4730 Mechanics 3

1 (i)	For triangle sketched with sides (0.5)2.5 and (0.5)6.3 and angle θ correctly marked OR Changes of velocity in i and j directions $2.5\cos\theta - 6.3$ and $2.5\sin\theta$, respectively.	B1	May be implied in subsequent working.
	For sides 0.5x2.5, 0.5x6.3 and 2.6 (or 2.5, 6.3 and 5.2) OR $-2.6\cos\alpha = 0.5(2.5\cos\theta - 6.3)$ and $2.6\sin\alpha = 0.5(2.5\sin\theta)$	B1ft	May be implied in subsequent working.
	$[5.2^2 = 2.5^2 + 6.3^2 - 2\text{x}2.5\text{x}6.3\cos\theta$ OR $2.6^2 = 0.5^2\{(2.5\cos\theta - 6.3)^2 + (2.5\sin\theta)^2]$	M1	For using cosine rule in triangle or eliminating α.
	$\cos\theta = 0.6$	A1 [4]	AG
(ii)		M1	For appropriate use of the sine rule or substituting for θ in one of the above equations in θ and α
	$\sin\alpha = 2.5\text{x}0.8/5.2$ OR $-2.6\cos\alpha = 0.5(2.5\text{x}0.6 - 6.3)$	A1	
		M1	For evaluating $(180 - \alpha)^o$ or $(\pi - \alpha)^c$
	Impulse makes angle of 157^o or 2.75^c with original direction of motion of P.	A1 [4]	SR (relating to previous 2 marks; max 1 mark out of 2) $\alpha = 23^o$ or 0.395^C B1

2 (i)	[70x2 = 4X – 4Y]	M1	For taking moments about A for AB (3 terms needed)
	X – Y = 35	A1 [2]	
(ii)	[110x3 = -4X + 6Y]	M1	For taking moments about C for BC (3 terms needed)
	2X – 3Y + 165 = 0	A1 [2]	AG
(iii)		M1	For attempting to solve for X and Y
	X = 270, Y = 235	A1ft	ft any (X, Y) satisfying the equation given in (ii)
		M1	For using magnitude = $\sqrt{X^2 + Y^2}$
	Magnitude is 358N	A1ft [4]	ft depends on all 4 Ms

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3 (i)	[T_A = (24x0.45)/0.6, T_B = (24x0.15)/0.6] $T_A - T_B = 18 - 6 = 12 = W \Rightarrow$ P in equil'm.	M1 A1 [2]	For using T = λ x/L for PA or PB
(ii)	Extensions are 0.45 + x and 0.15 – x Tensions are 18 + 40x and 6 – 40x	B1 B1 [2]	AG From T = λ x/L for PA and PB
(iii)	[12 + (6 – 40x) – (18 + 40x) = 12 $\ddot{x}$/g] $\ddot{x}$ = -80gx/12 $\Rightarrow$ SHM Period is 0.777s	M1 A1 A1 [3]	For using Newton's second law (4 terms required) AG From Period = $2\pi\sqrt{12/(80g)}$
(iv)	[$v_{max} = 0.15\sqrt{80g/12}$ or $v_{max} = 2\pi$x0.15/0.777 or ½ (12/g)v_{max}^2 +mg(0.15) +24{$0.45^2 + 0.15^2 - 0.6^2$}/(2x0.6) = 0] Speed is 1.21ms^{-1}	M1 A1 [2]	For using v_{max} = An or $v_{max} = 2\pi$A/T or conservation of energy (5 terms needed)

4 (i)	Loss in PE = mg(0.5sinθ) [½ mv^2 – ½ m3^2 = mg(0.5sinθ)] v^2 = 9 + 9.8sinθ	B1 M1 A1 [3]	For using KE gain = PE loss (3 terms required) AG
(ii)	a_r = 18 + 19.6sinθ [ma_t = mg cosθ] a_t = 9.8cosθ	B1 M1 A1 [3]	Using $a_r = v^2/0.5$ For using Newton's second law tangentially
(iii)	[T - mg sinθ = ma_r] T – 1.96sinθ = 0.2(18 + 19.6sinθ) T = 3.6 + 5.88sinθ θ = 3.8	M1 A1 A1 B1 [4]	For using Newton's second law radially (3 terms required) AG

