1(a)(i)	$\mathrm{M\,L\,T^{-2}}$	B1 **1**	Allow $\mathrm{kg\,m\,s^{-2}}$
(ii)	$(\mathrm{T}) = (\mathrm{M\,L\,T^{-2}})^{\alpha}\,(\mathrm{L})^{\beta}\,(\mathrm{M\,L^{-1}})^{\gamma}$ Powers of M: $\alpha + \gamma = 0$ of L: $\alpha + \beta - \gamma = 0$ of T: $-2\alpha = 1$ $\alpha = -\frac{1}{2}, \quad \beta = 1, \quad \gamma = \frac{1}{2}$	B1 M1 M2 A2 **6**	For $\mathrm{M\,L^{-1}}$ For three equations Give M1 for one equation Give A1 for one correct
(iii)	$kF_1^{\alpha}l_1^{\beta}\sigma^{\gamma} = kF_2^{\alpha}l_2^{\beta}\sigma^{\gamma}$ $F_1^{-\frac{1}{2}}\,l_1 = F_2^{-\frac{1}{2}}\,l_2$	M1 A1	 Equation relating F_1, F_2, l_1, l_2
	OR $F^{\alpha}l^{\beta}$ is constant M1 F is proportional to l^2 A1		or equivalent
	$F_2 = 90 \times \frac{2.0^2}{1.2^2}$ $= 250$ (N)	M1 A1 **4**	
(b)(i)	$\frac{2\pi}{\omega} = 0.01$ $\omega = 200\pi$ Maximum speed is $A\omega = 0.018 \times 200\pi$ $= 11.3$ $(\mathrm{m\,s^{-1}})$	B1 M1 A1 **3**	 Accept 3.6π
(ii)	Using $v^2 = \omega^2(A^2 - x^2)$ $8^2 = (200\pi)^2(0.018^2 - x^2)$ $x = 0.0127$ (m)	M1 M1 A1 A1 **4**	 Substituting values
	OR $v = 3.6\pi\cos(200\pi t) = 8$ M1 when $200\pi t = 0.785$ A1 $(t = 0.001249)$ $x = 0.018\sin(200\pi t) = 0.018\sin(0.785)$ M1 $= 0.0127$ A1		*Condone the use of degrees in this part*

2 (a)	$\omega = \frac{2\pi}{2.4\times10^6}$ $(=2.618\times10^{-6})$ Acceleration $a = r\omega^2$ (or $\frac{v^2}{r}$) $= 2.604\times10^{-3}$ Force is $ma = 7.5\times10^{22}\times2.604\times10^{-3}$ $=1.95\times10^{20}$ (N)	B1 M1 M1 A1 **4**	or $v = \frac{2\pi\times3.8\times10^8}{2.4\times10^6}$ $(=994.8)$ M0 for $F - mg = ma$ etc Accept 1.9×10^{20} or 2.0×10^{20}
(b)(i)	Change in PE is $mg(3.5-4\sin\theta)$ By conservation of energy $\frac{1}{2}mv^2 = mg(3.5-4\sin\theta)$ $v^2 = 68.6 - 78.4\sin\theta$	B1 M1 A1 **3**	or as separate terms Accept $7g - 8g\sin\theta$
(ii)	$0.2\times9.8\sin\theta - R = 0.2\times\frac{v^2}{4}$ $1.96\sin\theta - R = 0.05(68.6-78.4\sin\theta)$ $R = 5.88\sin\theta - 3.43$	M1 M1 A1 E1 **4**	Radial equation of motion (3 terms) Substituting from part (i) Correctly obtained
(iii)	When $\theta = 40°$, $v^2 = 18.21$ Radial acceleration is $\frac{v^2}{4} = 4.55$ $(\mathrm{m\,s^{-2}})$ Tangential acceleration is $9.8\cos40$ $= 7.51$ $(\mathrm{m\,s^{-2}})$	M1 A1 M1 A1 **4**	or $0.2g\sin40 - R = ma$ Accept 4.5 or 4.6 M0 for $a = mg\cos40$ etc
(iv)	Leaves surface when $R=0$ $\sin\theta = \frac{3.43}{5.88}$ $\theta = 35.7°$	M1 M1 A1 cao **3**	Accept $36°$, 0.62 rad

