

4763

Mark Scheme

January 2013

Question			Answer	Marks	Guidance
1	(a)		$A\omega = 5.1$ $4.5^2 = \omega^2(A^2 - 6^2)$ $4.5^2 = 5.1^2 - 36\omega^2$ $\omega = 0.4$ Period ($\frac{2\pi}{\omega}$) is $5\pi = 15.7$ s (3 sf) Amplitude (A) is 12.75 m	B1 M1 A1 M1 A1 A1 [6]	Using $v^2 = \omega^2(A^2 - x^2)$ Eliminating A or ω Allow 5π
1	(b)	(i)	$[F] = \text{MLT}^{-2}$ $[G] = \left[\frac{Fd^2}{m_1m_2} \right] = \frac{\text{MLT}^{-2}\text{L}^2}{\text{M}^2}$ $= \text{M}^{-1}\text{L}^3\text{T}^{-2}$	B1 M1 A1 [3]	Obtaining dimensions of G
1	(b)	(ii)	$\text{T}^{-1} = (\text{M}^{-1}\text{L}^3\text{T}^{-2})^\alpha \text{M}^\beta \text{L}^\gamma$ $\alpha = \frac{1}{2}$ $-\alpha + \beta = 0$ $\beta = \frac{1}{2}$ $3\alpha + \gamma = 0$ $\gamma = -\frac{3}{2}$	B1 M1 A1 M1 A1 [5]	Considering powers of M Considering powers of L <i>All marks FT from wrong [G] if comparable. No FT within part (ii).</i>

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1	(b)	(iii)	$\omega = 2.0 \times 10^{-6} \times \left(\frac{4.86 \times 10^{14}}{2500} \right)^{\frac{1}{2}} \times \left(\frac{30000}{50} \right)^{-\frac{3}{2}}$	M1M1 A1	For $\left(\frac{4.86 \times 10^{14}}{2500} \right)^{\pm\frac{1}{2}}$ and $\left(\frac{30000}{50} \right)^{\pm\frac{3}{2}}$ Correct equation for ω <i>Requires $\beta \neq 0, \gamma \neq 0$</i> <i>FT if comparable</i>
		OR	$2.0 \times 10^{-6} = k \times G^{\frac{1}{2}} \times 2500^{\frac{1}{2}} \times 50^{-\frac{3}{2}}$ $kG^{\frac{1}{2}} = 1.414 \times 10^{-5}$ $\omega = 1.414 \times 10^{-5} \times (4.86 \times 10^{14})^{\frac{1}{2}} \times 30000^{-\frac{3}{2}}$		M1 <i>Requires $\beta \neq 0$ or $\gamma \neq 0$</i> M1 <i>Requires $\beta \neq 0$ and $\gamma \neq 0$</i> A1 Correct equation for ω Condone the use of any value for G (including $G = 1$) <i>FT if comparable</i>
			Angular speed is $6.0 \times 10^{-5} \text{ rad s}^{-1}$	A1 [4]	CAO
2	(a)	(i)	$\frac{1}{2}mv^2 - \frac{1}{2}m(1.2)^2 = mg(0.8 - 0.8 \cos \frac{1}{6}\pi)$ $v^2 = 3.5407$ Radial component $\left(\frac{v^2}{0.8} \right)$ is 4.43 ms^{-2} (3 sf) $(\pm)mg \sin \frac{1}{6}\pi = ma_T$ Tangential component is 4.9 ms^{-2}	M1 A1 A1 M1 A1 [5]	Equation involving initial KE, final KE and attempt at PE Allow M1 for $\cos \frac{1}{6}\pi$ used instead of $\sin \frac{1}{6}\pi$; but M0 for $a_T = mg \sin \frac{1}{6}\pi$ Allow $\frac{1}{2}g$