5	Initial **i** components of velocity for A and B are $4ms^{-1}$ and $3ms^{-1}$ respectively. 3x4 + 4x3 = 3a + 4b 0.75(4 – 3) = b – a a = 3 Final **j** component of velocity for A is $3ms^{-1}$ Angle with l.o.c. is 45^o or 135^O	B1 M1 A1 M1 A1 M1 A1 B1 M1 A1ft [10]	May be implied. For using p.c.mmtm. parallel to l.o.c. For using NEL For attempting to find a Depends on all three M marks May be implied For using $\tan^{-1}(v_j/v_i)$ for A ft incorrect value of a ($\neq 0$) only
			SR for consistent sin/cos mix (max 8/10) 3x3 + 4x4 = 3a + 4b and b – a = 0.75(3 – 4) M1 M1 as scheme and A1 for *both* equ's a = 4 M1 as scheme A1 j component for A is $4ms^{-1}$ B1 Angle $\tan^{-1}(4/4) = 45^o$ M1 as scheme A1

6(i)	Initial speed in medium is $\sqrt{2g \times 10}$ (= 14) [0.125dv/dt = 0.125g – 0.025v] $\int \frac{5dv}{5g - v} = \int dt$ -5 ln(5g – v) = t (+A) [-5 ln35 = A] t = 5 ln{35/(49 – v)} $v = 49 - 35e^{-0.2t}$	B1 M1 M1 A1 M1 A1 M1 A1 [8]	 For using Newton's second law with a = dv/dt (3 terms required) For separating variables and attempt to integrate For using v(0) = 14 For method of transposition AG
(ii)	 $x = 49t + 175e^{-0.2t}$ (+B) $[x(3) = (49x3 + 175e^{-0.6}) – (0 + 175)]$ Distance is 68.0m	M1 A1 M1 A1 [4]	For integrating to find x(t) For using limits 0 to 3 or for using x(0) = 0 and evaluating x(3)

7(i)	Gain in EE = $20x^2/(2x2)$ Loss in GPE = $0.8g(2 + x)$ [½ $0.8v^2 = (15.68 + 7.84x) – 5x^2$] $v^2 = 39.2 + 19.6x – 12.5x^2$	B1 B1 M1 A1 [4]	Accept 0.8gx if gain in KE is ½ $0.8(v^2 – 19.6)$ For using the p.c.energy AG
(ii)	**(a)** Maximum extension is 2.72m	M1 A1 [2]	For attempting to solve $v^2 = 0$
	(b) [$19.6 – 25x = 0$, $v^2 = 46.8832 – 12.5(x – 0.784)^2$] $x = 0.784$ or $c = 46.9$ [$v_{max}{}^2 = 39.2 + 15.3664 – 7.6832$] Maximum speed is $6.85ms^{-1}$	 M1 A1 M1 A1 [4]	For solving $20x/2 = 0.8g$ or for differentiating and attempting to solve $d(v^2)/dx = 0$ or $dv/dx = 0$ or for expressing v^2 in the form $c – a(x – b)^2$. For substituting $x = 0.784$ in the expression for v^2 or for evaluating $\sqrt{c}$
	(c) $\pm(0.8g – 20x/2) = 0.8a$ or $2v\, dv/dx = 19.6 – 25x$ $a = \pm(9.8 – 12.5x)$ or $\ddot{y} = -12.5y$ where $y = x – 0.784$ [$\|a\|_{max} = \|9.8 – 12.5x2.72\|$ or $\|\ddot{y}_{max}\| = \|-12.5(2.72 – 0.784\|$] Maximum magnitude is $24.2ms^{-2}$	 M1 A1 A1 M1 A1 [5]	For using Newton's second law (3 terms required) or $a = v\, dv/dx$ For substituting x = ans(ii)(a) into a(x) or y = ans(ii)(a) – 0.784 into $\ddot{y}$ (y)