3 (i)	$\frac{\lambda}{15} \times 0.8 = 12 \times 9.8$ $\lambda = 2205$ (N)	M1 E1 **2**	
(ii)	$\frac{2205}{15} \times 5 - 12 \times 9.8 = 12a$ $a = 51.45$ $(\mathrm{m\,s^{-2}})$	M1 A1 A1 **3**	Equation of motion including tension Accept 51 or 52
(iii)	Loss of EE is $\frac{1}{2} \times \frac{2205}{15} \times 5^2$ $(= 1837.5)$	M1 A1	Calculating elastic energy
	By conservation of energy $12 \times 9.8 \times h = 1837.5$ $h = 15.625$ OA $= 20 - h = 4.375$ (m)	M1 F1 A1 **5**	Equation involving EE and PE
	OR $12 \times 9.8 \times 5 + \frac{1}{2} \times 12 \times v^2 = 1837.5$ M1 $v^2 = 208.25$ $0 = 208.25 - 2 \times 9.8 \times H$ F1 $H = 10.625$ OA $= 15 - H = 4.375$ (m) A1		Equation involving EE, PE and KE
(iv)	$T = \frac{2205}{15}(0.8 + x)$ $12 \times 9.8 - \frac{2205}{15}(0.8 + x) = 12\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -12.25x$	B1 M1 A1 E1 **4**	or $T = \frac{\lambda}{l}(x_0 + x)$ Equation of motion with three terms or $mg - \frac{\lambda}{l}(x_0 + x) = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ provided that $mg = \frac{\lambda}{l}x_0$ appears somewhere Correctly obtained *No marks for just writing* $-\frac{2205}{15}x = 12\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ *or just using the formula* $\omega^2 = \frac{\lambda}{ml}$ *If x is clearly measured upwards, treat as a mis-read*
(v)	$x = 4.2\cos(3.5t)$ Rope becomes slack when $x = -0.8$ $4.2\cos(3.5t) = -0.8$ $t = 0.504$ (s)	M1 A1 M1 A1 **4**	For $\cos(\sqrt{12.25}\,t)$ or $\sin(\sqrt{12.25}\,t)$ Accept 0.50 or 0.51

4 (i)	$\int y\,\mathrm{d}x = \int_0^2 (4-x^2)\,\mathrm{d}x \quad = \left[4x - \frac{1}{3}x^3\right]_0^2 \quad (=\frac{16}{3})$	B1	
	$\int xy\,\mathrm{d}x = \int_0^2 x(4-x^2)\,\mathrm{d}x$ $= \left[2x^2 - \frac{1}{4}x^4\right]_0^2 \quad (=4)$	M1	
		A1	
	$\bar{x} = \frac{4}{\frac{16}{3}}$ $= 0.75$	M1	
		E1	Correctly obtained
	$\int \frac{1}{2}y^2\,\mathrm{d}x = \int_0^2 \frac{1}{2}(16 - 8x^2 + x^4)\,\mathrm{d}x$	M1	
	$= \left[8x - \frac{4}{3}x^3 + \frac{1}{10}x^5\right]_0^2 \quad (=\frac{128}{15})$	A1	
	OR $\int yx\,\mathrm{d}y = \int_0^4 y\sqrt{4-y}\,\mathrm{d}y$ M1		Valid method of integration
	$= \left[-\frac{2}{3}y(4-y)^{\frac{3}{2}} - \frac{4}{15}(4-y)^{\frac{5}{2}}\right]_0^4$ A1		or $\left[-\frac{8}{3}(4-y)^{\frac{3}{2}} + \frac{2}{5}(4-y)^{\frac{5}{2}}\right]_0^4$
	$\bar{y} = \frac{\frac{128}{15}}{\frac{16}{3}}$	M1	
	$= 1.6$	E1 **9**	Correctly obtained *SR* If $\frac{1}{2}$ is omitted, marks for $\bar{y}$ are M1A0M0E0
(ii)	$\bar{x} = \frac{12\times 0 + 6.5\times 0.75 + 6.5\times 2.75}{12 + 6.5 + 6.5}$	M1 M1	For $6.5\times 0.75 + 6.5\times 2.75$ Using $\left(\sum m\right)\bar{x} = \sum mx$
	$= \frac{22.75}{25} = 0.91$	A1	
	$\bar{y} = \frac{12\times 0 + 6.5\times 1.6 + 6.5\times 1.6}{25}$	M1	Using $\left(\sum m\right)\bar{y} = \sum my$
	$= \frac{20.8}{25} = 0.832$	A1 **5**	
(iii)		M1 M1	For CM vertically below A For trig in a triangle containing θ, or finding the gradient of AG
	$\tan\theta = \frac{2-0.91}{4-0.832} \quad (=\frac{1.09}{3.168})$	A1	Correct expression for $\tan\theta$ or $\tan(90-\theta)$
	$\theta = 19.0°$	A1 **4**	Accept 0.33 rad

