GCE

Mathematics (MEI)

Advanced GCE **4763**

Mechanics 3

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

1(a)(i)	$\text{AP} = \sqrt{2.4^2 + 0.7^2} = 2.5$ Tension $T = 70 \times 0.35 \quad (= 24.5)$ Resultant vertical force on P is $2T\cos\theta - mg$ $= 2 \times 24.5 \times \frac{2.4}{2.5} - 4.8 \times 9.8$ $= 47.04 - 47.04 = 0$ Hence P is in equilibrium	M1 A1 M1 B1 B1 E1 **6**	 Attempting to resolve vertically For $T \times \frac{2.4}{2.5}$ (or $T\cos 16.3°$ *etc*) For 4.8×9.8 Correctly shown
(ii)	$\text{EE} = \frac{1}{2} \times 70 \times 0.35^2$ Elastic energy is 4.2875 J	M1 A1 **2**	(M0 for $\frac{1}{2} \times 70 \times 0.35$) *Note* If 70 is used as modulus instead of stiffness: (i) M1A0M1B1B1E0 (ii) M1 A1 for 1.99
(iii)	Initial $\text{KE} = \frac{1}{2} \times 4.8 \times 3.5^2$ By conservation of energy $4.8 \times 9.8h = 2 \times 4.2875 + \frac{1}{2} \times 4.8 \times 3.5^2$ $47.04h = 8.575 + 29.4$ Height is 0.807 m (3 sf)	B1 M1 F1 A1 **4**	 Equation involving EE, KE and PE (A0 for 0.8) ft is $\frac{2 \times (\text{ii}) + 29.4}{47.04}$
(b)(i)	$[\text{ Force }] = \text{M L T}^{-2}$ $[\text{ Stiffness }] = \text{M T}^{-2}$	B1 B1 **2**	*Deduct 1 mark if units are used*
(ii)	$\text{L T}^{-1} = \text{M}^{\alpha}(\text{M T}^{-2})^{\beta}\text{L}^{\gamma}$ $\gamma = 1$ $\beta = \frac{1}{2}$ $0 = \alpha + \beta$ $\alpha = -\frac{1}{2}$	 B1 B1 M1 A1 **4**	 Considering powers of M *When [Stiffness] is wrong in (i), allow all marks ft provided the work is comparable and answers are non-zero*

2 (i)	$R\cos\theta = mg$ [θ is angle between OB and vertical] $R \times 0.8 = 0.4 \times 9.8$ Normal reaction is 4.9 N	M1 A1 A1 **3**	Resolving vertically
(ii)	$R\sin\theta = m\dfrac{v^2}{r}$ $4.9 \times 0.6 = 0.4 \times \dfrac{v^2}{1.5}$ $v^2 = 11.025$ Speed is $3.32\ \mathrm{m\,s^{-1}}$ (3 sf)	M1 A1 A1 **3**	For acceleration $\dfrac{v^2}{r}$ or $r\omega^2$ or $4.9 \times 0.6 = 0.4 \times 1.5\omega^2$ ft is $1.5\sqrt{R}$
(iii)	By conservation of energy $\frac{1}{2}mu^2 = mg \times 2.5$ $u^2 = 5g$ ($u = 7$) $R - mg = m \times \dfrac{u^2}{2.5}$ $R - mg = 2mg$ $R = 3mg$	M1 A1 M1 E1 **4**	Equation involving KE and PE Vertical equation of motion (must have three terms) Correctly shown or $R = 11.76$ and $3 \times 0.4 \times 9.8 = 11.76$
(iv) **(v)**	$\frac{1}{2}mv^2 = mg \times 2.5\cos\theta$ $v^2 = 5g\cos\theta$ $R - mg\cos\theta = m \times \dfrac{v^2}{2.5}$ When $R = 2mg$ ($= 7.84$), $2mg - mg\cos\theta = \dfrac{mv^2}{2.5}$ $2mg - \dfrac{mv^2}{5} = \dfrac{mv^2}{2.5}$ $7.84 - 0.08v^2 = 0.16v^2$ $v^2 = \dfrac{98}{3}$ Speed is $5.72\ \mathrm{m\,s^{-1}}$ (3 sf) $\cos\theta = \dfrac{v^2}{5g} = \dfrac{2}{3}$ ($\theta = 48.2°$ or 0.841 rad) Tangential acceleration is $g\sin\theta$ Tangential acceleration is $7.30\ \mathrm{m\,s^{-2}}$ (3 sf)	M1 A1 M1 M1 M1 A1 M1 A1 **8**	*Mark (iv) and (v) as one part* Equation involving KE, PE and an angle (θ is angle with vertical) [$\frac{1}{2}mv^2 = mgh$ can earn M1A1, but only if $\cos\theta = h/2.5$ appears somewhere] Equation of motion towards O (must have three terms, and the weight must be resolved) Obtaining an equation for v Obtaining an equation for θ *These two marks are each dependent on M1M1 above* [$g\sin\theta$ in isolation only earns M1 if the angle θ is clearly indicated]

