OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4730

Mechanics 3

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

Registered Charity Number: 1066969
[Turn over

1	$0.2 = \frac{2\pi}{\omega} \Rightarrow \omega = 10\pi$	M1		For relevant use of $\frac{2\pi}{\omega}$
		A1		For correct value 10π
	Hence maximum speed is $0.3\times 10\pi = 3\pi \approx 9.42$ m s^{-1}	M1		For relevant use of $v = a\omega$
		A1√		For correct value 3π or 9.42
	Maximum acc is $0.3\times(10\pi)^2 = 30\pi^2 \approx 296$ m s^{-2}	M1		For relevant use of $a\omega^2$
		A1√	**6**	For correct value 30π or 296
			6	
3	**(i)** A and B move off $\perp$ and $\parallel$ resp. to line of centres	M1		For correct directions of motion after impact
	$2mv_B = mu\cos\theta$	A1		For correct momentum equation
	$v_B = eu\cos\theta$	A1		For correct restitution equation
	Hence $e = 0.5$	A1	**4**	For correct answer 0.5
	(ii) $v_A = u\sin\theta$	B1		For correct equation
	Hence $v_A = v_B \Rightarrow u\sin\theta = 0.5u\cos\theta$	M1		For forming the relevant equation for θ
	So $\theta = \tan^{-1} 0.5 \approx 26.6°$	A1	**3**	For correct value 26.6
			7	
3	$80\,000v\frac{\mathrm{d}v}{\mathrm{d}x} = -(27\,000 + 50v^2)$	M1		For using Newton II to form a DE
		A1		For correct equation including $v\frac{\mathrm{d}v}{\mathrm{d}x}$
	Hence $x = -\int \frac{1600v}{540+v^2}\,\mathrm{d}v$	M1		For separation of variables
	$= -800\ln(540+v^2) + k$	M1		For logarithmic form of integral
		A1√		For correct integration of (their) $\frac{av}{b+cv^2}$
	$v = 90$ when $x = 0 \Rightarrow k = 800\ln 8640$	M1		For use of initial condition to find k
	Hence when $v = 0$, $x = 800\ln 16$	M1		For evaluation of required distance (The previous two M marks can equivalently be earned by using definite integration)
	So distance is 2220 m approximately	A1	**8**	For correct value 2220
			8	
4	**(i)** Greatest tension $= \frac{1320\times 35}{25} = 1848$ N	M1		For use of $\frac{\lambda x}{l}$ at lowest point
		A1	**2**	For correct answer 1848
	(ii) **(a)** $mg\times 60 = \frac{1320}{2\times 25}(60-25)^2$	M1		For use of correct EPE formula $\frac{\lambda x^2}{2l}$
		A1		For correct unsimplified expression for EPE
	Hence the girl's mass is 55 kg	M1		For use of equation involving EPE and GPE
		A1	**4**	For correct answer 55
	(b) $55g\times 30 = \frac{1}{2}\times 55v^2 + \frac{1320}{2\times 25}\times(30-25)^2$	M1		For energy equation with KE, GPE and EPE
		A1√		For equation with all terms correct
	So $v^2 = 564$, hence speed is 23.7 m s^{-1}	A1	**3**	For correct answer 24.3
			9	

