Mark Scheme 4730
June 2007

1	(i)	[$\omega = 2\pi/6.1 = 1.03$]	M1		For using $T = 2\pi/\omega$
			M1		For using $v_{max} = a\omega$
		Speed is $3.09ms^{-1}$	A1	3	
	(ii)		M1		For using $v^2 = \omega^2(A^2 - x^2)$ or for using $v = A\omega\cos\omega t$ and $x = A\sin\omega t$
		$2.5^2 = 1.03^2(3^2 - x^2)$ or $x = 3\sin(1.03x0.60996..)$	A1ft		ft incorrect ω
		Distance is 1.76m	A1	3	

2	[Magnitudes 0.6, 0.057 x 7, 0.057 x 10]	M1		For triangle with magnitudes shown
	For magnitudes of 2 sides correctly marked	A1		
	For magnitudes of all 3 sides correctly marked	A1		
		M1		For attempting to find angle (α) opposite to the side of magnitude 0.057 x 7
		M1		For correct use of the cosine rule or equivalent
	$0.399^2 = 0.57^2 + 0.6^2 - 2 \times 0.57 \times 0.6\cos\alpha$	A1ft		
	Angle is 140^o	A1	7	$(180 - 39.8)^o$

2	ALTERNATIVE METHOD			
		M1		For using I= Δmv parallel to the initial direction of motion or parallel to the impulse
	$-0.6\cos\alpha = 0.057 \times 7\cos\beta - 0.057 \times 10$ or $0.6 = 0.057x10\cos\alpha + 0.057x7\cos\gamma$	A1		
		M1		For using I= Δmv perpendicular to the initial direction of motion or perpendicular to the impulse
	$0.6\sin\alpha = 0.057 \times 7\sin\beta$ or $0.057x10\sin\alpha = 0.057x 7\sin\gamma$	A1		
		M1		For eliminating β *or γ
	$0.399^2 = (0.57 - 0.6\cos\alpha)^2 + (0.6\sin\alpha)^2$ or $0.399^2 = (0.6 - 0.57\cos\alpha)^2 + (0.057\sin\alpha)^2$	A1ft		
	Angle is 140^o	A1	7	$(180 - 39.8)^o$

3	(i)	[0.2v dv/dx = -0.4v^2]	M1		For using Newton's second law with a = v dv/dx
		(1/v) dv/dx = -2	A1	2	AG
	(ii)	[$\int (1/v)dv = \int -2dx$]	M1		For separating variables and attempting to integrate
		ln v = -2x (+C)	A1		
		[ln v = -2x + ln u]	M1		For using v(0) = u
		v = ue^{-2x}	A1	4	AG
	(iii)	[$\int e^{2x}dx = \int udt$]	M1		For using v = dx/dt and separating variables
		e^{2x}/2 = ut (+C)	A1		
		[e^{2x}/2 = ut + ½]	M1		For using x(0) = 0
		u = 6.70	A1	4	Accept (e^4 – 1)/8

	ALTERNATIVE METHOD FOR PART (iii)			
	[$\int \frac{1}{v^2}dv = -2\int dt$ ➔ -1/v = -2t + A, and A = -1/u]	M1		For using a = dv/dt, separating variables, attempting to integrate and using v(0) = u
		M1		For substituting v = ue^{-2x}
	-e^{2x}/u = -2t – 1/u	A1		
	u = 6.70	A1	4	Accept (e^4 – 1)/8

4	y= 15sinα (=12)	B1		
	[4(15cosα) – 3 x 12 = 4a + 3b]	M1		For using principle of conservation of momentum in the direction of l.o.c.
	Equation complete with not more than one error	A1		
	4a + 3b = 0	A1		
		M1		For using NEL in the direction of l.o.c.
	0.5(15cosα + 12) = b - a	A1		
	[a = -4.5, b = 6]	M1		For solving for a and b
	[Speed = $\sqrt{(-4.5)^2 + 12^2}$, Direction tan^{-1}(12/(-4.50))]	M1		For correct method for speed or direction of A
	Speed of A is 12.8ms^{-1} and direction is 111^o anticlockwise from '**i**' direction	A1		Direction may be stated in any form , including θ = 69^o with θ clearly and appropriately indicated
	Speed of B is 6ms^{-1} to the right	A1	10	Depends on first three M marks

5	(i)	M1		For taking moments of forces on BC about B
	80 x 0.7cos60^o = 1.4T	A1		
	Tension is 20N	A1		
	[X = 20cos30^o]	M1		For resolving forces horizontally
	Horizontal component is 17.3N	A1ft		ft X = Tcos30^o
	[Y = 80 – 20sin30^o]	M1		For resolving forces vertically
	Vertical component is 70N	A1ft	7	ft Y = 80 – Tsin30^o
	(ii)	M1		For taking moments of forces on AB, or on ABC, about A
	17.3 x 1.4sinα = (80 x 0.7 + 70 x1.4)cosα or 80x0.7cosα + 80(1.4cosα + 0.7cos60^o) = 20cos60^o(1.4cosα +1.4cos60^o) + 20sin60^o(1.4sinα +14sin60^o)	A1ft		
	[tanα = (½ 80 + 70)/17.3= 11/$\sqrt{3}$]	M1		For obtaining a numerical expression for tanα
	α = 81.1^o	A1	4	

