OCR
RECOGNISING ACHIEVEMENT

Thursday 31 May 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4763 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4763
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The fixed point A is at a height $4b$ above a smooth horizontal surface, and C is the point on the surface which is vertically below A. A light elastic string, of natural length $3b$ and modulus of elasticity λ, has one end attached to A and the other end attached to a block of mass m. The block is released from rest at a point B on the surface where BC = $3b$, as shown in Fig. 1. You are given that the block remains on the surface and moves along the line BC.

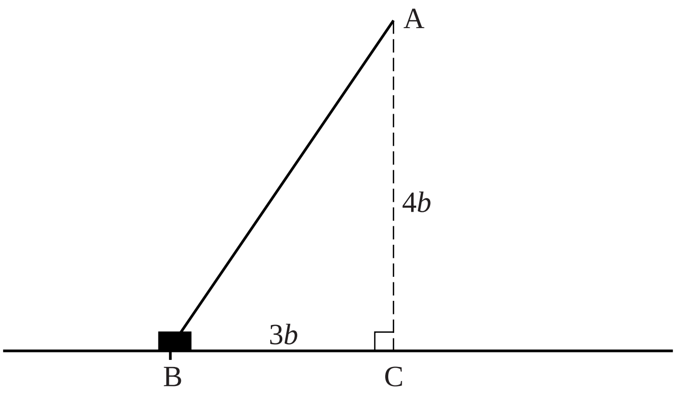

Fig. 1

(i) Show that immediately after release the acceleration of the block is $\frac{2\lambda}{5m}$. **[4]**

(ii) Show that, when the block reaches C, its speed v is given by $v^2 = \frac{\lambda b}{m}$. **[4]**

(iii) Show that the equation $v^2 = \frac{\lambda b}{m}$ is dimensionally consistent. **[3]**

The time taken for the block to move from B to C is given by $km^{\alpha}b^{\beta}\lambda^{\gamma}$, where k is a dimensionless constant.

(iv) Use dimensional analysis to find α, β and γ. **[4]**

When the string has natural length 1.2 m and modulus of elasticity 125 N, and the block has mass 8 kg, the time taken for the block to move from B to C is 0.718 s.

(v) Find the time taken for the block to move from B to C when the string has natural length 9 m and modulus of elasticity 20 N, and the block has mass 75 kg. **[3]**

2 **(a)** Fig. 2 shows a car of mass 800 kg moving at constant speed in a horizontal circle with centre C and radius 45 m, on a road which is banked at an angle of 18° to the horizontal. The forces shown are the weight *W* of the car, the normal reaction, *R*, of the road on the car and the frictional force *F*.

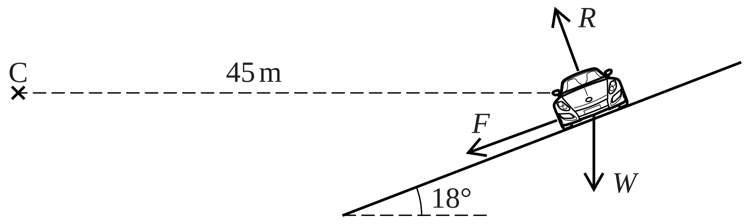

Fig. 2

(i) Given that the frictional force is zero, find the speed of the car. **[4]**

(ii) Given instead that the speed of the car is 15 m s^{-1}, find the frictional force and the normal reaction. **[7]**

(b) One end of a light inextensible string is attached to a fixed point O, and the other end is attached to a particle P of mass *m* kg. Starting with the string taut and P vertically below O, P is set in motion with a horizontal velocity of 7 m s^{-1}. It then moves in part of a vertical circle with centre O. The string becomes slack when the speed of P is 2.8 m s^{-1}.

Find the length of the string. Find also the angle that OP makes with the upward vertical at the instant when the string becomes slack. **[7]**

3 A particle Q is performing simple harmonic motion in a vertical line. Its height, x metres, above a fixed level at time t seconds is given by

$$x = c + A\cos(\omega t - \phi)$$

where c, A, ω and ϕ are constants.