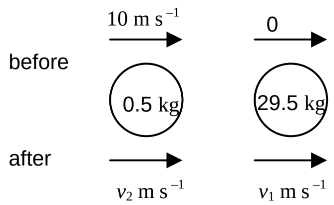
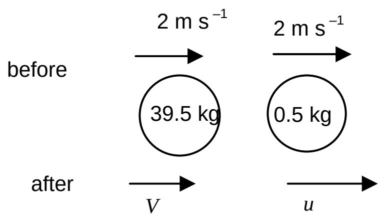
1(a)(i)	[Force] $= \mathrm{M\,L\,T^{-2}}$ [Power] = [Force] $\times$ [Distance] $\div$ [Time] = [Force] $\times \mathrm{L\,T^{-1}}$ $= \mathrm{M\,L^2\,T^{-3}}$	B1 M1 A1 **3**	or [Energy] $= \mathrm{M\,L^2\,T^{-2}}$ or [Energy] $\times \mathrm{T^{-1}}$
(ii)	$[\mathrm{RHS}] = \dfrac{(\mathrm{L})^3(\mathrm{L\,T^{-1}})^2(\mathrm{M\,L^{-3}})}{\mathrm{M\,L^2\,T^{-3}}}$ $= \mathrm{T}$ [LHS] = L so equation is not consistent	B1B1 M1 A1 E1 **5**	For $(\mathrm{L\,T^{-1}})^2$ and $(\mathrm{M\,L^{-3}})$ Simplifying dimensions of RHS With all working correct (cao) *SR* '$\ldots \mathrm{L} = \frac{28}{9}\pi\mathrm{T}$, so inconsistent ' can earn B1B1M1A1E0
(iii)	[RHS] needs to be multiplied by $\mathrm{L\,T^{-1}}$ which are the dimensions of *u* Correct formula is $x = \dfrac{28\pi r^3 u^3 \rho}{9P}$	M1 A1 A1 cao **3**	RHS must appear correctly
	OR $x = k r^{\alpha} u^{\beta} \rho^{\gamma} P^{\delta}$ $\beta = 3$ M1 A1 $x = \dfrac{28\pi r^3 u^3 \rho}{9P}$ A1		Equating powers of one dimension
(b)(i)	Elastic energy is $\frac{1}{2} \times 150 \times 0.8^2$ $= 48\ \mathrm{J}$	M1 A1 **2**	*Treat use of modulus* $\lambda = 150\ \mathrm{N}$ *as MR*
(ii)	In extreme position, length of string is $2\sqrt{1.2^2 + 0.9^2}$ $(= 3)$ elastic energy is $\frac{1}{2} \times 150 \times 1.4^2$ $(= 147)$ By conservation of energy, $147 - 48 = \frac{1}{2} \times m \times 10^2$ Mass is 1.98 kg	B1 M1 M1 A1 A1 **5**	for $\sqrt{1.2^2 + 0.9^2}$ or 1.5 or 3 allow M1 for $(2\times)\frac{1}{2} \times 150 \times 0.7^2$ Equation involving EE and KE

