4763 Mechanics 3

1 (i)	[Force] $= \mathrm{M\,L\,T^{-2}}$ [Density] $= \mathrm{M\,L^{-3}}$	B1 B1 **2**	
(ii)	$[\eta] = \dfrac{[F][d]}{[A][v_2 - v_1]} = \dfrac{(\mathrm{M\,L\,T^{-2}})(\mathrm{L})}{(\mathrm{L^2})(\mathrm{L\,T^{-1}})}$ $= \mathrm{M\,L^{-1}\,T^{-1}}$	B1 M1 A1 **3**	for $[A] = \mathrm{L^2}$ and $[v] = \mathrm{L\,T^{-1}}$ Obtaining the dimensions of η
(iii)	$\left[\dfrac{2a^2\rho g}{9\eta}\right] = \dfrac{\mathrm{L^2\,(M\,L^{-3})(L\,T^{-2})}}{\mathrm{M\,L^{-1}\,T^{-1}}} = \mathrm{L\,T^{-1}}$ which is same as the dimensions of v	B1 M1 E1 **3**	For $[g] = \mathrm{L\,T^{-2}}$ Simplifying dimensions of RHS Correctly shown
(iv)	$(\mathrm{M\,L^{-3}})\mathrm{L}^{\alpha}(\mathrm{L\,T^{-1}})^{\beta}(\mathrm{M\,L^{-1}\,T^{-1}})^{\gamma}$ is dimensionless $\gamma = -1$ $-\beta - \gamma = 0$ $-3 + \alpha + \beta - \gamma = 0$ $\alpha = 1, \quad \beta = 1$	B1 cao M1 M1A1 A1 cao **5**	
(v)	$R = \dfrac{\rho w v}{\eta} = \dfrac{0.4 \times 25 \times 150}{1.6 \times 10^{-5}} \quad (= 9.375 \times 10^7)$ $= \dfrac{1.3 \times 5v}{1.8 \times 10^{-5}}$ Required velocity is $260\ \mathrm{m\,s^{-1}}$	M1 A1 A1 cao **3**	Evaluating R Equation for v

2 (a)(i)	$T\cos\alpha = T\cos\beta + 0.27\times 9.8$ $\sin\alpha = \frac{1.2}{2.0} = \frac{3}{5},\ \cos\alpha = \frac{4}{5}\quad(\alpha = 36.87°)$ $\sin\beta = \frac{1.2}{1.3} = \frac{12}{13},\ \cos\beta = \frac{5}{13}\quad(\beta = 67.38°)$ $\frac{27}{65}T = 2.646$ Tension is 6.37 N	M1 A1 B1 M1 E1 **5**	Resolving vertically (weight and at least one resolved tension) Allow T_1 and T_2 For $\cos\alpha$ and $\cos\beta$ [or α and β] Obtaining numerical equation for T e.g. $T(\cos 36.9 - \cos 67.4) = 0.27\times 9.8$ *(Condone 6.36 to 6.38)*
(ii)	$T\sin\alpha + T\sin\beta = 0.27\times\frac{v^2}{1.2}$ $6.37\times\frac{3}{5} + 6.37\times\frac{12}{13} = 0.27\times\frac{v^2}{1.2}$ $v^2 = 43.12$ Speed is $6.57\ \text{m s}^{-1}$	M1 A1 M1 A1 **4**	Using $v^2/1.2$ Allow T_1 and T_2 Obtaining numerical equation for v^2
(b)(i)	$0.2\times 9.8 = 0.2\times\frac{u^2}{1.25}$ $u^2 = 9.8\times 1.25 = 12.25$ Speed is $3.5\ \text{m s}^{-1}$	M1 E1 **2**	Using acceleration $u^2/1.25$
(ii)	$\frac{1}{2}m(v^2 - 3.5^2) = mg(1.25 - 1.25\cos 60)$ $v^2 = 24.5$ Radial component is $\frac{24.5}{1.25}$ $= 19.6\ \text{m s}^{-2}$ Tangential component is $g\sin 60$ $= 8.49\ \text{m s}^{-2}$	M1 A1 M1 A1 M1 A1 **6**	Using conservation of energy With numerical value obtained by using energy (M0 if mass, or another term, included) For sight of $(m)g\sin 60°$ with no other terms
(iii)	$T + 0.2\times 9.8\cos 60 = 0.2\times 19.6$ Tension is 2.94 N	M1 A1 cao **2**	Radial equation (3 terms) *This M1 can be awarded in (ii)*

