4763 Mechanics 3

1(a) (i)	[Density] $= \mathrm{M\,L^{-3}}$ [Kinetic Energy] $= \mathrm{M\,L^2\,T^{-2}}$ [Power] $= \mathrm{M\,L^2\,T^{-3}}$	B1 B1 B1	*(Deduct B1 for* $\mathrm{kg\,m^{-3}}$ *etc)*	3
(ii)	$\mathrm{M\,L^2\,T^{-3}} = [\eta]\mathrm{L}(\mathrm{L\,T^{-1}})^2$ $[\eta] = \mathrm{M\,L^{-1}\,T^{-1}}$	B1 M1 A1	For $[v] = \mathrm{L\,T^{-1}}$ *Can be earned in (iii)* Obtaining the dimensions of η	3
(iii)	$\mathrm{M\,L^2\,T^{-3}} = (\mathrm{M\,L^{-3}})^\alpha \mathrm{L}^\beta (\mathrm{L\,T^{-1}})^\gamma$ $\alpha = 1$ $-3 = -\gamma$ $\gamma = 3$ $2 = -3\alpha + \beta + \gamma$ $\beta = 2$	B1 cao M1 A1 M1 A1 A1	Considering powers of T *(No ft if* $\gamma = 0$ *)* Considering powers of L Correct equation *(ft requires 4 terms)* *(No ft if* $\beta = 0$ *)*	6
(b)	EE at start is $\frac{1}{2}k \times 0.8^2$ EE at end is $\frac{1}{2}k \times 0.3^2$ $\frac{1}{2}k \times 0.8^2 = \frac{1}{2}k \times 0.3^2 + 5.5 \times 9.8 \times 3.5$ Stiffness is $686\ \mathrm{N\,m^{-1}}$	M1 A1 A1 M1 F1 A1	Calculating elastic energy k may be $\frac{\lambda}{l}$ or $\frac{\lambda}{1.2}$ Equation involving EE and PE *(must have three terms)* (A0 for $\lambda = 823.2$)	6
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2 (a)	$\int \pi x y^2 \, dx = \int_0^a \pi x(a^2 - x^2) \, dx$	M1	*Limits not required*	
	$= \pi \left[\frac{1}{2}a^2x^2 - \frac{1}{4}x^4 \right]_0^a$	A1	For $\frac{1}{2}a^2x^2 - \frac{1}{4}x^4$	
	$= \frac{1}{4}\pi a^4$	A1		
	$\bar{x} = \dfrac{\frac{1}{4}\pi a^4}{\frac{2}{3}\pi a^3}$	M1		
	$= \frac{3}{8}a$	E1		5
(b) (i)	Area is $\int_1^4 (2 - \sqrt{x}) \, dx$	M1	*Limits not required*	
	$= \left[2x - \frac{2}{3}x^{3/2} \right]_1^4 \quad (= \frac{4}{3})$	A1	For $2x - \frac{2}{3}x^{\frac{3}{2}}$	
	$\int x y \, dx = \int_1^4 x(2 - \sqrt{x}) \, dx$	M1	*Limits not required*	
	$= \left[x^2 - \frac{2}{5}x^{\frac{5}{2}} \right]_1^4 \quad (= \frac{13}{5})$	A1	For $x^2 - \frac{2}{5}x^{\frac{5}{2}}$	
	$\bar{x} = \dfrac{13/5}{4/3} = \dfrac{39}{20} = 1.95$	A1		
	$\int \frac{1}{2}y^2 \, dx = \int_1^4 \frac{1}{2}(2 - \sqrt{x})^2 \, dx$	M1	$\int (2 - \sqrt{x})^2 \, dx$ *or* $\int \left((2 - y)^2 - 1\right) y \, dy$	
	$= \left[2x - \frac{4}{3}x^{3/2} + \frac{1}{4}x^2 \right]_1^4 \quad (= \frac{5}{12})$	A2	For $2x - \frac{4}{3}x^{\frac{3}{2}} + \frac{1}{4}x^2$ *or* $\frac{3}{2}y^2 - \frac{4}{3}y^3 + \frac{1}{4}y^4$ Give A1 for two terms correct, or all correct with ½ omitted	
	$\bar{y} = \dfrac{5/12}{4/3} = \dfrac{5}{16} = 0.3125$	A1		9
(ii)	Taking moments about A $T_C \times 3 - W \times 0.95 = 0$	M1 A1	Moments equation (no force omitted) Any correct moments equation (May involve both T_A and T_C) Accept Wg or $W = \frac{4}{3}, \frac{4}{3}g$ here	
	$T_A + T_C = W$	M1	Resolving vertically (or a second moments equation)	
	$T_A = \dfrac{41}{60}W, \quad T_C = \dfrac{19}{60}W$	A1	*Accept 0.68W, 0.32W*	4
