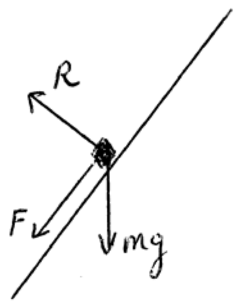
1 (i)	[Velocity] $= \mathrm{L\,T^{-1}}$ [Acceleration] $= \mathrm{L\,T^{-2}}$ [Force] $= \mathrm{M\,L\,T^{-2}}$	B1 B1 B1 **3**	*Deduct 1 mark if answers given as* $\mathrm{m\,s^{-1}},\ \mathrm{m\,s^{-2}},\ \mathrm{kg\,m\,s^{-2}}$
(ii)	$[G] = \dfrac{[F][r^2]}{[m_1][m_2]} = \dfrac{(\mathrm{M\,L\,T^{-2}})(\mathrm{L^2})}{\mathrm{M^2}}$ $= \mathrm{M^{-1}\,L^3\,T^{-2}}$	M1 E1 **2**	
(iii)	$G = 6.67\times10^{-11}\times0.4536\times\dfrac{1}{(0.3048)^3}$ $= 1.07\times10^{-9}\quad(\ \mathrm{lb^{-1}\,ft^3\,s^{-2}}\ )$	M1M1 A1 **3**	For $\times0.4536$ and $\times\dfrac{1}{(0.3048)^3}$ *SC* Give M1 for $6.67\times10^{-11}\times\dfrac{1}{0.4536}\times(0.3048)^3$ $(\ =4.16\times10^{-12}\ )$
(iv)	$[\mathrm{RHS}] = \sqrt{\dfrac{(\mathrm{M^{-1}\,L^3\,T^{-2}})(\mathrm{M})}{\mathrm{L}}}$ $= \sqrt{\mathrm{L^2\,T^{-2}}} = \mathrm{L\,T^{-1}}$ which is the same as [LHS]	M1A1 E1 **3**	
(v)	$\mathrm{T} = (\mathrm{M^{-1}\,L^3\,T^{-2}})^{\alpha}\,\mathrm{M}^{\beta}\,\mathrm{L}^{\gamma}$ Powers of M: $-\alpha+\beta=0$ of L: $3\alpha+\gamma=0$ of T: $-2\alpha=1$ $\alpha=-\frac{1}{2},\ \ \beta=-\frac{1}{2},\ \ \gamma=\frac{3}{2}$	M1 M1 A1 M1 A1 **5**	At least two equations Three correct equations Obtaining at least one of $\alpha,\ \beta,\ \gamma$

2(a)(i)	At the highest point, $T + 5\times 9.8 = 5\times \frac{v^2}{1.8}$ For least speed, $T = 0$, $v^2 = 1.8\times 9.8$ Speed is at least $4.2\ \mathrm{m\,s^{-1}}$	M1 A1 E1 **3**	Using acceleration $v^2/1.8$ *T* may be omitted
(ii)	For least tension, speed at top is $4.2\ \mathrm{m\,s^{-1}}$ By conservation of energy, $\frac{1}{2}\times 5\times (w^2 - 4.2^2) = 5\times 9.8\times 3.6$ $w^2 = 88.2 \quad (w = 9.39)$ $T - 5\times 9.8 = 5\times \frac{88.2}{1.8}$ Tension is at least 294 N	M1 A1 M1 A1 ft A1 **5**	Energy equation with 3 terms Equation of motion with 3 terms
(b)(i)	$R\sin\theta = 0.02\times 9.8$ $R\cos\theta = 0.02\times 0.32\times 8.75^2$ $\tan\theta = \frac{0.02\times 9.8}{0.02\times 0.32\times 8.75^2} = 0.4$	B1 M1 A1 E1 **4**	Using acceleration 0.32×8.75^2 *SC* If $\sin\theta$ and $\cos\theta$ interchanged, award B0M1A1E0
(ii)		B1 B1 **2**	For *R* and *mg* For *F* acting down the slope
(iii)	$R\sin\theta = 0.02\times 9.8 + F\cos\theta$ $R\cos\theta + F\sin\theta = 0.02\times 0.32\,\omega^2$ For maximum ω, $F = \mu R$ $R(\sin\theta - \mu\cos\theta) = 0.02\times 9.8$ $R(\cos\theta + \mu\sin\theta) = 0.02\times 0.32\,\omega^2$ $\omega^2 = \frac{9.8(\cos\theta + \mu\sin\theta)}{0.32(\sin\theta - \mu\cos\theta)} = \frac{9.8(1+\mu\tan\theta)}{0.32(\tan\theta - \mu)}$ $= \frac{9.8(1+0.11\times 0.4)}{0.32(0.4-0.11)}$ $\omega = 10.5$	M1 A1 A1 M1 M1 A1 cao **6**	Resolving *F* and *R* [or *mg* and accn] *Can give A1A1 for sin / cos interchanged consistent with (i)* *Dependent on first M1* Obtaining a numerical value for ω^2 *Dependent on M1M1*

