ADVANCED GCE

MATHEMATICS (MEI) 4763

Mechanics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Thursday 24 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 **(a)** Two light elastic strings, each having natural length 2.15 m and stiffness $70\,\mathrm{N\,m^{-1}}$, are attached to a particle P of mass 4.8 kg. The other ends of the strings are attached to fixed points A and B, which are 1.4 m apart at the same horizontal level. The particle P is placed 2.4 m vertically below the midpoint of AB, as shown in Fig. 1.

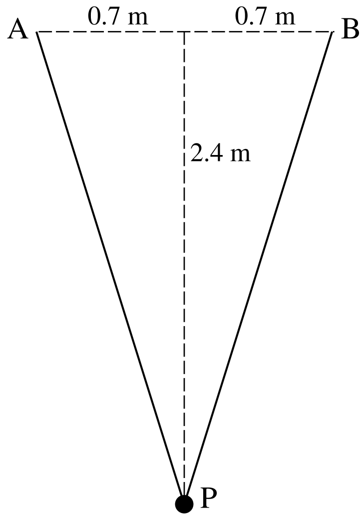

Fig. 1

(i) Show that P is in equilibrium in this position. **[6]**

(ii) Find the energy stored in the string AP. **[2]**

Starting in this equilibrium position, P is set in motion with initial velocity $3.5\,\mathrm{m\,s^{-1}}$ vertically upwards. You are given that P first comes to instantaneous rest at a point C where the strings are slack.

(iii) Find the vertical height of C above the initial position of P. **[4]**

(b) **(i)** Write down the dimensions of force and stiffness (of a spring). **[2]**

A particle of mass m is performing oscillations with amplitude a on the end of a spring with stiffness k. The maximum speed v of the particle is given by $v = cm^{\alpha}k^{\beta}a^{\gamma}$, where c is a dimensionless constant.

(ii) Use dimensional analysis to find α, β and γ. **[4]**

2 A hollow hemisphere has internal radius 2.5 m and is fixed with its rim horizontal and uppermost. The centre of the hemisphere is O. A small ball B of mass 0.4 kg moves in contact with the smooth inside surface of the hemisphere.

At first, B is moving at constant speed in a horizontal circle with radius 1.5 m, as shown in Fig. 2.1.

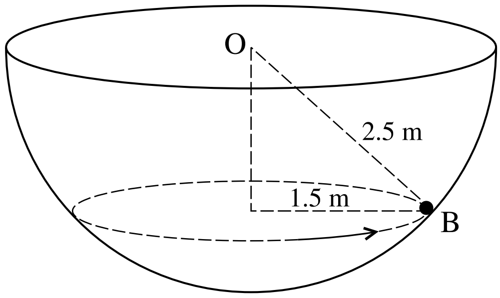

Fig. 2.1

(i) Find the normal reaction of the hemisphere on B. **[3]**

(ii) Find the speed of B. **[3]**

The ball B is now released from rest on the inside surface at a point on the same horizontal level as O. It then moves in part of a vertical circle with centre O and radius 2.5 m, as shown in Fig. 2.2.

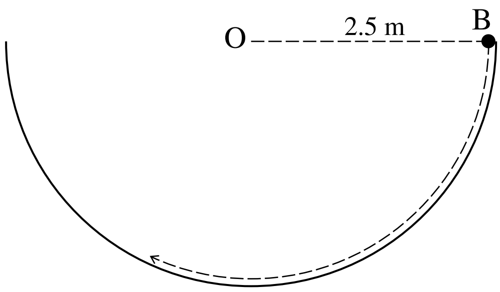

Fig. 2.2

(iii) Show that, when B is at its lowest point, the normal reaction is three times the weight of B. **[4]**

For an instant when the normal reaction is twice the weight of B, find

(iv) the speed of B, **[5]**

(v) the tangential component of the acceleration of B. **[3]**

3 In this question, give your answers in an exact form.

The region R_1 (shown in Fig. 3) is bounded by the x-axis, the lines $x = 1$ and $x = 5$, and the curve $y = \dfrac{1}{x}$ for $1 \leqslant x \leqslant 5$.

(i) A uniform solid of revolution is formed by rotating the region R_1 through 2π radians about the x-axis. Find the x-coordinate of the centre of mass of this solid. **[5]**

(ii) Find the coordinates of the centre of mass of a uniform lamina occupying the region R_1. **[7]**

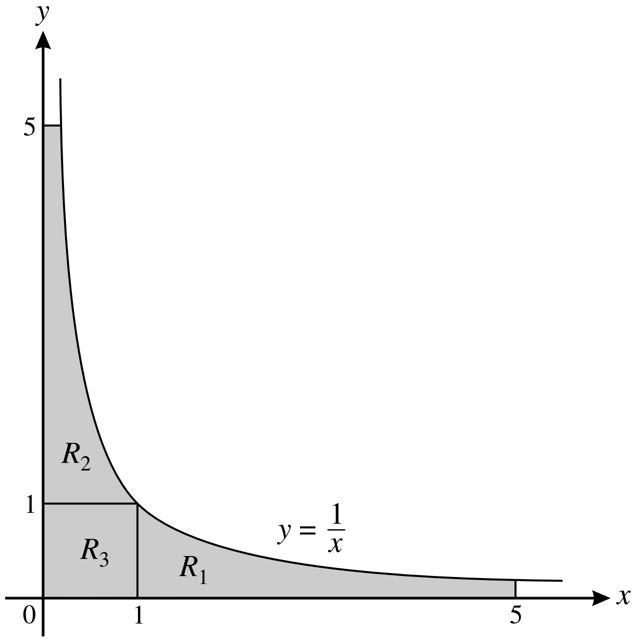

Fig. 3

The region R_2 is bounded by the y-axis, the lines $y = 1$ and $y = 5$, and the curve $y = \dfrac{1}{x}$ for $\frac{1}{5} \leqslant x \leqslant 1$.
The region R_3 is the square with vertices (0, 0), (1, 0), (1, 1) and (0, 1).

(iii) Write down the coordinates of the centre of mass of a uniform lamina occupying the region R_2. **[2]**

(iv) Find the coordinates of the centre of mass of a uniform lamina occupying the region consisting of R_1, R_2 and R_3 (shown shaded in Fig. 3). **[4]**

4 A particle P is performing simple harmonic motion in a vertical line. At time t s, its displacement x m above a fixed point O is given by

$$x = A \sin \omega t + B \cos \omega t$$

where A, B and ω are constants.

(i) Show that the acceleration of P, in $\mathrm{m\,s^{-2}}$, is $-\omega^2 x$. **[3]**

When $t = 0$, P is 16 m *below* O, moving with velocity $7.5\ \mathrm{m\,s^{-1}}$ *upwards*, and has acceleration $1\ \mathrm{m\,s^{-2}}$ *upwards*.

(ii) Find the values of A, B and ω. **[4]**

(iii) Find the maximum displacement, the maximum speed, and the maximum acceleration of P. **[5]**

(iv) Find the speed and the direction of motion of P when $t = 15$. **[2]**

(v) Find the distance travelled by P between $t = 0$ and $t = 15$. **[4]**