OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4763

Mechanics 3

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 22 June 2011
Morning

Duration: 1 hour 30 minutes

4763

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 A particle is moving in a straight line. At time t its displacement x from a fixed point O on the line is given by

$$x = A \sin \omega t$$

where A and ω are constants.

(i) Show that $\frac{d^2x}{dt^2} = -\omega^2 x$ and $\left(\frac{dx}{dt}\right)^2 = \omega^2(A^2 - x^2)$. **[5]**

A ball floats on the surface of the sea. Waves cause the ball to rise and fall in a vertical line, and the ball is executing simple harmonic motion. The centre of the oscillations is 8 m above the sea-bed. The ball has speed $1.2\,\mathrm{m\,s^{-1}}$ when it is 7.3 m above the sea-bed, and it has speed $0.75\,\mathrm{m\,s^{-1}}$ when it is 10 m above the sea-bed.

(ii) Show that the amplitude of the oscillations is 2.5 m, and find the period. **[6]**

(iii) Find the maximum speed of the ball. **[1]**

(iv) Find the magnitude and direction of the acceleration of the ball when it is 6.4 m above the sea-bed. **[3]**

(v) Find the time taken for the ball to move upwards from 6 m above the sea-bed to 9 m above the sea-bed. **[3]**

2 **(a)** A particle P of mass 0.2 kg is connected to a fixed point O by a light inextensible string of length 3.2 m, and is moving in a vertical circle with centre O and radius 3.2 m. Air resistance may be neglected. When P is at the highest point of the circle, the tension in the string is 0.6 N.

(i) Find the speed of P when it is at the highest point. **[3]**

(ii) For an instant when OP makes an angle of 60° with the downward vertical, find

(A) the radial and tangential components of the acceleration of P, **[5]**

(B) the tension in the string. **[3]**

(b) A solid cone is fixed with its axis of symmetry vertical and its vertex V uppermost. The semi-vertical angle of the cone is 36°, and its surface is smooth. A particle Q of mass 0.2 kg is connected to V by a light inextensible string, and Q moves in a horizontal circle at constant speed, in contact with the surface of the cone, as shown in Fig. 2.

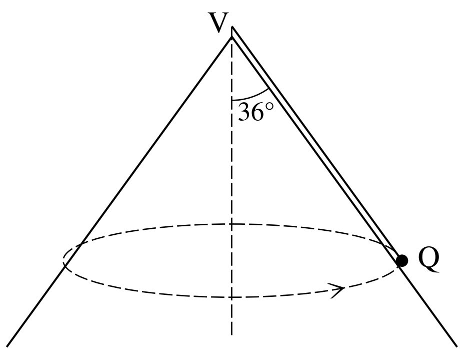

Fig. 2

The particle Q makes one complete revolution in 1.8 s, and the normal reaction of the cone on Q has magnitude 0.75 N.

(i) Find the tension in the string. **[2]**

(ii) Find the length of the string. **[5]**

3 Fixed points A and B are 4.8 m apart on the same horizontal level. The midpoint of AB is M. A light elastic string, with natural length 3.9 m and modulus of elasticity 573.3 N, has one end attached to A and the other end attached to B.

(i) Find the elastic energy stored in the string. **[2]**

A particle P is attached to the midpoint of the string, and is released from rest at M. It comes instantaneously to rest when P is 1.8 m vertically below M.

(ii) Show that the mass of P is 15 kg. **[5]**

(iii) Verify that P can rest in equilibrium when it is 1.0 m vertically below M. **[4]**

In general, a light elastic string, with natural length a and modulus of elasticity λ, has its ends attached to fixed points which are a distance d apart on the same horizontal level. A particle of mass m is attached to the midpoint of the string, and in the equilibrium position each half of the string has length h, as shown in Fig. 3.

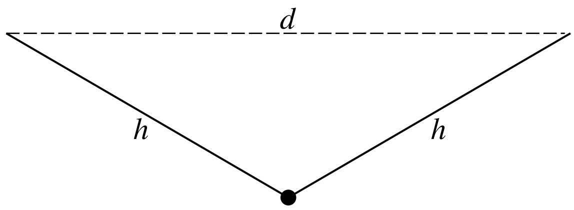

Fig. 3

When the particle makes small oscillations in a vertical line, the period of oscillation is given by the formula

$$\sqrt{\frac{8\pi^2 h^3}{8h^3 - ad^2}}\, m^{\alpha} a^{\beta} \lambda^{\gamma}.$$

(iv) Show that $\dfrac{8\pi^2 h^3}{8h^3 - ad^2}$ is dimensionless. **[1]**

(v) Use dimensional analysis to find α, β and γ. **[4]**

(vi) Hence find the period when the particle P makes small oscillations in a vertical line centred on the position of equilibrium given in part **(iii)**. **[2]**

4 The region A is bounded by the curve $y = x^2 + 5$ for $0 \leqslant x \leqslant 3$, the x-axis, the y-axis and the line $x = 3$. The region B is bounded by the curve $y = x^2 + 5$ for $0 \leqslant x \leqslant 3$, the y-axis and the line $y = 14$. These regions are shown in Fig. 4.

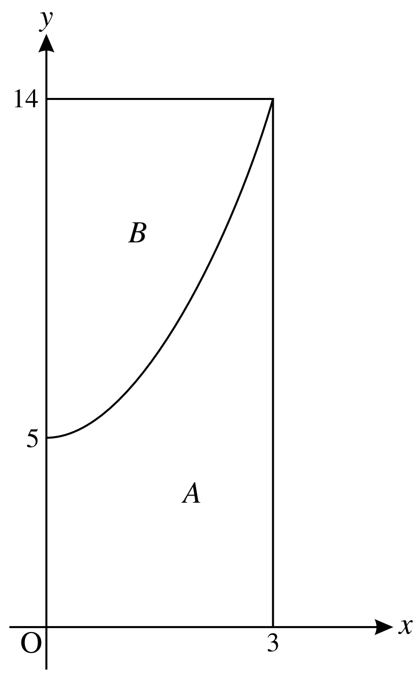

Fig. 4

(i) Find the coordinates of the centre of mass of a uniform lamina occupying the region A. **[9]**

(ii) The region B is rotated through 2π radians about the y-axis to form a uniform solid of revolution R. Find the y-coordinate of the centre of mass of the solid R. **[6]**

(iii) The region A is rotated through 2π radians about the y-axis to form a uniform solid of revolution S. Using your answer to part **(ii)**, or otherwise, find the y-coordinate of the centre of mass of the solid S. **[3]**