3 (i)	$\frac{980}{25}y = 5\times 9.8$ Extension is 1.25 m	M1 A1 **2**	Using $\frac{\lambda y}{l_0}$ (Allow M1 for $980y = mg$)
(ii)	$T = \frac{980}{25}(1.25+x)$ $5\times 9.8 - 39.2(1.25+x) = 5\frac{d^2x}{dt^2}$ $-39.2x = 5\frac{d^2x}{dt^2}$ $\frac{d^2x}{dt^2} = -7.84x$	B1 (ft) M1 F1 E1 **4**	*(ft) indicates ft from previous parts as for A marks* Equation of motion with three terms Must have $\ddot{x}$ In terms of x only
(iii)	$8.4^2 = 7.84(A^2 - 1.25^2)$ Amplitude is 3.25 m	M2 A1 A1 **4**	Using $v^2 = \omega^2(A^2 - x^2)$
	OR M2 $\frac{980}{2\times 25}y^2 = 5\times 9.8y + \frac{1}{2}\times 5\times 8.4^2$ A1 $y = 4.5$ Amplitude is $4.5 - 1.25 = 3.25$ m A1		Equation involving EE, PE and KE
	OR $x = A\sin 2.8t + B\cos 2.8t$ $x = -1.25,\ v = 8.4$ when $t = 0$ M2 $\Rightarrow A = 3,\ B = -1.25$ A1 Amplitude is $\sqrt{A^2+B^2} = 3.25$ A1		Obtaining A and B Both correct
(iv)	Maximum speed is $A\omega = 3.25\times 2.8$ $= 9.1\ \text{m s}^{-1}$	M1 A1 **2**	or equation involving EE, PE and KE ft only if answer is greater than 8.4
(v)	$x = 3.25\cos 2.8t$ $-1.25 = 3.25\cos 2.8t$ Time is 0.702 s	B1 (ft) M1 M1 A1 cao **4**	or $x = 3.25\sin 2.8t$ or $v = 9.1\cos 2.8t$ or $v = 9.1\sin 2.8t$ or $x = 3.25\sin(2.8t+\varepsilon)$ etc or $x = \pm 3\sin 2.8t \pm 1.25\cos 2.8t$ Obtaining equation for t or ε by setting $x = (\pm)1.25$ or $v = (\pm)8.4$ or solving $\pm 3\sin 2.8t \pm 1.25\cos 2.8t = 3.25$ Strategy for finding the required time e.g. $\frac{1}{2.8}\sin^{-1}\frac{1.25}{3.25} + \frac{1}{4}\times\frac{2\pi}{2.8}$ $2.8t - 0.3948 = \frac{1}{2}\pi$ or $2.8t - 1.966 = 0$

(vi)	e.g. Rope is light Rock is a particle No air resistance / friction / external forces Rope obeys Hooke's law / Perfectly elastic / Within elastic limit / No energy loss in rope	B1B1B1 **3**	Three modelling assumptions

4 (a)	$\int \frac{1}{2}y^2\,\mathrm{d}x = \int_{-a}^{a} \frac{1}{2}(a^2 - x^2)\,\mathrm{d}x$	M1	For integral of $(a^2 - x^2)$
	$= \left[\frac{1}{2}(a^2x - \frac{1}{3}x^3) \right]_{-a}^{a}$		
	$= \frac{2}{3}a^3$	A1	
	$\bar{y} = \dfrac{\frac{2}{3}a^3}{\frac{1}{2}\pi a^2}$	M1	*Dependent on previous M1*
	$= \dfrac{4a}{3\pi}$	E1 **4**	
(b)(i)			*π may be omitted throughout*
	$V = \int \pi y^2\,\mathrm{d}x = \int_0^h \pi (mx)^2\,\mathrm{d}x$	M1	For integral of x^2 or use of $V = \frac{1}{3}\pi r^2 h$ and $r = mh$
	$= \left[\frac{1}{3}\pi m^2 x^3 \right]_0^h = \frac{1}{3}\pi m^2 h^3$	A1	
	$\int \pi x y^2\,\mathrm{d}x = \int_0^h \pi x (mx)^2\,\mathrm{d}x$	M1	For integral of x^3
	$= \left[\frac{1}{4}\pi m^2 x^4 \right]_0^h = \frac{1}{4}\pi m^2 h^4$	A1	
	$\bar{x} = \dfrac{\frac{1}{4}\pi m^2 h^4}{\frac{1}{3}\pi m^2 h^3}$	M1	*Dependent on M1 for integral of x^3*
	$= \frac{3}{4}h$	E1 **6**	
(ii)	$m_1 = \frac{1}{3}\pi \times 0.7^2 \times 2.4\rho = \frac{1}{3}\pi\rho \times 1.176$ $\mathrm{VG}_1 = 1.8$		
	$m_2 = \frac{1}{3}\pi \times 0.4^2 \times 1.1\rho = \frac{1}{3}\pi\rho \times 0.176$	B1	For m_1 and m_2 (or volumes)
	$\mathrm{VG}_2 = 1.3 + \frac{3}{4}\times 1.1 = 2.125$	B1	or $\frac{1}{4}\times 1.1$ from base
	$(m_1 - m_2)(\mathrm{VG}) + m_2(\mathrm{VG}_2) = m_1(\mathrm{VG}_1)$	M1	Attempt formula for composite body
	$(\mathrm{VG}) + 0.176 \times 2.125 = 1.176 \times 1.8$	F1	
	Distance (VG) is 1.74 m	A1 **5**	
(iii)	VQG is a right-angle	M1	
	$\mathrm{VQ} = \mathrm{VG}\cos\theta$ where $\tan\theta = \dfrac{0.7}{2.4}$ $(\theta = 16.26°)$	M1	
	$\mathrm{VQ} = 1.7428 \times \dfrac{24}{25}$		
	$= 1.67$ m	A1 **3**	ft is $\mathrm{VG} \times 0.96$