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3 (i)	By conservation of energy, $\frac{1}{2}\times 0.6\times 6^2 - \frac{1}{2}\times 0.6v^2 = 0.6\times 9.8(1.25-1.25\cos\theta)$ $36 - v^2 = 24.5 - 24.5\cos\theta$ $v^2 = 11.5 + 24.5\cos\theta$	M1 A1 E1	Equation involving KE and PE	3
(ii)	$T - 0.6\times 9.8\cos\theta = 0.6\times\frac{v^2}{1.25}$ $T - 5.88\cos\theta = 0.48(11.5 + 24.5\cos\theta)$ $T = 5.52 + 17.64\cos\theta$	M1 A1 M1 A1	For acceleration $\frac{v^2}{r}$ Substituting for v^2	4
(iii)	String becomes slack when $T = 0$ $\cos\theta = -\frac{5.52}{17.64}$ ($\theta = 108.2°$ or 1.889 rad) $v^2 = 11.5 - 24.5\times\frac{5.52}{17.64}$ Speed is 1.96 m s^{-1} (3 sf)	M1 A1 M1 A1 cao	 *May be implied* or $0.6\times 9.8\times\frac{5.52}{17.64} = 0.6\times\frac{v^2}{1.25}$ or $-0.6\times 9.8\times\frac{v^2-11.5}{24.5} = 0.6\times\frac{v^2}{1.25}$	4
(iv)	$T_1\cos\theta = mg$ $T_1\times\frac{1.2}{1.25} = 0.6\times 9.8$ (where θ is angle COP) Tension in OP is 6.125 N $T_1\sin\theta + T_2 = \frac{mv^2}{0.35}$ $6.125\times\frac{0.35}{1.25} + T_2 = \frac{0.6\times 1.4^2}{0.35}$ Tension in CP is 1.645 N	M1 A1 A1 M1 F1B1 A1	Resolving vertically Horizontal equation (three terms) For LHS and RHS	7
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4(i)	$T_{AP} = \frac{7.35}{1.5} \times 0.05 \quad (= 0.245)$ $T_{BP} = \frac{7.35}{2.5} \times 0.5 \quad (= 1.47)$ Resultant force up the plane is $T_{BP} - T_{AP} - mg\sin 30°$ $= 1.47 - 0.245 - 0.25 \times 9.8 \sin 30°$ $= 1.47 - 0.245 - 1.225$ $= 0$ Hence there is no acceleration	M1 A1 A1 M1 E1	Using Hooke's law or $\frac{7.35}{1.5}(\text{AP} - 1.5)$ or $\frac{7.35}{2.5}(2.05 - \text{AP})$ Correctly shown	5
(ii)	$T_{AP} = \frac{7.35}{1.5}(0.05 + x) \quad (= 0.245 + 4.9x)$ $T_{BP} = \frac{7.35}{2.5}(4.55 - 1.55 - x - 2.5)$ $= 2.94(0.5 - x)$ $= 1.47 - 2.94x$	B1 M1 E1		3
(iii)	$T_{BP} - T_{AP} - mg\sin 30° = m\frac{d^2x}{dt^2}$ $(1.47 - 2.94x) - (0.245 + 4.9x) - 1.225 = 0.25\frac{d^2x}{dt^2}$ $\frac{d^2x}{dt^2} = -31.36x$ Hence the motion is simple harmonic Period is $\frac{2\pi}{\sqrt{31.36}} = \frac{2\pi}{5.6}$ Period is 1.12 s (3 sf)	M1 A2 E1 B1 cao	Equation of motion parallel to plane Give A1 for an equation which is correct apart from sign errors Must state conclusion. Working must be fully correct (cao) *If a is used for accn down plane, then $a = 31.36x$ can earn M1A2; but E1 requires comment about directions* *Accept* $\frac{5\pi}{14}$	5
(iv)	$x = -0.05\cos 5.6t$ $v = 0.28\sin 5.6t$ $-0.2 = 0.28\sin 5.6t$ OR $0.2^2 = 31.36(0.05^2 - x^2)$ $x = (\pm)\ 0.035$ $0.035 = -0.05\cos 5.6t$ M1 $5.6t = \pi + 0.7956$ Time is 0.703 s (3 sf)	M1 A1 M1 M1 A1cao	For $A\sin\omega t$ or $A\cos\omega t$ Allow $\pm 0.05\sin/\cos 5.6t$ *Implied by* $v = \pm 0.28\sin/\cos 5.6t$ Using $v = \pm 0.2$ to obtain an equation for t Fully correct strategy for finding the required time	5
				[18]