2t}\mathbf{i} + 2\mathbf{j} = k(4\mathbf{i} + \mathbf{j})$, comparing **j** components, $k = 2$ — M1 A1

$\therefore\ 2\mathrm{e}^{2t} = 8,\ t = \frac{1}{2}\ln 4 = 0.69$ s (2sf) — M1 A1

(c) e.g. predicts **v**, **a** increasing to very large values — B1 **(7)**

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2. *(a)* work done $= \int_0^1 5 + 4\mathrm{e}^{-x}\ \mathrm{d}x = [5x - 4\mathrm{e}^{-x}]_0^1$ — M1 A1

$= (5 - 4\mathrm{e}^{-1}) - (0 - 4) = 9 - 4\mathrm{e}^{-1} = 7.53$ J (3sf) — M1 A1

(b) work done = change in KE — M1

$\therefore\ 9 - 4\mathrm{e}^{-1} = \frac{1}{2} \times 0.4 \times (v^2 - 2^2)$ — M1 A1

giving $v = 6.45\ \mathrm{m\,s^{-1}}$ (3sf) — A1 **(8)**

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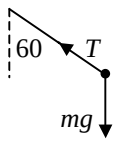
3. *(a)*



resolve ↑: $T\cos 60° - mg = 0$ — M1

$\therefore\ \frac{1}{2}T = 0.1 \times 9.8$ — A1

giving $T = 1.96$ N — A1

(b) resolve ←: $T\sin 60° = mr\omega^2$ — M1 A1

let length of string $= l$ $\therefore\ r = l\sin 60°$ so $1.96 = 0.1 \times l \times 4^2$ — M1 A1

giving $l = 1.225$ m — A1 **(8)**

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4. *(a)* $\ddot{x} = -\omega^2 x\ \therefore\ 0.12 = |-\omega^2 \times 0.03|$ giving $\omega = 2$ — M1 A1

period $= \frac{2\pi}{\omega} = \pi$ seconds — M1 A1

(b) $v^2 = \omega^2(a^2 - x^2)\ \therefore\ 0.08^2 = 2^2(a^2 - 0.03^2)$ — M1 A1

giving $a = 0.05$ m or 5 cm — A1

(c) $v_{\max} = \omega a = 2 \times 0.05 = 0.1\ \mathrm{m\,s^{-1}}$ or $10\ \mathrm{cm\,s^{-1}}$ — M1 A1 **(9)**

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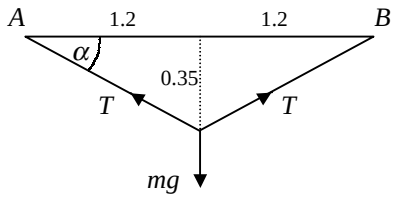
5. *(a)* by symmetry — B1

(b)



by Pythag. distance of object from $A = 1.25$ — M1

$T = \frac{\lambda x}{l} = \frac{280 \times 0.25}{1} = 70$ — M1 A1

resolve ↑: $2T\sin\alpha - mg = 0$ — M1 A1

$\sin\alpha = \frac{7}{25}\ \therefore\ 2 \times 70 \times \frac{7}{25} = m \times 9.8$ — M1

giving $m = 4$ kg — A1

(c) EPE $= 2 \times \frac{\lambda x^2}{2l} = 2 \times \frac{280 \times 0.25^2}{2 \times 1} = 17.5$ J — M1 A1 **(10)**

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6. *(a)* $\bar{x} \times \int_0^1 \rho\pi y^2 \, dx = \int_0^1 \rho\pi y^2 x \, dx \quad \therefore \bar{x} \times \int_0^1 y^2 \, dx = \int_0^1 y^2 x \, dx$ B1

$\int_0^1 y^2 \, dx = \int_0^1 (x^2+1)^2 \, dx = \int_0^1 (x^4 + 2x^2 + 1) \, dx$ M1 A1

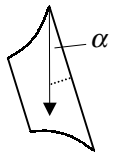
$= [\frac{1}{5}x^5 + \frac{2}{3}x^3 + x]_0^1 = \frac{28}{15}$ M1 A1

$\int_0^1 y^2 x \, dx = \int_0^1 x(x^2+1)^2 \, dx = \int_0^1 (x^5 + 2x^3 + x) \, dx$ M1

$= [\frac{1}{6}x^6 + \frac{1}{2}x^4 + \frac{1}{2}x^2]_0^1 = \frac{7}{6}$ M1 A1

$\therefore \bar{x} = \frac{7}{6} \div \frac{28}{15} = \frac{5}{8}$, $\bar{y} = 0$ by symmetry $\therefore$ coords are $(\frac{5}{8}, 0)$ M1 A1

(b)



$\tan\alpha = \frac{1-\frac{5}{8}}{2} = \frac{3}{16}$ M1 A1

$\therefore \alpha = 11°$ (nearest degree) A1 **(13)**

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7. *(a)* full circles if $T \geq 0$ at max height B1

resolve ↓: $T + mg = \frac{mv^2}{r} \quad \therefore \frac{mv^2}{0.6} \geq mg$ M1 A1

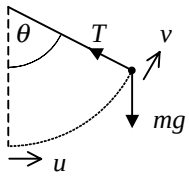
con. of ME: $\frac{1}{2}m(u^2 - v^2) = mg \times 1.2$ M1 A1

$\therefore v^2 = u^2 - 2.4g$ M1

combining, $\frac{u^2 - 2.4g}{0.6} \geq g$ M1

giving $u^2 \geq 3g \quad \therefore u \geq \sqrt{3g}$ A1

(b)



resolve ↖: $T - mg\cos\theta = \frac{mv^2}{r}$ M1

when slack, $T = 0 \quad \therefore v^2 = {}^-0.6g\cos\theta$ A1

con. of ME: $\frac{1}{2}m(u^2 - v^2) = 0.6mg(1 - \cos\theta)$ M1 A1

$\therefore v^2 = 25 - 1.2g(1 - \cos\theta)$ M1

combining, ${}^-0.6g\cos\theta = 25 - 1.2g(1 - \cos\theta)$ M1

giving $\cos\theta = \frac{1.2g - 25}{1.8g} \quad \therefore \theta = 139°$ (nearest deg) A1

(c) $v^2 = {}^-0.6g(\frac{1.2g-25}{1.8g})$ giving $v^2 = 4.413$ M1

vertically: $u = \sqrt{4.413}\sin 41°$, $a = {}^-g$, $v = 0$ (for greatest height) M1

using $v^2 = u^2 + 2as$, $0 = 4.413\sin^2 41° - 2gs$ M1 A1

giving $s = 0.098$ m $= 10$ cm (nearest cm) A1 **(20)**

Total **(75)**

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Performance Record – M3 Paper E

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	variable accel.	variable force, work done	circular motion	SHM	elastic string, EPE	centre of mass by integr., equilm.	motion in a vertical circle	
Marks	7	8	8	9	10	13	20	75
Student								

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