4763 Mechanics 3

1 (i)	$\frac{1}{2}m(v^2-1.4^2)=m\times 9.8(2.6-2.6\cos\theta)$ $v^2-1.96=50.96-50.96\cos\theta$ $v^2=52.92-50.96\cos\theta$	M1 A1 E1 **3**	Equation involving KE and PE
(ii)	$0.65\times 9.8\cos\theta-R=0.65\times\frac{v^2}{2.6}$ $6.37\cos\theta-R=0.25(52.92-50.96\cos\theta)$ $6.37\cos\theta-R=13.23-12.74\cos\theta$ $R=19.11\cos\theta-13.23$	M1 A1 M1 A1 **4**	Radial equation involving v^2/r Substituting for v^2 *Dependent on previous M1* *Special case:* $R=13.23-19.11\cos\theta$ *earns* M1A0M1SC1
(iii)	Leaves surface when $R=0$ $\cos\theta=\frac{13.23}{19.11}\ \ (=\frac{9}{13})\quad(\theta=46.19°)$ $v^2=52.92-50.96\times\frac{9}{13}$ Speed is $4.2\ \text{m s}^{-1}$	M1 A1 M1 A1 **4**	(ft if $R=a+b\cos\theta$ and $0<-\frac{a}{b}<1$) *Dependent on previous M1*
(iv)	$T\sin\alpha+R\cos\alpha=0.65\times 9.8$ $T\cos\alpha-R\sin\alpha=0.65\times\frac{1.2^2}{2.4}$	M1 A1 M1 A1	Resolving vertically (3 terms) Horiz eqn involving v^2/r or $r\omega^2$
	OR $T-mg\sin\alpha=m\left(\frac{1.2^2}{2.4}\right)\cos\alpha$ M1A1 $mg\cos\alpha-R=m\left(\frac{1.2^2}{2.4}\right)\sin\alpha$ M1A1		
	$\sin\alpha=\frac{2.4}{2.6}=\frac{12}{13},\quad\cos\alpha=\frac{5}{13}\quad(\alpha=67.38°)$ Tension is 6.03 N Normal reaction is 2.09 N	M1 M1 A1 A1 **8**	Solving to obtain a value of *T* or *R* *Dependent on necessary M1s* *(Accept 6 , 2.1)* *Treat* $\omega=1.2$ *as a misread, leading to* $T=6.744,\ R=0.3764$ *for 7 / 8*

2 (i)	$\frac{1}{2}\times 5000x^2 = \frac{1}{2}\times 400\times 3^2$ Compression is 0.849 m	M1 A1 A1 **3**	Equation involving EE and KE Accept $\frac{3\sqrt{2}}{5}$
(ii)	Change in PE is $400\times 9.8\times(7.35+1.4)\sin\theta$ $= 400\times 9.8\times 8.75\times\frac{1}{7}$ $= 4900\text{ J}$ Change in EE is $\frac{1}{2}\times 5000\times 1.4^2$ $= 4900\text{ J}$ Since Loss of PE = Gain of EE, car will be at rest	M1 A1 M1 E1 **4**	Or $400\times 9.8\times 1.4\sin\theta$ *and* $\frac{1}{2}\times 400\times 4.54^2$ Or $784+4116$ M1M1A1 can also be given for a correct equation in *x* (compression): $2500x^2-560x-4116=0$ Conclusion required, or solving equation to obtain $x=1.4$
(iii)	WD against resistance is $7560(24+x)$ Change in EE is $\frac{1}{2}\times 5000x^2$ Change in KE is $\frac{1}{2}\times 400\times 30^2$ Change in PE is $400\times 9.8\times(24+x)\times\frac{1}{7}$	B1 B1 B1 B1	($=181440+7560x$) ($=2500x^2$) ($=180000$) ($=13440+560x$)
	OR Speed 7.75 m s^{-1} when it hits buffer, then WD against resistance is $7560x$ B1 Change in EE is $\frac{1}{2}\times 5000x^2$ B1 Change in KE is $\frac{1}{2}\times 400\times 7.75^2$ B1 Change in PE is $400\times 9.8\times x\times\frac{1}{7}$ B1		($=2500x^2$) ($=12000$) ($=560x$)
	$-7560(24+x)=\frac{1}{2}\times 5000x^2-\frac{1}{2}\times 400\times 30^2$ $-400\times 9.8\times(24+x)\times\frac{1}{7}$ $-7560(24+x)=2500x^2-180\,000-560(24+x)$ $-3.024(24+x)=x^2-72-0.224(24+x)$ $x^2+2.8x-4.8=0$ $x=\frac{-2.8+\sqrt{2.8^2+19.2}}{2}$ $=1.2$	M1 F1 M1 A1 M1 A1 **10**	Equation involving WD, EE, KE, PE Simplification to three term quadratic