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2	(a) (ii)	$\frac{1}{2}mv^2 - \frac{1}{2}m(1.2)^2 = mg(0.8 - 0.8\cos\theta)$ $mg\cos\theta - R = \frac{mv^2}{0.8}$ <p>Leaves surface when $R = 0$</p> $v^2 - 1.44 = 2 \times 9.8 \times 0.8 \left(1 - \frac{v^2}{7.84}\right)$ <p>Speed is 2.39 ms^{-1}</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>Equation involving initial KE, final KE and attempt at PE in general position</p> <p>Equation involving resolved component of weight and v^2 / r</p> <p>R may be omitted</p> <p>May be implied</p> <p>Obtaining equation in v or θ <i>Dependent on previous M1M1M1</i></p>	<p>θ between OP and upward vertical Allow mgh for PE if h is linked to θ in later work</p> <p>e.g. Implied by $mg\cos\theta = \frac{mv^2}{0.8}$</p> <p>$\cos\theta = \frac{107}{147} = 0.728$ $\theta = 0.756 \text{ rad}$ or 43.3°</p>
2	(b)	$T_R \sin\alpha + T_S \sin\beta = mg$ $0.8T_R + 0.28T_S = 0.9 \times 9.8 \quad (= 8.82)$ $T_R \cos\alpha + T_S \cos\beta = m \frac{v^2}{r}$ $0.6T_R + 0.96T_S = 0.9 \times \frac{5^2}{2.4} \quad (= 9.375)$ <p>Tension in string RQ is 9.737 N Tension in string SQ is 3.68 N</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[7]</p>	<p>Resolving vertically (three terms)</p> <p>Allow $\sin 53.1^\circ$, etc</p> <p>Horizontal equation of motion</p> <p>Obtaining T_R or T_S</p>	<p>$\alpha = \hat{RQC} = 53.1^\circ$, $\beta = \hat{SQC} = 16.3^\circ$</p> <p>Three terms, and v^2 / r</p> <p><i>Dependent on previous M1M1</i></p>

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3	(i)	Length of each string is 7.8 m $T = \frac{728}{6.4}(7.8 - 6.4)$ Tension is 159.25 N	B1 M1 A1 [3]	Using Hooke's law	Must use extension
3	(ii)	$2T \cos \theta = mg$ $2 \times 159.25 \times \frac{5}{13} = m \times 9.8$ $m = \frac{122.5}{9.8} = 12.5 \text{ kg}$	M1 A1 E1 [3]	Resolving vertically FT <i>Working must lead to 12.5 to 3 sf</i>	$\theta = \widehat{XPM} = 67.4^\circ$
3	(iii)	New length of each string is 7.5 m $T = \frac{728}{6.4}(7.5 - 6.4) (= 125.125)$ $mg - 2T \cos \theta = ma$ $12.5 \times 9.8 - 2 \times 125.125 \times 0.28 = 12.5a$ Acceleration is 4.19 ms^{-2} downwards (3 sf)	M1 A1 M1 A1 A1 [5]	Hooke's law with new extension Vertical equation of motion (3 terms) FT for incorrect T <i>Some indication of downwards required</i>	
3	(iv)	At maximum speed, acceleration is zero Acceleration is zero in equilibrium position	B1 [1]	Mention of zero acceleration	<i>Reference to $v^2 = \omega^2(A^2 - x^2)$, SHM, etc, will usually be B0</i>