2 (a)(i)	Vertically, $T\cos 55° = 0.6 \times 9.8$ Tension is $10.25\ \text{N}$	M1 A1 **2**	
(ii)	Radius of circle is $r = 2.8\sin 55°\quad (= 2.294)$	B1	
	Towards centre, $T\sin 55° = 0.6 \times \dfrac{v^2}{2.8\sin 55°}$	M2	Give M1 for one error
	OR $T\sin 55° = 0.6 \times (2.8\sin 55°) \times \omega^2$ M1 $\omega = 2.47$ $v = (2.8\sin 55°)\,\omega$ M1		or $T = 0.6 \times 2.8 \times \omega^2$ *Dependent on previous M1*
	Speed is $5.67\ \text{m s}^{-1}$	A1 **4**	
(b)(i)	Tangential acceleration is $r\alpha = 1.4 \times 1.12$ $F_1 = 0.5 \times 1.4 \times 1.12$ $= 0.784\ \text{N}$ Radial acceleration is $r\omega^2 = 1.4\omega^2$ $F_2 = 0.5 \times 1.4\omega^2$ $= 0.7\omega^2\ \text{N}$	M1 A1 M1 A1 **4**	*SR* $F_1 = -0.784,\ F_2 = -0.7\omega^2$ *penalise once only*
(ii)	Friction $F = \sqrt{F_1^{\,2} + F_2^{\,2}}$ Normal reaction $R = 0.5 \times 9.8$ About to slip when $F = \mu \times 0.5 \times 9.8$ $\sqrt{0.784^2 + 0.49\omega^4} = 0.65 \times 0.5 \times 9.8$ $\omega = 2.1$	M1 M1 A1 A1 A1 cao **5**	For LHS and RHS *Both dependent on M1M1*
(iii)	$\tan\theta = \dfrac{F_1}{F_2}$ $= \dfrac{0.784}{0.7 \times 2.1^2}$ Angle is $14.25°$	M1 A1 A1 **3**	Allow M1 for $\tan\theta = \dfrac{F_2}{F_1}$ etc Accept 0.249 rad

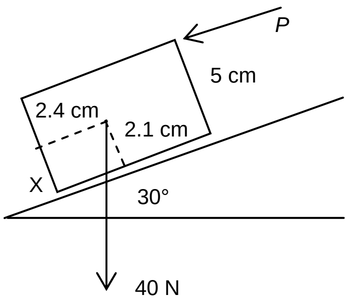
3 (i)	$T_{AP} = \frac{1323}{3} \times 2 \quad (= 882)$ $T_{BP} = \frac{1323}{4.5} \times 2.5 \quad (= 735)$ $T_{AP} - mg - T_{BP} = 882 - 15 \times 9.8 - 735 = 0$ so P is in equilibrium	B1 B1 E1 **3**	
	OR $\frac{1323}{3}(AP - 3) = \frac{1323}{4.5}(BP - 4.5) + 15 \times 9.8$ B2 $AP + BP = 12$ and solving, $AP = 5$ E1		Give B1 for one tension correct
(ii)	Extension of AP is $5 - x - 3 = 2 - x$ $T_{AP} = \frac{1323}{3}(2 - x) = 441(2 - x)$ Extension of BP is $7 + x - 4.5 = 2.5 + x$ $T_{BP} = \frac{1323}{4.5}(2.5 + x) = 294(2.5 + x)$	E1 B1 B1 **3**	
(iii)	$441(2 - x) - 15 \times 9.8 - 294(2.5 + x) = 15\frac{d^2x}{dt^2}$ $\frac{d^2x}{dt^2} = -49x$ Motion is SHM with period $\frac{2\pi}{\omega} = \frac{2\pi}{7} = 0.898\text{ s}$	M1 A1 M1 A1 **4**	Equation of motion involving 3 forces Obtaining $\frac{d^2x}{dt^2} = -\omega^2 x \ (+c)$ Accept $\frac{2}{7}\pi$
(iv)	Centre of motion is AP = 5 If minimum value of AP is 3.5, amplitude is 1.5 Maximum value of AP is 6.5 m	B1 **1**	
(v)	When $AP = 4.1, \quad x = 0.9$ Using $v^2 = \omega^2(A^2 - x^2)$ $v^2 = 49(1.5^2 - 0.9^2)$ Speed is 8.4 m s^{-1}	M1 A1 A1 **3**	Accept ± 8.4 or -8.4
	OR $x = 1.5\sin 7t$ When $x = 0.9,\ 7t = 0.6435 \quad (t = 0.0919)$ $v = 7 \times 1.5\cos 7t$ M1 $= 10.5\cos(0.6435)$ A1 $= 8.4$ A1		or $x = 1.5\cos 7t$ or $7t = 0.9273 \quad (t = 0.1325)$ or $v = -7 \times 1.5\sin 7t$ $= (-)\,10.5\sin(0.9273)$

(vi)	$x = 1.5\cos 7t$	M1 A1	For $\cos(\sqrt{49}\,t)$ or $\sin(\sqrt{49}\,t)$ or $x = 1.5\sin 7t$ *M1A1 above can be awarded in (v) if not earned in (vi)*
	When $1.5\cos 7t = 0.5$	M1	or other fully correct method to find the required time e.g. $0.400 - 0.224$ or $0.224 - 0.049$
	Time taken is 0.176 s	A1 **4**	Accept 0.17 or 0.18

