

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education  
Advanced General Certificate of Education**

**MEI STRUCTURED MATHEMATICS**

**4763**

Mechanics 3

Tuesday **10 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:  
8 page answer booklet  
Graph paper  
MEI Examination Formulae and Tables (MF2)

**TIME** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

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**This question paper consists of 4 printed pages.**

## 2

- 1 (a) (i) Write down the dimensions of force. [1]

The period,  $t$ , of a vibrating wire depends on its tension,  $F$ , its length,  $l$ , and its mass per unit length,  $\sigma$ .

- (ii) Assuming that the relationship is of the form  $t = kF^\alpha l^\beta \sigma^\gamma$ , where  $k$  is a dimensionless constant, use dimensional analysis to determine the values of  $\alpha$ ,  $\beta$  and  $\gamma$ . [6]

Two lengths are cut from a reel of uniform wire. The first has length 1.2 m, and it vibrates under a tension of 90 N. The second has length 2.0 m, and it vibrates with the same period as the first wire.

- (iii) Find the tension in the second wire. (You may assume that changing the tension does not significantly change the mass per unit length.) [4]

- (b) The midpoint M of a vibrating wire is moving in simple harmonic motion in a straight line, with amplitude 0.018 m and period 0.01 s.

- (i) Find the maximum speed of M. [3]

- (ii) Find the distance of M from the centre of the motion when its speed is  $8 \text{ m s}^{-1}$ . [4]

3

- 2 (a) A moon of mass  $7.5 \times 10^{22}$  kg moves round a planet in a circular path of radius  $3.8 \times 10^8$  m, completing one orbit in a time of  $2.4 \times 10^6$  s. Find the force acting on the moon. [4]
- (b) Fig. 2 shows a fixed solid sphere with centre O and radius 4 m. Its surface is smooth. The point A on the surface of the sphere is 3.5 m vertically above the level of O. A particle P of mass 0.2 kg is placed on the surface at A and is released from rest. In the subsequent motion, when OP makes an angle  $\theta$  with the horizontal and P is still on the surface of the sphere, the speed of P is  $v \text{ m s}^{-1}$  and the normal reaction acting on P is  $R \text{ N}$ .

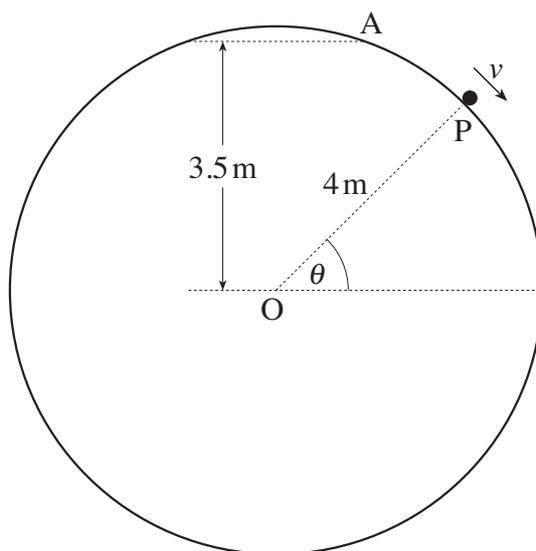


Fig. 2

- (i) Express  $v^2$  in terms of  $\theta$ . [3]
- (ii) Show that  $R = 5.88 \sin \theta - 3.43$ . [4]
- (iii) Find the radial and tangential components of the acceleration of P when  $\theta = 40^\circ$ . [4]
- (iv) Find the value of  $\theta$  at the instant when P leaves the surface of the sphere. [3]

## 4

- 3 A light elastic rope has natural length 15 m. One end of the rope is attached to a fixed point O and the other end is attached to a small rock of mass 12 kg.

When the rock is hanging in equilibrium vertically below O, the length of the rope is 15.8 m.

- (i) Show that the modulus of elasticity of the rope is 2205 N. [2]

The rock is pulled down to the point 20 m vertically below O, and is released from rest in this position. It moves upwards, and comes to rest instantaneously, with the rope slack, at the point A.

- (ii) Find the acceleration of the rock immediately after it is released. [3]  
 (iii) Use an energy method to find the distance OA. [5]

At time  $t$  seconds after release, the rope is still taut and the displacement of the rock *below the equilibrium position* is  $x$  metres.

- (iv) Show that  $\frac{d^2x}{dt^2} = -12.25x$ . [4]

- (v) Write down an expression for  $x$  in terms of  $t$ , and hence find the time between releasing the rock and the rope becoming slack. [4]

- 4 The region between the curve  $y = 4 - x^2$  and the  $x$ -axis, from  $x = 0$  to  $x = 2$ , is occupied by a uniform lamina. The units of the axes are metres.

- (i) Show that the coordinates of the centre of mass of this lamina are (0.75, 1.6). [9]

This lamina and another exactly like it are attached to a uniform rod PQ, of mass 12 kg and length 8 m, to form a rigid body as shown in Fig. 4. Each lamina has mass 6.5 kg. The ends of the rod are at P(-4, 0) and Q(4, 0). The rigid body lies entirely in the  $(x, y)$  plane.

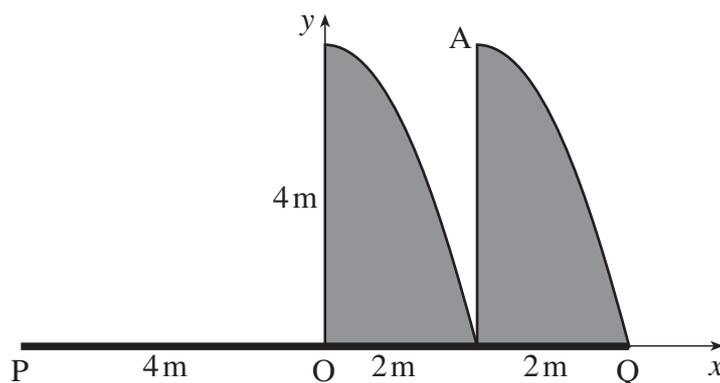


Fig. 4

- (ii) Find the coordinates of the centre of mass of the rigid body. [5]

The rigid body is freely suspended from the point A(2, 4) and hangs in equilibrium.

- (iii) Find the angle that PQ makes with the horizontal. [4]