3 (i)	Volume is $\int_1^5 \pi\left(\frac{1}{x}\right)^2 \mathrm{d}x$	M1	π may be omitted throughout *Limits not required*
	$= \pi\left[-\frac{1}{x}\right]_1^5 \quad (= \frac{4}{5}\pi)$	A1	For $-\frac{1}{x}$
	$\int \pi x y^2 \,\mathrm{d}x = \int_1^5 \pi x\left(\frac{1}{x}\right)^2 \mathrm{d}x$	M1	*Limits not required*
	$= \pi\left[\ln x\right]_1^5 \quad (= \pi \ln 5)$	A1	For $\ln x$
	$\bar{x} = \frac{\pi \ln 5}{\frac{4}{5}\pi} = \frac{5\ln 5}{4}$ *(2.012)*	A1 **5**	*SR* If exact answers are not seen, deduct only the first A1 affected
(ii)	Area is $\int_1^5 \frac{1}{x}\mathrm{d}x$	M1	*Limits not required*
	$= \left[\ln x\right]_1^5 \quad (= \ln 5)$	A1	For $\ln x$
	$\int x y \,\mathrm{d}x = \int_1^5 x\left(\frac{1}{x}\right)\mathrm{d}x \quad \left(= \left[x\right]_1^5 = 4\right)$	M1	*Limits not required*
	$\bar{x} = \frac{4}{\ln 5}$ *(2.485)*	A1	
	$\int \frac{1}{2} y^2 \,\mathrm{d}x = \int_1^5 \frac{1}{2}\left(\frac{1}{x}\right)^2 \mathrm{d}x$	M1	For $\int \left(\frac{1}{x}\right)^2 \mathrm{d}x$
	$= \left[-\frac{1}{2x}\right]_1^5 \quad (= \frac{2}{5})$	A1	For $-\frac{1}{2x}$
	$\bar{y} = \frac{2/5}{\ln 5} = \frac{2}{5\ln 5}$ *(0.2485)*	A1 **7**	
(iii)	CM of R_2 is $\left(\frac{2}{5\ln 5}, \frac{4}{\ln 5}\right)$	B1B1 ft **2**	*Do not penalise inexact answers in this part*
(iv)		B1	For CM of R_3 is $(\frac{1}{2}, \frac{1}{2})$ (one coordinate is sufficient)
		M1	Using $\sum mx$ with three terms
	$\bar{x} = \frac{(\ln 5)\left(\frac{4}{\ln 5}\right) + (\ln 5)\left(\frac{2}{5\ln 5}\right) + (1)\left(\frac{1}{2}\right)}{\ln 5 + \ln 5 + 1}$	M1	Using $\frac{\sum mx}{\sum m}$ with at least two terms in each sum
	CM is $\left(\frac{4.9}{2\ln 5 + 1}, \frac{4.9}{2\ln 5 + 1}\right)$ *(1.161 , 1.161)*	A1 cao **4**	

4 (i)	$v = \frac{dx}{dt} = A\omega\cos\omega t - B\omega\sin\omega t$ $a = \frac{d^2x}{dt^2} = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$ $= -\omega^2(A\sin\omega t + B\cos\omega t) = -\omega^2 x$	B1 M1 E1 **3**	 Finding the second derivative Correctly shown
(ii)	$B = -16$ $\omega = 0.25$ $A = 30$	B1 B1 B2 **4**	When A is wrong, give B1 for a correct equation involving A [e.g. $A\omega = 7.5$ or $7.5^2 = \omega^2(A^2 + B^2 - 16^2)$] or for $A = -30$
(iii)	Maximum displacement is $(\pm)\sqrt{A^2 + B^2}$ Maximum displacement is 34 m Maximum speed is $(\pm)\ 34\omega$ Maximum acceleration is $(\pm)\ 34\omega^2$ Maximum speed is $8.5\ \mathrm{m\,s^{-1}}$ Maximum acceleration is $2.125\ \mathrm{m\,s^{-2}}$	M1 A1 M1 F1 F1 **5**	Or $7.5^2 = \omega^2(\text{amp}^2 - 16^2)$ Or finding t when $v = 0$ and substituting to find x For either (any valid method) Only ft from $\omega \times$ amp Only ft from $\omega^2 \times$ amp
(iv)	$v = 7.5\cos 0.25t + 4\sin 0.25t$ When $t = 15$, $v = 7.5\cos 3.75 + 4\sin 3.75$ $= -8.44$ Speed is $8.44\ \mathrm{m\,s^{-1}}$ (3 sf); downwards	M1 A1 **2**	
(v)	Period $\frac{2\pi}{\omega} \approx 25$ s , so $t = 0$ to $t = 15$ is less than one period When $t = 15$, $x = 30\sin 3.75 - 16\cos 3.75$ $= -4.02$ Distance travelled is $16 + 34 + 34 + 4.02$ Distance travelled is 88.0 m (3 sf)	M1 M1 M1 A1 cao **4**	 Take account of change of direction Fully correct strategy for distance