4 (i)	$\int \pi y^2\,dx = \int_1^4 \pi x\,dx$ $= \left[\frac{1}{2}\pi x^2\right]_1^4 = 7.5\pi$ $\int \pi x y^2\,dx$ $= \int_1^4 \pi x^2\,dx \quad = \left[\frac{1}{3}\pi x^3\right]_1^4 \quad (= 21\pi)$ $\bar{x} = \frac{21\pi}{7.5\pi}$ $= 2.8$	M1 A1 M1 A1 M1 A1 **6**	*π may be omitted throughout*
(ii)	Cylinder has mass $3\pi\rho$ Cylinder has CM at $x = 2.5$ $(4.5\pi\rho)\bar{x} + (3\pi\rho)(2.5) = (7.5\pi\rho)(2.8)$ $\bar{x} = 3$	B1 B1 M1 A1 E1 **5**	Or volume 3π Relating three CMs *(ρ and / or π may be omitted)* or equivalent, e.g. $\bar{x} = \frac{(7.5\pi\rho)(2.8) - (3\pi\rho)(2.5)}{7.5\pi\rho - 3\pi\rho}$ Correctly obtained
(iii)(*A*)	Moments about A, $S \times 3 - 96 \times 2 = 0$ $S = 64\text{ N}$ Vertically, $R + S = 96$ $R = 32\text{ N}$	M1 A1 M1 A1 **4**	Moments equation or another moments equation *Dependent on previous M1*
(*B*)	Moments about A, $S \times 3 - 96 \times 2 - 6 \times 1.5 = 0$ Vertically, $R + S = 96 + 6$ $R = 35\text{ N}, \quad S = 67\text{ N}$	M1 A1 A1 **3**	Moments equation Both correct
	OR Add 3 N to each of *R* and *S* M1 $R = 35\text{ N}, \quad S = 67\text{ N}$ A2		*Provided* $R \neq S$ Both correct

Q 1		mark		sub
(i)				
	$10 \times 0.5 = 0.5v_2 + 29.5v_1$	M1 A1	PCLM and two terms on RHS All correct. Any form.	
	$\frac{v_1 - v_2}{0 - 10} = -0.8$	M1 A1	NEL Any form	
	$v_1 = 0.3$ so $V_1 = 0.3$ $v_2 = -7.7$ so $V_2 = 7.7$ m s^{-1} in opposite to original direction	A1 A1 F1	 Speed. Accept $\pm$. Must be correct interpretation of clear working	7
(ii) (A)	$10 \times 0.5 = 30V$ so $V = \frac{1}{6}$	M1 A1 A1	PCLM and coalescence All correct. Any form. Clearly shown. Accept decimal equivalence. Accept no direction.	3
(B)	Same velocity No force on sledge in direction of motion	E1 E1	Accept speed	2
(iii)		B1		
	$2 \times 40 = 0.5u + 39.5V$ $u - V = 10$ Hence $V = 1.875$	M1 A1 B1 A1	PCLM, masses correct Any form May be seen on the diagram. Accept no reference to direction.	5
		17		

