

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education**

MEI STRUCTURED MATHEMATICS

4763

Mechanics 3

Tuesday

10 JANUARY 2006

Afternoon

1 hour 30 minutes

Additional materials:

8 page answer booklet

Graph paper

MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

2

- 1 (a) (i) Write down the dimensions of force. [1]

The period, t , of a vibrating wire depends on its tension, F , its length, l , and its mass per unit length, σ .

- (ii) Assuming that the relationship is of the form $t = kF^\alpha l^\beta \sigma^\gamma$, where k is a dimensionless constant, use dimensional analysis to determine the values of α , β and γ . [6]

Two lengths are cut from a reel of uniform wire. The first has length 1.2 m, and it vibrates under a tension of 90 N. The second has length 2.0 m, and it vibrates with the same period as the first wire.

- (iii) Find the tension in the second wire. (You may assume that changing the tension does not significantly change the mass per unit length.) [4]

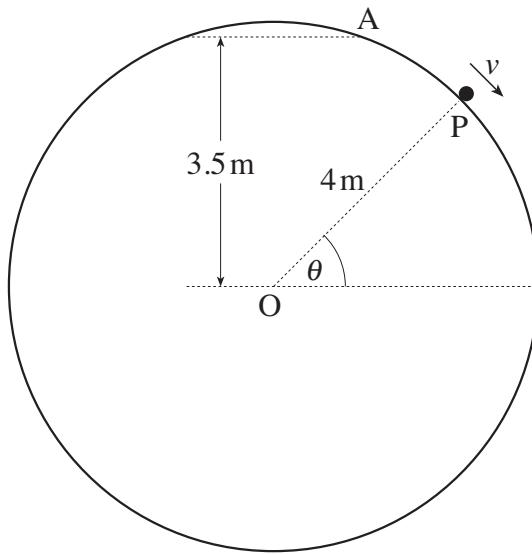
- (b) The midpoint M of a vibrating wire is moving in simple harmonic motion in a straight line, with amplitude 0.018 m and period 0.01 s.

- (i) Find the maximum speed of M. [3]

- (ii) Find the distance of M from the centre of the motion when its speed is 8 m s^{-1} . [4]

3

- 2 (a) A moon of mass 7.5×10^{22} kg moves round a planet in a circular path of radius 3.8×10^8 m, completing one orbit in a time of 2.4×10^6 s. Find the force acting on the moon. [4]
- (b) Fig. 2 shows a fixed solid sphere with centre O and radius 4 m. Its surface is smooth. The point A on the surface of the sphere is 3.5 m vertically above the level of O. A particle P of mass 0.2 kg is placed on the surface at A and is released from rest. In the subsequent motion, when OP makes an angle θ with the horizontal and P is still on the surface of the sphere, the speed of P is v m s $^{-1}$ and the normal reaction acting on P is R N.

**Fig. 2**

- (i) Express v^2 in terms of θ . [3]
- (ii) Show that $R = 5.88 \sin \theta - 3.43$. [4]
- (iii) Find the radial and tangential components of the acceleration of P when $\theta = 40^\circ$. [4]
- (iv) Find the value of θ at the instant when P leaves the surface of the sphere. [3]

- 3 A light elastic rope has natural length 15 m. One end of the rope is attached to a fixed point O and the other end is attached to a small rock of mass 12 kg.

When the rock is hanging in equilibrium vertically below O, the length of the rope is 15.8 m.

- (i) Show that the modulus of elasticity of the rope is 2205 N. [2]

The rock is pulled down to the point 20 m vertically below O, and is released from rest in this position. It moves upwards, and comes to rest instantaneously, with the rope slack, at the point A.

- (ii) Find the acceleration of the rock immediately after it is released. [3]

- (iii) Use an energy method to find the distance OA. [5]

At time t seconds after release, the rope is still taut and the displacement of the rock *below the equilibrium position* is x metres.

- (iv) Show that $\frac{d^2x}{dt^2} = -12.25x$. [4]

- (v) Write down an expression for x in terms of t , and hence find the time between releasing the rock and the rope becoming slack. [4]

- 4 The region between the curve $y = 4 - x^2$ and the x -axis, from $x = 0$ to $x = 2$, is occupied by a uniform lamina. The units of the axes are metres.

- (i) Show that the coordinates of the centre of mass of this lamina are $(0.75, 1.6)$. [9]

This lamina and another exactly like it are attached to a uniform rod PQ, of mass 12 kg and length 8 m, to form a rigid body as shown in Fig. 4. Each lamina has mass 6.5 kg. The ends of the rod are at P(-4, 0) and Q(4, 0). The rigid body lies entirely in the (x, y) plane.

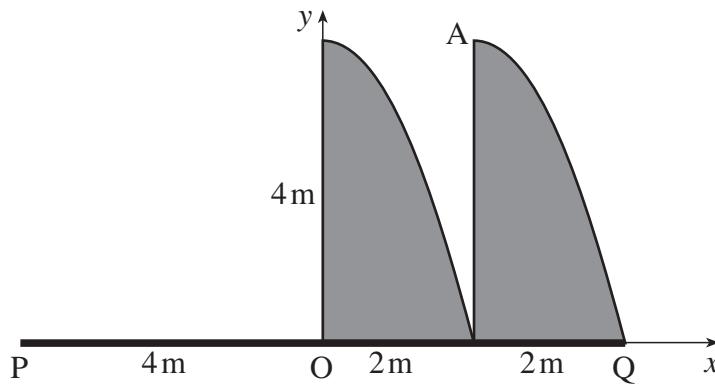


Fig. 4

- (ii) Find the coordinates of the centre of mass of the rigid body. [5]

The rigid body is freely suspended from the point A(2, 4) and hangs in equilibrium.

- (iii) Find the angle that PQ makes with the horizontal. [4]