4763 Mechanics 3

1(a)(i)	[Force] $= \mathrm{M\,L\,T^{-2}}$ [Density] $= \mathrm{M\,L^{-3}}$	B1 B1 **2**	
(ii)	$[E] = \dfrac{[F][l_0]}{[A][l-l_0]} = \dfrac{(\mathrm{M\,L\,T^{-2}})(\mathrm{L})}{(\mathrm{L^2})(\mathrm{L})}$ $= \mathrm{M\,L^{-1}\,T^{-2}}$	B1 M1 A1 **3**	for $[A] = \mathrm{L}^2$ Obtaining the dimensions of *E*
(iii)	$\mathrm{T} = \mathrm{L}^{\alpha}(\mathrm{M\,L^{-3}})^{\beta}(\mathrm{M\,L^{-1}\,T^{-2}})^{\gamma}$ $-2\gamma = 1, \quad \beta + \gamma = 0$ $\gamma = -\frac{1}{2}$ $\beta = \frac{1}{2}$ $\alpha - 3\beta - \gamma = 0$ $\alpha = 1$	B1 cao F1 M1 A1 A1 **5**	Obtaining equation involving α, β, γ
(b)	$AP = 1.7\ \mathrm{m}$ $F = T\cos\theta$ $R + T\sin\theta = 5 \times 9.8$ $T\cos\theta = 0.4(49 - T\sin\theta)$ $\frac{8}{17}T = 0.4(49 - \frac{15}{17}T)$ $T = 23.8$ $T = k(1.7 - 1.5)$ Stiffness is $119\ \mathrm{N\,m^{-1}}$	B1 M1 M1 M1 A1 A1 M1 A1 **8**	Resolving in any direction Resolving in another direction *(M1 for resolving requires no force omitted, with attempt to resolve all appropriate forces)* Using $F = 0.4R$ to obtain an equation involving just one force (or *k*) Correct equation *Allow* $T\cos 61.9$ *etc* or $R = 28$ or $F = 11.2$ *May be implied* *Allow M1 for* $T = \dfrac{\lambda}{1.5} \times 0.2$ If $R = 49$ is assumed, max marks are B1M1M0M0A0A0M1A0

2(a)(i)	$0.1 + 0.01 \times 9.8 = 0.01 \times \frac{u^2}{0.55}$ Speed is $3.3\ \mathrm{m\,s^{-1}}$	M1 A1 A1 **3**	Using acceleration $u^2/0.55$
(ii)	$\frac{1}{2}m(v^2 - u^2) = mg(2 \times 0.55 - 0.15)$ $\frac{1}{2}(v^2 - 3.3^2) = 9.8 \times 0.95$ $v^2 = 29.51$ $R - mg\cos\theta = m\frac{v^2}{a}$ $R - 0.01 \times 9.8 \times \frac{0.4}{0.55} = 0.01 \times \frac{29.51}{0.55}$ Normal reaction is 0.608 N	M1 A1 M1 A1 A1 **5**	Using conservation of energy *(ft is* $v^2 = u^2 + 18.62$*)* Forces and acceleration towards centre *(ft is* $\frac{u^2 + 22.54}{55}$ *)*
(b)(i)	$T = 0.8r\omega^2$ $T = \frac{160}{2}(r-2)$ $\omega^2 = \frac{80(r-2)}{0.8r} = \frac{100(r-2)}{r}$ $\omega^2 = 100 - \frac{200}{r} < 100$, so $\omega < 10$	B1 B1 E1 E1 **4**	
(ii)	$\mathrm{EE} = \frac{1}{2} \times \frac{160}{2} \times (r-2)^2 = 40(r-2)^2$ $\mathrm{KE} = \frac{1}{2}m(r\omega)^2$ $= \frac{1}{2} \times 0.8 \times r^2 \times \frac{100(r-2)}{r}$ $= 40r(r-2)$ Since $r > r-2$, $40r(r-2) > 40(r-2)^2$ i.e. KE > EE	B1 M1 A1 E1 **4**	Use of $\frac{1}{2}mv^2$ with $v = r\omega$ From fully correct working only
(iii)	When $\omega = 6$, $36 = \frac{100(r-2)}{r}$ $r = 3.125$ $T = 80(r-2) = 80(3.125-2)$ Tension is 90 N	M1 M1 A1 cao **3**	Obtaining *r*