Q 2		mark	comment	sub
(i)	$X = R\cos 30$ (1) $Y + R\sin 30 = L$ (2)	B1 M1 A1	 Attempt at resolution	3
(ii)	ac moments about A $R - 2L = 0$ Subst in (1) and (2) $X = 2L\dfrac{\sqrt{3}}{2}$ so $X = \sqrt{3}L$ $Y + 2L \times \dfrac{1}{2} = L$ so $Y + L = L$ and $Y = 0$	B1 M1 E1 E1	 Subst **their** $R = 2L$ into **their** (1) or (2) Clearly shown Clearly shown	4
(iii)	(Below all are taken as tensions e. g. T_{AB} in AB)	B1 B1	Attempt at all forces (allow one omitted) Correct. Accept internal forces set as tensions or thrusts or a mix	2
(iv)	↓ A $T_{AD}\cos 30\ (-Y) = 0$ so $T_{AD} = 0$	M1 E1	Vert equilibrium at A attempted. $Y = 0$ need not be explicit	2
(v)	Consider the equilibrium at pin-joints A → $T_{AB} - X = 0$ so $T_{AB} = \sqrt{3}L$ (T) C ↓ $L + T_{CE}\cos 30 = 0$ so $T_{CE} = \dfrac{-2L}{\sqrt{3}}$ so $\dfrac{2L}{\sqrt{3}}\left(= \dfrac{2L\sqrt{3}}{3}\right)$ (C) C ← $T_{BC} + T_{CE}\cos 60 = 0$ so $T_{BC} = -\left(-\dfrac{2\sqrt{3}L}{3}\right) \times \dfrac{1}{2} = \dfrac{\sqrt{3}L}{3}$ (T)	M1 B1 B1 B1 B1 B1 F1	At least one relevant equilib attempted (T) not required Or equiv from **their** diagram Accept any form following from **their** equation. (C) not required. Or equiv from **their** diagram FT **their** T_{CE} or equiv but do not condone inconsistent signs even if right answer obtained. (T) not required. T and C consistent with **their** answers and **their** diagram	7
(vi)	↓ B $T_{BD}\cos 30 + T_{BE}\cos 30 = 0$ so $T_{BD} = -T_{BE}$ so mag equal and opp sense	M1 E1	Resolve vert at B A statement required	2
		20		

Q 3		mark		sub
(i)	(10, 2, 2.5)	B1		1
(ii)	By symmetry $\bar{x} = 10$, $\bar{y} = 2$ $(240+80)\bar{z} = 80\times 0 + 240\times 2.5$ so $\bar{z} = 1.875$	B1 B1 B1 M1 A1	 Total mass correct Method for c.m. Clearly shown	5
(iii)	$\bar{x} = 10$ by symmetry $(320+80)\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = 320\begin{pmatrix}10\\ 2\\ 1.875\end{pmatrix} + 80\begin{pmatrix}10\\ 4\\ 3\end{pmatrix}$ $\bar{y} = 2.4$ $\bar{z} = 2.1$	E1 M1 B1 B1 E1 E1	Could be derived Method for c.m. *y* coord c.m. of lid *z* coord c.m. of lid shown shown	6
(iv)	 c.w moments about X $40\times 0.024\cos 30 - 40\times 0.021\sin 30$ = 0.41138… so 0.411 N m (3 s. f.)	B1 B1 B1 E1	Award for correct use of dimensions 2.1 and 2.4 or equivalent 1st term o.e. (allow use of 2.4 and 2.1) 2nd term o.e. (allow use of 2.4 and 2.1) Shown [Perpendicular method: M1 Complete method: A1 Correct lengths and angles E1 Shown]	4
(v)	0.41138… - 0.05*P* = 0 *P* = 8.22768…… so 8.23 (3 s. f.)	M1 A1	Allow use of 5 Allow if cm used consistently	2
		18		

Q 4		mark		sub
(i)	$F_{max} = \mu R$ $R = 2g \cos 30$ so $F_{max} = 0.75 \times 2 \times 9.8 \times \cos 30 = 12.730...$ so 12.7 N (3 s. f.) **either** Weight cpt down plane is $2g\sin 30 = 9.8$ N so no as 9.8 < 12.7 **or** Slides if $\mu < \tan 30$ But 0.75 > 0.577… so no	M1 B1 A1 B1 E1 B1 E1	Must have attempt at R with mg resolved [Award 2/3 retrospectively for limiting friction seen below] The inequality must be properly justified The inequality must be properly justified	5
(ii) (A)	Increase in GPE is $2 \times 9.8 \times (6 + 4\sin 30) = 156.8$ J	M1 B1 A1	Use of mgh 6 + 4 sin 30	3
(B)	WD against friction is $4 \times 0.75 \times 2 \times 9.8 \times \cos 30 = 50.9222...$ J	M1 A1	Use of WD = Fd	2
(C)	Power is $10 \times (156.8 + 50.9222\ldots)/60$ = 34.620… so 34.6 W (3 s. f.)	M1 A1	Use P = WD/t	2
(iii)	$0.5 \times 2 \times 9^2$ $= 2 \times 9.8 \times (6 + x\sin 30)$ $+ \ 0.5 \times 2 \times 4^2$ -90 so x = 3.8163…. so 3.82 (3 s. f.)	M1 B1 A1 A1 A1	Equating KE to GPE and WD term. Allow sign errors and one KE term omitted. Allow ‘old’ friction as well. Both KE terms. Allow wrong signs. All correct but allow sign errors All correct, including signs. cao	5
		17		

4762 - Mechanics 2

General Comments

Many excellent scripts were seen in response to this paper with the majority of candidates able to make some progress worthy of credit on every question. The majority of candidates seemed to understand the principles required. However, diagrams in many cases were poor and not as helpful to the candidate as they could have been and some candidates did not clearly identify the principle or process being used. As has happened in previous sessions, those parts of the questions that were least well done were those that required an explanation or interpretation of results or that required the candidate to *show* a given answer. In the latter case some candidates failed to include all of the relevant steps in the working.