3 (i)	$k \times 0.8 = 60 \times 9.8$ Stiffness is $735\ \mathrm{N\,m^{-1}}$	M1 A1 **2**	
(ii)	Loss of PE is $60 \times 9.8(32 + x)$ Gain in EE is $\frac{1}{2} \times 735x^2$ $\frac{1}{2} \times 735x^2 = 60 \times 9.8(32 + x)$ $x^2 = 1.6(32 + x)$ $x^2 - 1.6x - 51.2 = 0$ $(x - 8)(x + 6.4) = 0$ $x = 8$ Length of rope is 40 m	B1 B1 M1 E1 M1 A1 **6**	*If x is measured from equilibrium position, treat as MR* Obtaining a value of *x*
(iii)	Tension $T = 735x$ $mg - T = m\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}$ $60 \times 9.8 - 735x = 60\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}$ $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + 12.25x = 9.8$	B1 M1 A1 E1 **4**	 Equation of motion with 3 terms
(iv)	SHM with $\omega^2 = 12.25$ ($\omega = 3.5$) Time taken is $\dfrac{1}{4} \times \dfrac{2\pi}{\omega}$ $= \frac{1}{7}\pi = 0.449\ \mathrm{s}$	M1 M1 A1 **3**	 or $\omega t = \frac{1}{2}\pi$
(v)	When $x = 8$, $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = 9.8 - 12.25 \times 8$ $= -88.2$ Acceleration is $88.2\ \mathrm{m\,s^{-2}}$ (upwards) This acceleration (9*g*) is too large for comfort	M1 A1 B1 **3**	or $735 \times 8 - 60 \times 9.8 = 60a$

4 (i)	Area is $\int_1^a \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^a$	M1	
	$= 1 - \frac{1}{a}$	A1	
	$\int x\,y\,dx = \int_1^a \frac{1}{x}\,dx \quad (= \ln a)$	M1	
	$\bar{x} = \frac{\int x\,y\,dx}{\int y\,dx}$	M1	
	$= \frac{\ln a}{1-\frac{1}{a}} \quad \left(= \frac{a \ln a}{a-1} \right)$	A1	
	$\int \frac{1}{2}y^2\,dx = \int_1^a \frac{1}{2x^4}\,dx = \left[-\frac{1}{6x^3}\right]_1^a$ $= \frac{1}{6}\left(1-\frac{1}{a^3}\right)$	M1	Condone omission of $\frac{1}{2}$
	$\bar{y} = \frac{\int \frac{1}{2}y^2\,dx}{\int y\,dx}$	M1	($\frac{1}{2}$ needed for this mark)
	$= \frac{\frac{1}{6}\left(1-\frac{1}{a^3}\right)}{1-\frac{1}{a}} = \frac{a^3-1}{6(a^3-a^2)}$	E1 **8**	
(ii)	When $a=2$, $\bar{x} = 2\ln 2$, $\bar{y} = \frac{7}{24}$		
	$\tan\theta = \frac{\bar{x}-1}{1-\bar{y}}$	M1	CM vertically below A
	$= \frac{2\ln 2 - 1}{1-\frac{7}{24}}$	A1	Correct expression for $\tan\theta$ or $\tan(90-\theta)$
	$\theta = 28.6°$	A1 **3**	

(iii)	Volume is $\int \pi y^2 \, dx \;= \pi \int_1^a \frac{1}{x^4} dx$ $= \pi \left[-\frac{1}{3x^3} \right]_1^a = \frac{\pi}{3}\left(1 - \frac{1}{a^3}\right)$ $\int \pi x y^2 \, dx \;= \pi \int_1^a \frac{1}{x^3} dx = \pi \left[-\frac{1}{2x^2} \right]_1^a$ $= \frac{\pi}{2}\left(1 - \frac{1}{a^2}\right)$ $\bar{x} = \frac{\int \pi x y^2 \, dx}{\int \pi y^2 \, dx}$ $= \frac{\frac{\pi}{2}\left(1 - \frac{1}{a^2}\right)}{\frac{\pi}{3}\left(1 - \frac{1}{a^3}\right)} = \frac{3(a^3 - a)}{2(a^3 - 1)}$ Since $a > 1, \;\; a^3 - a < a^3 - 1$ Hence $\bar{x} < \frac{3}{2}$, i.e. $\bar{x} < 1.5$	M1 A1 M1 M1 A1 M1 E1 **7**	π may be omitted throughout Any correct form or $\bar{x} \to 1.5$ as $a \to \infty$ Fully convincing argument