OCR
RECOGNISING ACHIEVEMENT

Monday 28 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4763/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4763/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(a)** A particle P is executing simple harmonic motion, and the centre of the oscillations is at the point O. The maximum speed of P during the motion is $5.1\,\mathrm{m\,s^{-1}}$. When P is 6 m from O, its speed is $4.5\,\mathrm{m\,s^{-1}}$. Find the period and the amplitude of the motion. **[6]**

(b) The force F of gravitational attraction between two objects of masses m_1 and m_2 at a distance d apart is given by $F = \dfrac{Gm_1m_2}{d^2}$, where G is the universal gravitational constant.

(i) Find the dimensions of G. **[3]**

Three objects, each of mass m, are moving in deep space under mutual gravitational attraction. They move round a single circle with constant angular speed ω, and are always at the three vertices of an equilateral triangle of side R. You are given that $\omega = kG^{\alpha}m^{\beta}R^{\gamma}$, where k is a dimensionless constant.

(ii) Find α, β and γ. **[5]**

For three objects of mass 2500 kg at the vertices of an equilateral triangle of side 50 m, the angular speed is $2.0 \times 10^{-6}\,\mathrm{rad\,s^{-1}}$.

(iii) Find the angular speed for three objects of mass 4.86×10^{14} kg at the vertices of an equilateral triangle of side 30 000 m. **[4]**

2 **(a)** A fixed solid sphere with a smooth surface has centre O and radius 0.8 m. A particle P is given a horizontal velocity of $1.2\,\mathrm{m\,s^{-1}}$ at the highest point on the sphere, and it moves on the surface of the sphere in part of a vertical circle of radius 0.8 m.

(i) Find the radial and tangential components of the acceleration of P at the instant when OP makes an angle $\frac{1}{6}\pi$ radians with the upward vertical. (You may assume that P is still in contact with the sphere.) **[5]**

(ii) Find the speed of P at the instant when it leaves the surface of the sphere. **[6]**

(b) Two fixed points R and S are 2.5 m apart with S vertically below R. A particle Q of mass 0.9 kg is connected to R and to S by two light inextensible strings; Q is moving in a horizontal circle at a constant speed of $5\,\mathrm{m\,s^{-1}}$ with both strings taut. The radius of the circle is 2.4 m and the centre C of the circle is 0.7 m vertically below S, as shown in Fig. 2.

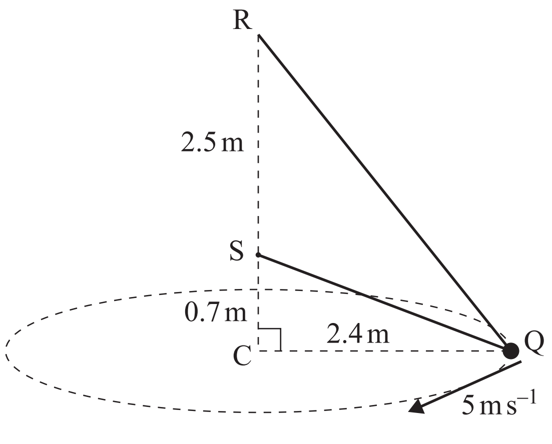

Fig. 2

Find the tension in the string RQ and the tension in the string SQ. **[7]**

3 Two fixed points X and Y are 14.4 m apart and XY is horizontal. The midpoint of XY is M. A particle P is connected to X and to Y by two light elastic strings. Each string has natural length 6.4 m and modulus of elasticity 728 N. The particle P is in equilibrium when it is 3 m vertically below M, as shown in Fig. 3.

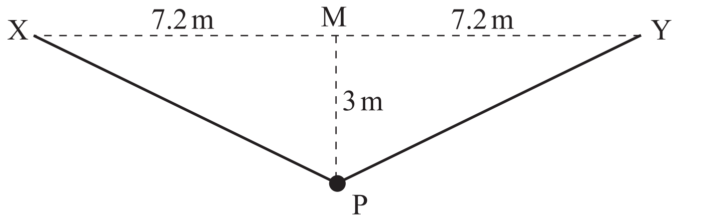

Fig. 3

(i) Find the tension in each string when P is in the equilibrium position. **[3]**

(ii) Show that the mass of P is 12.5 kg. **[3]**

The particle P is released from rest at M, and moves in a vertical line.

(iii) Find the acceleration of P when it is 2.1 m vertically below M. **[5]**

(iv) Explain why the maximum speed of P occurs at the equilibrium position. **[1]**

(v) Find the maximum speed of P. **[6]**

4 **(a)** The region enclosed between the curve $y = x^4$ and the line $y = h$ (where h is positive) is rotated about the y-axis to form a uniform solid of revolution. Find the y-coordinate of the centre of mass of this solid. **[5]**

(b) The region A is bounded by the x-axis, the curve $y = x + \sqrt{x}$ for $0 \leqslant x \leqslant 4$, and the line $x = 4$. The region B is bounded by the y-axis, the curve $y = x + \sqrt{x}$ for $0 \leqslant x \leqslant 4$, and the line $y = 6$. These regions are shown in Fig. 4.

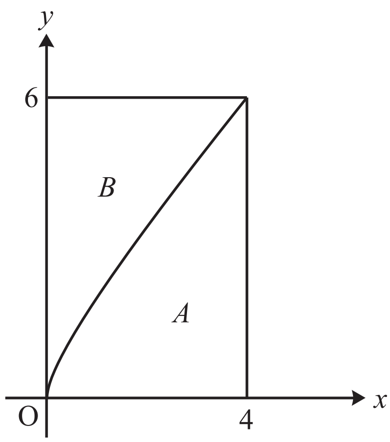

Fig. 4

(i) A uniform lamina occupies the region A. Show that the x-coordinate of the centre of mass of this lamina is 2.56, and find the y-coordinate. **[9]**

(ii) Using your answer to part **(i)**, or otherwise, find the coordinates of the centre of mass of a uniform lamina occupying the region B. **[4]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

