OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4763**: Mechanics 3 (M3)

Advanced GCE

Mark Scheme for June 2015

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a.   Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b.   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c.   The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d.   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e.   The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f.   Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (eg lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	Guidance	
1	**(i)**		$[\text{ Force }] = M L T^{-2}$	B1		
			$[\text{ Work }] = M L^2 T^{-2}$	B1		
			$[\text{ Power }] = M L^2 T^{-3}$	B1		
				[3]		*Deduct one mark if kg, m, s used consistently for M, L, T*
1	**(ii)**		$[\lambda] = \left[\dfrac{F}{v^2}\right] = \dfrac{M L T^{-2}}{(L T^{-1})^2}$	M1	Obtaining dimensions of λ	M0 if $P = \lambda U^3$ used
			$= M L^{-1}$	A1	FT $[\text{ Force }] \times L^{-2} T^2$	B2 (BOD) for correct answer with no working
				[2]		
1	**(iii)**		$[\lambda U^3] = (M L^{-1})(L T^{-1})^3 = M L^2 T^{-3}$	M1	Obtaining dimensions of λU^3	Must be simplified
			Same as power, so dimensionally consistent	E1	Correctly shown	
				[2]		
1	**(iv)**		$T = M^\alpha (M L^2 T^{-3})^\beta (M L^{-1})^\gamma$			
			$\beta = -\tfrac{1}{3}$	B1		Equation from powers of M or L
			$\alpha + \beta + \gamma = 0$, $\quad 2\beta - \gamma = 0$	M1	One equation correct (FT)	If A0 give SC1 for non-zero values with $\gamma = 2\beta$ OR $\alpha + \beta + \gamma = 0$
			$\alpha = 1$, $\quad \gamma = -\tfrac{2}{3}$	A1A1	CAO	*(SC1 will usually imply M1)*
				[4]		
1	**(v)**	(*A*)	$\lambda = \dfrac{P}{U^3} = \dfrac{35000}{45^3} = 0.384 \ (\text{kg m}^{-1}) \quad$ (3 sf)	B1		
		(*B*)	$0.3841 \times \dfrac{1}{0.454} \times 1609$	M1	For conversion factor $\dfrac{1609}{0.454}$	OR changing *both* $P \ (= 1.389 \times 10^9)$ *and* $U \ (= 100.7)$ to new units
			$= 1360 \ (\text{lb mi}^{-1}) \quad$ (3 sf)	A1	CAO	
				[3]		

Question			Answer	Marks	Guidance	
1	(vi)		$\lambda_D = \dfrac{75000}{54^3}$ $(=0.476)$	M1		
			$t_D = 9.18 \times \dfrac{1250}{800} \times \left(\dfrac{75}{35}\right)^{-\frac{1}{3}} \times \left(\dfrac{0.4763}{0.3841}\right)^{-\frac{2}{3}}$	M1 M1	Two ratios and powers correct Correct expression for new time	Or equation for k $(k=0.1983)$
		OR	$t = kmP^{-\frac{1}{3}}(PU^{-3})^{-\frac{2}{3}} = kmP^{-1}U^2$		M1 Expressing t in terms of m, P, U	
			$t_D = 9.18 \times \dfrac{1250}{800} \times \left(\dfrac{75}{35}\right)^{-1} \times \left(\dfrac{54}{45}\right)^{2}$		M1 Two ratios and powers correct M1 Correct expression for new time	Or equation for k $(k=0.1983)$
			Time is 9.639 s	A1 **[4]**	CAO	
2	(a)		$T_1 + mg = m\dfrac{v_1^2}{a}$	B1		*Condone r for a*
				M1	Equation involving KE and PE	
			$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 + mga(1-\cos\theta)$	A1		
			$T_2 + mg\cos\theta = m\dfrac{v_2^2}{a}$	B1		*Condone same v for B1B1*
			$T_2 - T_1 = \dfrac{mv_2^2}{a} - \dfrac{mv_1^2}{a} + mg(1-\cos\theta)$			
			$T_2 - T_1 = 2mg(1-\cos\theta) + mg(1-\cos\theta)$	M1	Eliminating v_1 and v_2	*Dependent on previous M1*
			$T_2 = T_1 + 3mg(1-\cos\theta)$	E1 **[6]**		

Question			Answer	Marks	Guidance	
2	(b)	(i)	$T_{QR}\cos 60° = 1.5 \times 9.8$	M1	Resolving vertically for R	
			Tension in QR is 29.4 N	A1		
			$T_{AQ}\cos\theta = 0.9 \times 9.8 + T_{QR}\cos 60°$	M1	Resolving vertically for Q	θ is the angle CAQ
			$\frac{12}{13}T_{AQ} = 8.82 + 14.7$	A1	FT e.g.	$T\cos 22.6 = 0.9 \times 9.8 + 29.4\cos 60°$
			Tension in AQ is 25.48 N	A1	CAO	
				[5]		
2	(b)	(ii)		M1	Using $1.5r\omega^2$	Allow $1.5\dfrac{v^2}{r}$ provided $v = r\omega$ also seen
			$T_{QR}\sin 60° = 1.5(0.5 + 1.8\sin 60°)\omega^2$	A1		
			Angular speed is $2.87\,\text{rad s}^{-1}$ (3 sf)	A1	FT is $\sqrt{0.2804\,T_{QR}}$	
				[3]		
2	(b)	(iii)	$T_{CQ} + T_{AQ}\sin\theta - T_{QR}\sin 60° = (0.9)(0.5)\omega^2$	B1B1	For LHS and RHS (CAO)	
			$T_{CQ} + 25.48 \times \frac{5}{13} - 29.4\sin 60° = 0.45 \times 2.871^2$	M1	Numerical equation for T_{CQ}	At most one error (FT including in B1's) and no missing terms
			Tension in CQ is 19.4 N (3 sf)	A1	CAO	
				[4]		
3	(i)			M1	Using stiffness × extension to find tension in *both* strings	Must use extension. Condone use of modulus for full marks
			If $BC = y$, $12y = 30(3.5 - y)$	A1	Correct equation for a distance	
			Distance BC is 2.5 m	A1		
				[3]		