5	**(i)**	Radial acc is $\frac{6.5^2}{1.5} = 28.2$ m s^{-2}	B1		For correct value 28.2
		Transverse acc is $g\sin 40° = 6.30$ m s^{-2}	B1	**2**	For correct value 6.30
	(ii)	$\frac{1}{2}\times 0.3\times(6.5^2 - v^2) = 0.3\times 9.8\times 1.5(\cos 40° - \cos\theta°)$	M1		For equating PE gain to KE loss, or equiv
			B1		For correct expression for PE gain
			B1		For correct expression for KE loss
		Hence $42.25 - v^2 = 29.4(\cos 40° - \cos\theta°)$			
		i.e. $v^2 \approx 19.7 + 29.4\cos\theta°$	A1	**4**	For showing given answer correctly
	(iii)	$T - 0.3g\cos\theta° = 0.3\times\frac{v^2}{1.5}$	M1		For use of Newton II, including $\frac{mv^2}{r}$ term
			A1		For correct (unsimplified) equation
		Hence $T = 2.94\cos\theta° + 0.2(19.7 + 29.4\cos\theta°)$	M1		For substitution, to obtain expression for T
		$= 3.95 + 8.82\cos\theta°$	A1	**4**	For correct answer
	(iv)	$T = 0$ when $3.95 + 8.82\cos\theta° = 0$	M1		For equating T to zero to find $\cos\theta$
		Hence $v^2 = 19.7 + 29.4\times\left(-\frac{3.95}{8.82}\right) \Rightarrow v \approx 2.56$	M1		For using this $\cos\theta$ to find v
			A1	**3**	For correct answer 2.56
				13	
6	**(i)**	Mom @ B for BAC: $V_C\times 1.6 = 150\times 0.4 + 270\times 1.2$	M1		For suitable moments equation for BAC
		Hence $V_C = 240$	A1		For correct value for V_C (or equivalent)
		Mom @ C for AC: $V_A\times 0.8 + H_A\times 2.5 = 270\times 0.4$	M1		For a moments equation for one rod with all required forces included
			A1		For a correct equation
		Res $\updownarrow$ for AC: $V_A + V_C = 270$	M1		For another equation leading to V_A
		Hence $V_A = 270 - 240 = 30$ N (upwards)	A1		For correct magnitude and direction
		and $2.5H_A = 108 - 0.8\times 30 \Rightarrow H_A = 33.6$ N (right)	M1		For substituting back to find H_A
			A1	**8**	For correct magnitude and direction
	(ii)	$V_B = 270 + 150 - V_C = 180$	M1		For finding all of V_B, H_B and H_C
			A1√		For correct V_B
		$H_B = H_C = H_A = 33.6$	A1√		For both H_B and H_C correct
		$\frac{H_B}{V_B} = \frac{33.6}{180} = 0.187$, $\frac{H_C}{V_C} = \frac{33.6}{240} = 0.14$	M1		For considering ratios at B and C, or equiv
		Hence friction is limiting at B	A1√		For identifying point with larger ratio
		Value of μ is 0.187	A1√	**6**	For identifying the larger ratio as μ
				14	

7	**(i)** $T_{AP} = \frac{196}{0.8} \times (1.5 - 0.8) = 171.5$	M1		For using Hook's law to find either tension
	$T_{BP} = \frac{196}{0.8} \times (2.6 - 1.5 - 0.8) = 73.5$	A1		For both tensions correct
	$T_{AP} - T_{BP} = 98 = 10g$, hence equilibrium	M1		For considering $T_{AP} = mg + T_{BP}$, or equiv
		A1	**4**	For showing given result correctly
	(ii) Extension of PA is $1.5 + x - 0.8 = 0.7 + x$	M1		For finding either extension in terms of x
	Hence $T_{AP} = \frac{196}{0.8}(0.7 + x) = 245(0.7 + x)$	A1		For showing one given answer correctly
	and $T_{BP} = \frac{196}{0.8}(1.1 - x - 0.8) = 245(0.3 - x)$	A1	**3**	For showing the other given answer correctly
	(iii) $245(0.3 - x) + 10g - 245(07. + x) = 10\ddot{x}$	M1		For use of Newton II, at a general position
		A1		For a correct equation
	Hence $\ddot{x} = -49x$, so the motion is SHM	A1	**3**	For showing the given result correctly
	(iv) $0.2 = 0.25\cos(7t)$	M1		For use of ± 0.2 in SHM equation involving t
		A1		For a correct equation for a relevant time
	Hence half of time above mid-pt is $t = 0.0919...$	A1		For correct value for a relevant time
	Proportion is $\frac{t}{\pi / \omega} = 0.205$	M1		For relating t to period of oscillation
		A1	**5**	For correct proportion 0.205
			15	