ALTERNATIVE METHOD FOR PART (i)			
	M1		For taking moments of forces on BC about B
Hx1.4sin60^o + Vx1.4cos60^o = 80x0.7cos60^o	A1		Where H and V are components of T
	M1		For using H = V$\sqrt{3}$ and solving simultaneous equations
Tension is 20N	A1		
Horizontal component is 17.3N	B1ft		ft value of H used to find T
[Y = 80 – V]	M1		For resolving forces vertically
Vertical component is 70N	A1ft	7	ft value of V used to find T

6	(i) [T = 2058x/5.25]	M1		For using T = λx/L
	2058x/5.25 = 80 x 9.8 (x = 2)	A1		
	OP = 7.25m	A1	3	AG From 5.25 + 2
	(ii) Initial PE = (80 + 80)g(5) (= 7840) or (80 + 80)gX used in energy equation	B1		
	Initial KE = ½ (80 + 80)3.5^2 (= 980)	B1		
	[Initial EE = $2058x2^2/(2x5.25)$ (= 784), Final EE = $2058x7^2/(2x5.25)$ (= 9604), or $2058(X + 2)^2/(2x5.25)$]	M1		For using EE = $\lambda x^2/2L$
	[Initial energy = 7840 + 980 + 784, final energy = 9604 or 1568X + 980 + 784 = $196(X^2 + 4X + 4)$ ➔ $196X^2 – 784X – 980 = 0$]	M1		For attempting to verify compatibility with the principle of conservation of energy, or using the principle and solving for X
	Initial energy = final energy or X = 5 ➔P&Q just reach the net	A1	5	AG
	(iii) [PE gain = 80g(7.25 + 5)]	M1		For finding PE gain from net level to O
	PE gain = 9604	A1		
	PE gain = EE at net level ➔ P just reaches O	A1	3	AG
	(iv) For any one of 'light rope', 'no air resistance', 'no energy lost in rope'	B1		
	For any other of the above	B1	2	

FIRST ALTERNATIVE METHOD FOR PART (ii)			
[160g – 2058x/5.25 = 160v dv/dx]	M1		For using Newton's second law with a = v dv/dx, separating the variables and attempting to integrate
$v^2/2 = gx – 1.225x^2$ (+ C)	A1		Any correct form
	M1		For using v(2) = 3.5
C = -8.575	A1		
$[v(7)^2]/2$ = 68.6 – 60.025 – 8.575 = 0 ➔P&Q just reach the net	A1	5	AG

SECOND ALTERNATIVE METHOD FOR PART (ii)			
$\ddot{x} = g - 2.45x$ (= -2.45(x – 4))	B1		
	M1		For using $n^2 = 2.45$ and $v^2 = n^2(A^2 – (x – 4)^2)$
$3.5^2 = 2.45(A^2 – (-2)^2)$ (A = 3)	A1		
[(4 – 2) + 3]	M1		For using 'distance travelled downwards by P and Q = distance to new equilibrium position + A
distance travelled downwards by P and Q = 5 ➔P&Q just reach the net	A1	5	AG

7	(i) [$a = 0.7^2/0.4$]	M1		For using $a = v^2/r$
	For not more than one error in $T - 0.8g\cos60^o = 0.8\text{x}0.7^2/0.4$	A1		
	Above equation complete and correct	A1		
	Tension is 4.9N	A1	4	
	(ii)	M1		For using the principle of conservation of energy
	$\frac{1}{2}\, 0.8v^2 = \frac{1}{2}\, 0.8(0.7)^2 + 0.8g0.4 - 0.8g0.4\cos60^o$	A1		($v = 2.1$)
	$(2.1 - 0)/7 = 2u$	M1		For using NEL
	Q's initial speed is 0.15ms^{-1}	A1	4	AG
	(iii)	M1		For using Newton's second law transversely
	$(m)0.4\ddot{\theta} = -(m)g\sin\theta$	A1		*Allow m = 0.8 (or any other numerical value)
	[$0.4\ddot{\theta} \approx -g\theta$]	M1		For using $\sin\theta \approx \theta$
	[$\frac{1}{2}\, m0.15^2 = mg0.4(1 - \cos\theta_{max})$ ➔ $\theta_{max} = 4.34^o$ (0.0758rad)]	M1		For using the principle of conservation of energy to find θ_{max}
	θ_{max} small justifies $0.4\ddot{\theta} \approx -g\theta$, and this implies SHM	A1	5	
	(iv) [$T = 2\pi/\sqrt{24.5} = 1.269..$] [$\sqrt{24.5}\, t = \pi$]	M1		For using $T = 2\pi/n$ or for solving either $\sin nt = 0$ (non-zero t) (considering displacement) or $\cos nt = -1$ (considering velocity)
	Time interval is 0.635s	A1ft	2	From $t = \frac{1}{2}T$