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3	(v)	Change of PE is $12.5 \times 9.8 \times 3$ (= 367.5) Initial EE is $2 \times \frac{728 \times 0.8^2}{2 \times 6.4}$ (= 72.8) Final EE is $2 \times \frac{728 \times 1.4^2}{2 \times 6.4}$ (= 222.95) $\frac{1}{2}(12.5)v^2 - 367.5 + 222.95 = 72.8$ Maximum speed is 5.90 ms^{-1} (3 sf)	B1 B1 B1 M1 A1 A1 [6]	Allow one string (36.4) Allow one string (111.475) Equation involving KE, PE and EE FT from any B0 above All signs must be correct CAO <i>All terms must be non-zero</i>
4	(a)	$V = \int_0^h \pi(y^{\frac{1}{4}})^2 dy$ $= \pi \left[\frac{2}{3} y^{\frac{3}{2}} \right]_0^h \quad \left(= \frac{2}{3} \pi h^{\frac{3}{2}} \right)$ $V \bar{y} = \int \pi x^2 y dy = \int_0^h \pi y^{\frac{1}{2}} y dy$ $= \pi \left[\frac{2}{5} y^{\frac{5}{2}} \right]_0^h \quad \left(= \frac{2}{5} \pi h^{\frac{5}{2}} \right)$ $\bar{y} = \frac{\frac{2}{5} \pi h^{\frac{5}{2}}}{\frac{2}{3} \pi h^{\frac{3}{2}}} = \frac{3}{5} h$	M1 A1 M1 A1 A1 [5]	For $\int \dots (y^{\frac{1}{4}})^2 dy$ For $\frac{2}{3} y^{\frac{3}{2}}$ For $\int x^2 y dy$ For $\frac{2}{5} y^{\frac{5}{2}}$

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4	(b) (i)	$A = \int_0^4 (x + \sqrt{x}) dx$ $= \left[\frac{1}{2}x^2 + \frac{2}{3}x^{\frac{3}{2}} \right]_0^4 \quad (= \frac{40}{3})$ $A\bar{x} = \int xy dx = \int_0^4 x(x + \sqrt{x}) dx$ $= \left[\frac{1}{3}x^3 + \frac{2}{5}x^{\frac{5}{2}} \right]_0^4 \quad (= \frac{512}{15})$ $\bar{x} = \frac{\frac{512}{15}}{\frac{40}{3}} = \frac{64}{25} = 2.56$ $A\bar{y} = \int \frac{1}{2}y^2 dx = \int_0^4 \frac{1}{2}(x + \sqrt{x})^2 dx$ $= \left[\frac{1}{6}x^3 + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{4}x^2 \right]_0^4 \quad (= \frac{412}{15})$ $\bar{y} = \frac{\frac{412}{15}}{\frac{40}{3}} = \frac{103}{50} = 2.06$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>E1</p> <p>M1</p> <p>A1A1</p> <p>A1</p> <p>[9]</p>	<p>For $\int (x + \sqrt{x}) dx$</p> <p>For $\frac{1}{2}x^2 + \frac{2}{3}x^{\frac{3}{2}}$</p> <p>For $\int xy dx$</p> <p>For $\frac{1}{3}x^3 + \frac{2}{5}x^{\frac{5}{2}}$</p> <p>For $\int \dots y^2 dx$</p> <p>For $\frac{1}{6}x^3 + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{4}x^2$</p> <p>Give A1 for two correct terms</p>

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4	(b)	(ii)	Area of B is $24 - \frac{40}{3} = \frac{32}{3}$ $\frac{32}{3} \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} + \frac{40}{3} \begin{pmatrix} 2.56 \\ 2.06 \end{pmatrix} = 24 \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 1.3 \\ 4.175 \end{pmatrix}$	M1 M1 A1 A1	CM of composite body Correct strategy CAO FT requires $0 < \bar{y} < 6$ (One coordinate sufficient) <i>FT is $6.75 - 1.25\bar{y}_A$</i> <i>No FT from wrong area</i>
		OR	$\int \frac{1}{4}(\sqrt{1+4y}-1)^2 y dy$ or $\int x(6-x-\sqrt{x}) dx$ or $\int \frac{1}{32}(\sqrt{1+4y}-1)^4 dy$ or $\int \frac{1}{2}(6-x-\sqrt{x})(6+x+\sqrt{x}) dx$ $\bar{x} = 1.3, \bar{y} = 4.175$	M1 For any one of these M1 For one successful integration A1A1	
				[4]	