(i) Show that $\ddot{x} = -\omega^2(x - c)$. **[3]**

Fig. 3 shows the displacement-time graph of Q for $0 \leqslant t \leqslant 14$.

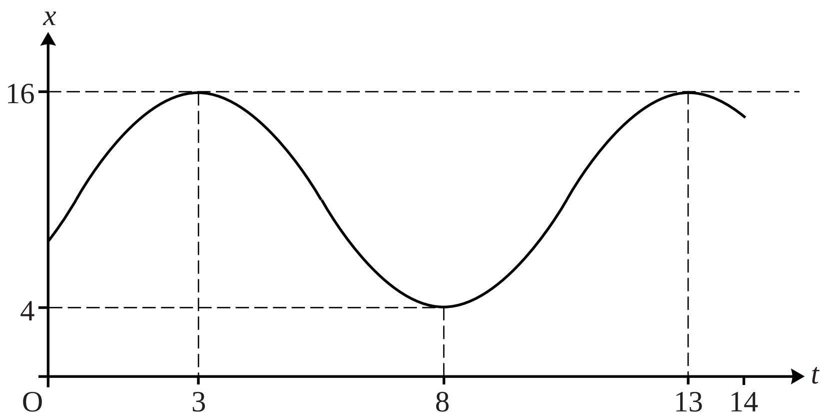

Fig. 3

(ii) Find exact values for c, A, ω and ϕ. **[6]**

(iii) Find the maximum speed of Q. **[2]**

(iv) Find the height and the velocity of Q when $t = 0$. **[3]**

(v) Find the distance travelled by Q between $t = 0$ and $t = 14$. **[4]**

4 **(a)** A uniform lamina occupies the region bounded by the x-axis, the y-axis and the curve $y = 3 - \sqrt{x}$ for $0 \leqslant x \leqslant 9$. Find the coordinates of the centre of mass of this lamina. **[9]**

(b) Fig. 4.1 shows the region bounded by the line $x = 2$ and the part of the circle $y^2 = 25 - x^2$ for which $2 \leqslant x \leqslant 5$. This region is rotated about the x-axis to form a uniform solid of revolution S.

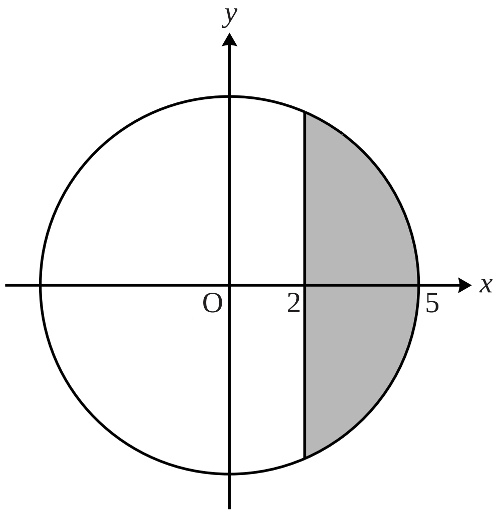

Fig. 4.1

(i) Find the x-coordinate of the centre of mass of S. **[5]**

The solid S rests in equilibrium with its curved surface in contact with a rough plane inclined at 25° to the horizontal. Fig. 4.2 shows a vertical section containing AB, which is a diameter and also a line of greatest slope of the flat surface of S. This section also contains XY, which is a line of greatest slope of the plane.

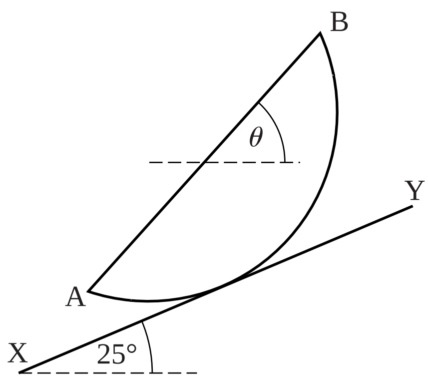

Fig. 4.2

(ii) Find the angle θ that AB makes with the horizontal. **[4]**

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