Comments on Individual Questions

1 Many candidates gained significant credit on this question. Those that drew a diagram were usually more successful than those who did not. Many candidates did not indicate which direction was to be positive and this did lead to some errors in signs or inconsistencies between equations.

(i) The majority of candidates were able to gain some credit for this part of the question. Sign errors occurred in a few cases in the use of Newton's experimental law and many candidates forgot to indicate the direction in which the ball was travelling after the impact.

(ii) (A) Almost all the candidates could gain full credit for this part of the question.

(B) Very few candidates could obtain any credit for this part. Many did not appreciate the vector nature of the problem and merely stated (incorrectly) that the sledge would speed up because the mass had decreased and momentum had to be conserved. A small number of candidates appreciated that there would be no change in the velocity of the sledge but could not give a valid reason for this. Few mentioned that there was no force on the sledge in the direction of motion.

(iii) Many candidates were able to gain full credit for this part of the question. Of those who did not, a significant minority drew an inadequately labelled diagram or made errors with the masses. A small number of candidates did not understand the significance of the velocity of the snowball being relative to the sledge and assumed that the snowball had a speed of 10 m s^{-1}.

2 Some excellent answers to this question were seen, with many candidates gaining almost full credit. It was pleasing to see that there were fewer mistakes made with inconsistent equations than has been the case in previous sessions.

(i) This part was well done by almost all the candidates.

(ii) Most candidates did well on this part of the question. Some arithmetic errors were seen and, in a few cases, some 'creative' algebra to try to establish the given results.

(iii) In most cases the standard of the diagrams was satisfactory and worth some credit but a significant minority of candidates did not label the internal forces and/or omitted one or more of the external forces. A few candidates obviously changed the diagram in response to their answers as they worked through the following parts of the question and this then led to mistakes.

(iv) This part of the question caused few problems.

(v) Many candidates scored well on this part of the question. Those who did not were usually those who drew an inadequate diagram or who ignored their diagram; these made mistakes with signs or produced equations that were inconsistent with each other and with the diagram drawn in part (iii).

(vi) This part of the question was not as well done as previous parts. Arguments based purely on symmetry at B were common but few appreciated that the vertical equilibrium had to be considered. Those candidates who attempted to look at the vertical equilibrium often forgot to resolve the forces in BD and BE. A common answer was to simply write down $T_{BD} = \pm T_{BE}$ without any supporting argument or interpretation of the result.

3 Only the last two parts of this question caused any problems to the vast majority of candidates. The principles behind the calculation of centres of mass appeared to be well understood and candidates who adopted column vector notation made fewer mistakes than those who calculated the co-ordinates separately.

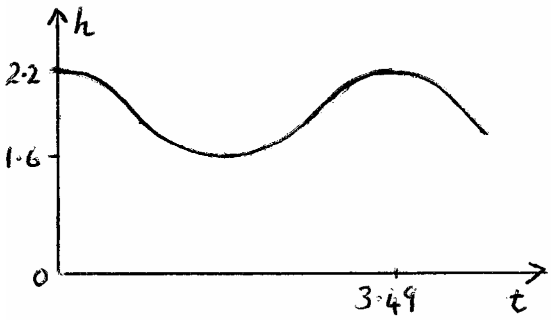
(i) Most obtained full credit for this part.

(ii) Few candidates had difficulty with this part.

(iii) Most candidates were able to obtain full credit for this part. A minority of them wrongly assigned a mass of 100 units to the lid and it was common to see the z co-ordinate of the centre of mass of the lid assumed to be 2.5 cm. However, many who made this error realised the mistake and clearly corrected it. Unfortunately, there were some candidates who made both of the above errors, completed the working and still stated 2.1 as the z component of the centre of mass.