3(a)(i)	[Velocity] $= \mathrm{L\,T^{-1}}$ [Force] $= \mathrm{M\,L\,T^{-2}}$ [Density] $= \mathrm{M\,L^{-3}}$	B1 B1 B1 **3**	*Deduct 1 mark for* $\mathrm{m\,s^{-1}}$ *etc*
(ii)	$\mathrm{M\,L\,T^{-2}} = (\mathrm{M\,L^{-3}})^{\alpha}(\mathrm{L\,T^{-1}})^{\beta}(\mathrm{L^2})^{\gamma}$ $\alpha = 1$ $\beta = 2$ $-3\alpha + \beta + 2\gamma = 1$ $\gamma = 1$	B1 B1 M1A1 A1 **5**	(ft if equation involves α, β and γ)
(b)(i)	$\frac{2\pi}{\omega} = 4.3$ $\omega = \frac{2\pi}{4.3}$ $(=1.4612)$	M1 A1	
	$\dot{\theta}^2 = 1.4612^2(0.08^2 - 0.05^2)$ Angular speed is $0.0913\ \mathrm{rad\,s^{-1}}$	M1 F1 A1 **5**	Using $\omega^2(A^2 - \theta^2)$ For RHS (b.o.d. for $v = 0.0913\ \mathrm{m\,s^{-1}}$)
	OR $\dot{\theta} = 0.08\omega\cos\omega t$ M1 $= 0.08 \times 1.4612\cos 0.6751$ F1 $= 0.0913$ A1		Or $\dot{\theta} = (-)\ 0.08\omega\sin\omega t$ $= (-)\ 0.08 \times 1.4612\sin 0.8957$
(ii)	$\theta = 0.08\sin\omega t$ When $\theta = 0.05$, $0.08\sin\omega t = 0.05$ $\omega t = 0.6751$ $t = 0.462$ Time taken is 2×0.462 $= 0.924\ \mathrm{s}$	B1 M1 A1 cao M1 A1 cao **5**	or $\theta = 0.08\cos\omega t$ Using $\theta = (\pm)0.05$ to obtain an equation for *t* *B1M1 above can be earned in (i)* or $t = 0.613$ from $\theta = 0.08\cos\omega t$ or $t = 1.537$ from $\theta = 0.08\cos\omega t$ Strategy for finding the required time (2×0.462 or $\frac{1}{2} \times 4.3 - 2 \times 0.613$ or $1.537 - 0.613$) *Dep on first M1* *For* $\theta = 0.05\sin\omega t$ *, max B0M1A0M0* *(for* $0.05 = 0.05\sin\omega t$ *)*

4 (a)	Area is $\int_0^{\ln 3} e^x \, dx = \left[e^x \right]_0^{\ln 3}$	M1	
	$= 2$	A1	
	$\int x\,y\,dx = \int_0^{\ln 3} x e^x \, dx$	M1	
	$= \left[x e^x - e^x \right]_0^{\ln 3}$	M1 A1	Integration by parts For $x e^x - e^x$
	$= 3\ln 3 - 2$		
	$\bar{x} = \dfrac{3\ln 3 - 2}{2} = \frac{3}{2}\ln 3 - 1$	A1	ww full marks (B4) Give B3 for 0.65
	$\int \frac{1}{2} y^2 \, dx = \int_0^{\ln 3} \frac{1}{2}(e^x)^2 \, dx$	M1	For integral of $(e^x)^2$
	$= \left[\frac{1}{4} e^{2x} \right]_0^{\ln 3}$	A1	For $\frac{1}{4} e^{2x}$
	$= 2$		
	$\bar{y} = \dfrac{2}{2} = 1$	A1 **9**	If area wrong, SC1 for $\bar{x} = \dfrac{3\ln 3 - 2}{area}$ and $\bar{y} = \dfrac{2}{area}$
(b)(i)	Volume is $\int \pi y^2 \, dx = \int_2^a \pi \dfrac{36}{x^4} dx$	M1	*π may be omitted throughout*
	$= \pi \left[-\dfrac{12}{x^3} \right]_2^a = \pi \left(\dfrac{3}{2} - \dfrac{12}{a^3} \right)$	A1	
	$\int \pi x y^2 \, dx = \int_2^a \pi \dfrac{36}{x^3} dx$	M1	
	$= \pi \left[-\dfrac{18}{x^2} \right]_2^a = \pi \left(\dfrac{9}{2} - \dfrac{18}{a^2} \right)$	A1	
	$\bar{x} = \dfrac{\int \pi x y^2 \, dx}{\int \pi y^2 \, dx}$	M1	
	$= \dfrac{\pi \left(\frac{9}{2} - \frac{18}{a^2} \right)}{\pi \left(\frac{3}{2} - \frac{12}{a^3} \right)} = \dfrac{3(a^3 - 4a)}{a^3 - 8}$	E1 **6**	
(ii)	Since $a > 2$, $4a > 8$	M1	*Condone $\geq$ instead of $>$ throughout*
	so $a^3 - 4a < a^3 - 8$	A1	
	Hence $\bar{x} = \dfrac{3(a^3 - 4a)}{a^3 - 8} < 3$ i.e. CM is less than 3 units from O	E1 **3**	Fully acceptable explanation *Dependent on M1A1*
	OR As $a \to \infty$, $\bar{x} = \dfrac{3(1 - 4a^{-2})}{1 - 8a^{-3}} \to 3$ M1A1 Since $\bar{x}$ increases as *a* increases, $\bar{x}$ is less than 3 E1		Accept $\bar{x} \approx \dfrac{3a^3}{a^3} \to 3$, etc (M1 for $\bar{x} \to 3$ stated, but A1 requires correct justification)