PMT

Question			Answer	Marks	Guidance	
3	**(ii)**		$$30(1.0-x)-12(2.5+x)=10.5\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}$$ $$\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}=-4x$$	B1 M1 A1 E1 **[4]**	For $30(1.0-x)$ _or_ $12(2.5+x)$ Equation of motion Allow $(\pm)\,10.5a$ on RHS	FT if BC is wrong Two forces in terms of x, and acc'n FT if BC is wrong _When necessary, replacing a by_ $-\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}$ _requires some explanation_
3	**(iii)**		Maximum speed is $A\omega$ Maximum speed is $5\,\mathrm{m\,s}^{-1}$	M1 A1 **[2]**	OR Using energy to obtain an equation for v at C FT is $2\times$ BC	$\frac{1}{2}(30)(3.5^2)=\frac{1}{2}(30)(1.0^2)$ $+\frac{1}{2}(12)(2.5^2)+\frac{1}{2}(10.5)v^2$
3	**(iv)**		$4.8^2=4(2.5^2-x^2)$ Distance from C is 0.7 m	M1 A1 **[2]**	Using $v^2=\omega^2(A^2-x^2)$ CAO	OR Using energy to obtain an equation for a distance Condone $\pm\,0.7$
3	**(v)**		$x=-2.5\cos 2t$ $1.0=-2.5\cos 2t$ Time is 0.991 s (3 sf)	B1 M1 M1 A1 **[4]**	$\pm 2.5\sin 2t$ _or_ $\pm 2.5\cos 2t$ Using $x=1.0$ to obtain a time Fully correct strategy for finding the required time CAO	FT if BC is wrong (or A or ω) e.g. $0.2058+\dfrac{1}{4}\times\dfrac{2\pi}{2}$

Question			Answer	Marks	Guidance	
3	(vi)			M1	Using change of elastic energy to obtain an equation for a distance	OR Fully correct strategy using two stages of SHM
			$\frac{1}{2}(30)(3.5^2) = \frac{1}{2}(12)(BE^2)$	A1		Correct (FT) equation for a distance
			$\qquad BE = 5.534$			
			Distance DE is 2.03 m (3 sf)	A1	CAO	
				[3]		
4	(a)		Area is $\displaystyle\int_0^a \frac{x^2(a-x)}{a^2}\,dx$	M1		
			$= \left[\dfrac{x^3}{3a} - \dfrac{x^4}{4a^2} \right]_0^a \quad \left(= \dfrac{a^2}{12} \right)$	A1		
			$\displaystyle\int xy\,dx$	M1		
			$= \displaystyle\int_0^a \frac{x^3(a-x)}{a^2}\,dx = \left[\dfrac{x^4}{4a} - \dfrac{x^5}{5a^2} \right]_0^a \quad \left(= \dfrac{a^3}{20} \right)$	A1		
			$\bar{x} = \dfrac{\frac{1}{20}a^3}{\frac{1}{12}a^2} = \dfrac{3a}{5}$	A1		
			$\displaystyle\int \tfrac{1}{2}y^2\,dx = \int_0^a \frac{x^4(a-x)^2}{2a^4}\,dx$	M1	For $\displaystyle\int \ldots y^2\,dx$	
			$= \left[\dfrac{x^5}{10a^2} - \dfrac{x^6}{6a^3} + \dfrac{x^7}{14a^4} \right]_0^a \quad \left(= \dfrac{a^3}{210} \right)$	A2	Give A1 if just one error (e.g. omission of factor ½)	
			$\bar{y} = \dfrac{\frac{1}{210}a^3}{\frac{1}{12}a^2} = \dfrac{2a}{35} \quad (\approx 0.0571a)$	A1		
				[9]		

Question			Answer	Marks	Guidance	
4	(b)	(i)	Volume is $\int_0^3 \pi(x^2+16)\,dx$	M1	π may be omitted throughout	*Condone consistent use of $2\pi y^2$ etc*
			$= \pi \left[\dfrac{x^3}{3} + 16x \right]_0^3 \quad (=57\pi)$	A1		
			$\int \pi x y^2\,dx$	M1		
			$= \int_0^3 \pi x(x^2+16)\,dx \quad = \pi \left[\dfrac{x^4}{4} + 8x^2 \right]_0^3 \quad (=\dfrac{369}{4}\pi)$	A1		
			$\bar{x} = \dfrac{\frac{369}{4}\pi}{57\pi} = \dfrac{123}{76} \quad (\approx 1.62)$	A1		
				[5]		
4	(b)	(ii)	Volume of A and B combined is $\pi \times 5^2 \times 3 = 75\pi$	M1	CM of composite body	
			$(18\pi)\bar{x}_B + (57\pi)\left(\dfrac{123}{76}\right) = (75\pi)(1.5)$	A2	Give A1 if just one error	FT values from (i)
		OR	$\int_0^3 \pi x \left(25 - (x^2+16)\right)\,dx$	M1		
			$= \dfrac{81}{4}\pi$	A1		
			$(18\pi)\bar{x}_B = \dfrac{81}{4}\pi$	A1 FT		
			$\bar{x}_B = \dfrac{9}{8} \quad (=1.125)$	A1	CAO	
				[4]		

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