3 (i)	$\frac{dx}{dt} = A\omega\cos\omega t - B\omega\sin\omega t$ $\frac{d^2x}{dt^2} = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$ $= -\omega^2(A\sin\omega t + B\cos\omega t) = -\omega^2 x$	B1 B1 ft E1 **3**	*Must follow from their* $\dot{x}$ Fully correct completion *SR* For $\dot{x} = -A\omega\cos\omega t + B\omega\sin\omega t$ $\ddot{x} = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$ award B0B1E0
(ii)	$B = 2$ $A\omega = -1.44$ $-B\omega^2 = -0.18$ *or* $-0.18 = -\omega^2(2)$ $\omega = 0.3,\quad A = -4.8$	B1 M1 A1 cao M1 A1 cao A1 cao **6**	Using $\frac{dx}{dt} = -1.44$ when $t = 0$ $\frac{d^2x}{dt^2} = -0.18$ when $t = 0$ (or $x = 2$)
(iii)	Period is $\frac{2\pi}{\omega} = \frac{2\pi}{0.3} = 20.94 = 20.9$ s (3 sf) Amplitude is $\sqrt{A^2 + B^2} = \sqrt{4.8^2 + 2^2}$ $= 5.2$ m	E1 M1 A1 **3**	or $1.44^2 = 0.3^2(a^2 - 2^2)$
(iv)	$x = -4.8\sin 0.3t + 2\cos 0.3t$ $v = -1.44\cos 0.3t - 0.6\sin 0.3t$ When $t = 12,\ x = 0.3306$ ($v = 1.56$) When $t = 24,\ x = -2.5929$ ($v = -1.35$) Distance travelled is $(5.2 - 0.3306) + 5.2 + 2.5929$ $= 12.7$ m	M1 A1 M1 M1 A1 **5**	Finding *x* when $t = 12$ and $t = 24$ Both displacements correct Considering change of direction Correct method for distance ft from their A, B, ω and amplitude: *Third M1 requires the method to be comparable to the correct one* *A1A1 both require* $\omega \approx 0.3,\ A \neq 0,\ B \neq 0$ *Note* ft from $A = +4.8$ is $x_{12} = -3.92$ ($v < 0$) $x_{24} = 5.03$ ($v > 0$) Distance is $(5.2 - 3.92) + 5.2 + 5.03$ $= 11.5$

4 (i)	$V=\int_1^8 \pi(x^{-\frac{1}{3}})^2\,dx$	M1	π *may be omitted throughout*
	$=\pi\left[\,3x^{\frac{1}{3}}\,\right]_1^8=3\pi$	A1	
	$V\bar{x}=\int_1^8 \pi x(x^{-\frac{1}{3}})^2\,dx$	M1	
	$=\pi\left[\,\frac{3}{4}x^{\frac{4}{3}}\,\right]_1^8=\frac{45}{4}\pi$	A1	
	$\bar{x}=\dfrac{\frac{45}{4}\pi}{3\pi}$	M1	*Dependent on previous M1M1*
	$=\frac{15}{4}=3.75$	A1	
		6	
(ii)	$A=\int_1^8 x^{-\frac{1}{3}}\,dx$	M1	
	$=\left[\,\frac{3}{2}x^{\frac{2}{3}}\,\right]_1^8=\frac{9}{2}=4.5$	A1	
	$A\bar{x}=\int_1^8 x(x^{-\frac{1}{3}})\,dx$	M1	
	$=\left[\,\frac{3}{5}x^{\frac{5}{3}}\,\right]_1^8=\frac{93}{5}=18.6$	A1	
	$\bar{x}=\frac{18.6}{4.5}=\frac{62}{15}\quad(\approx 4.13)$	A1	
	$A\bar{y}=\int_1^8 \frac{1}{2}(x^{-\frac{1}{3}})^2\,dx$	M1	If $\frac{1}{2}$ omitted, award M1A0A0
	$=\left[\,\frac{3}{2}x^{\frac{1}{3}}\,\right]_1^8=\frac{3}{2}=1.5$	A1	
	$\bar{y}=\frac{1.5}{4.5}=\frac{1}{3}$	A1	
		8	

(iii)	$(1)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}+(3.5)\begin{pmatrix}4.5\\ 0.25\end{pmatrix}=(4.5)\begin{pmatrix}62/15\\ 1/3\end{pmatrix}=\begin{pmatrix}18.6\\ 1.5\end{pmatrix}$ $\bar{x}=2.85$ $\bar{y}=0.625$	M1 M1 A1 A1 **4**	Attempt formula for CM of composite body (one coordinate sufficient) Formulae for both coordinates; signs must now be correct, but areas (1 and 3.5) may be wrong. ft only if $1<\bar{x}<8$ ft only if $0.5<\bar{y}<1$ *Other methods:* M1A1 for $\bar{x}$ M1A1 for $\bar{y}$ *(In each case, M1 requires a complete and correct method leading to a numerical value)*