(iv) Few candidates made much progress here. The most common mistake was to ignore one of the components of the weight. Trigonometric errors were also common.

(v) Very few correct responses to this part were seen. Many of those candidates who appreciated that the moment of P had to be equated to the clockwise moment of the weight from the previous part had inconsistent units for the distance of P from the pivot.

1(a)(i)	[Velocity] $= \mathrm{L\,T^{-1}}$ [Acceleration] $= \mathrm{L\,T^{-2}}$ [Force] $= \mathrm{M\,L\,T^{-2}}$ [Density] $= \mathrm{M\,L^{-3}}$ [Pressure] $= \mathrm{M\,L^{-1}\,T^{-2}}$	B1 B1 B1 B1 B1 **5**	*(Deduct 1 mark if answers given as* $\mathrm{m\,s^{-1}}$, $\mathrm{m\,s^{-2}}$, $\mathrm{kg\,m\,s^{-2}}$ *etc)*
(ii)	$[P] = \mathrm{M\,L^{-1}\,T^{-2}}$ $[\frac{1}{2}\rho v^2] = (\mathrm{M\,L^{-3}})(\mathrm{L\,T^{-1}})^2$ $= \mathrm{M\,L^{-1}\,T^{-2}}$ $[\rho g h] = (\mathrm{M\,L^{-3}})(\mathrm{L\,T^{-2}})(\mathrm{L}) = \mathrm{M\,L^{-1}\,T^{-2}}$ All 3 terms have the same dimensions	M1 A1 A1 E1 **4**	Finding dimensions of 2nd or 3rd term Allow e.g. ‘Equation is dimensionally consistent’ following correct work
(b)(i)		M1 A1 **2**	For a ‘cos’ curve (starting at the highest point) Approx correct values marked on both axes
(ii)	Period $\frac{2\pi}{\omega} = 3.49$ $\omega = 1.8$ $h = 1.9 + 0.3\cos 1.8t$	M1 A1 M1 F1 **4**	Accept $\frac{2\pi}{3.49}$ For $h = c + a\cos/\sin$ with either $c = \frac{1}{2}(1.6 + 2.2)$ or $a = \frac{1}{2}(2.2 - 1.6)$
(iii)	When $h = 1.7$, float is 0.2 m below centre Acceleration is $\omega^2 x = 1.8^2 \times 0.2$ $= 0.648\ \mathrm{m\,s^{-2}}$ upwards	M1A1 A1 cao **3**	Award M1 if there is at most one error
	OR When $h = 1.7$, $\cos 1.8t = -\frac{2}{3}$ ($1.8t = 2.30$, $t = 1.28$) Acceleration $\ddot{h} = -0.3 \times 1.8^2 \cos 1.8t$ M1 $= -0.3 \times 1.8^2 \times (-\frac{2}{3})$ A1 $= 0.648\ \mathrm{m\,s^{-2}}$ upwards A1 cao		

2 (i)	$R\cos 60 = 0.4\times 9.8$ Normal reaction is 7.84 N	M1 A1 **2**	Resolving vertically (e.g. $R\sin 60 = mg$ is M1A0 $R = mg\cos 60$ is M0)
(ii)	$R\sin 60 = 0.4\times \dfrac{v^2}{2.7\sin 60}$ Speed is $6.3\ \mathrm{m\,s^{-1}}$	M1 M1 A1 A1 cao **4**	Horizontal equation of motion Acceleration $\dfrac{v^2}{r}$ (M0 for $\dfrac{v^2}{2.7}$)
	OR $R\sin 60 = 0.4\times(2.7\sin 60)\omega^2$ M1 $\omega = 2.694$ A1 $v = (2.7\sin 60)\omega$ M1 Speed is $6.3\ \mathrm{m\,s^{-1}}$ A1 cao		Horizontal equation of motion or $R = 0.4\times 2.7\times\omega^2$ For $v = r\omega$ (M0 for $v = 2.7\omega$)
(iii)	By conservation of energy, $\frac{1}{2}\times 0.4\times(9^2 - v^2) = 0.4\times 9.8\times(2.7 + 2.7\cos\theta)$ $81 - v^2 = 52.92 + 52.92\cos\theta$ $v^2 = 28.08 - 52.92\cos\theta$	M1 A1 A1 **3**	Equation involving KE and PE Any (reasonable) correct form e.g. $v^2 = 81 - 52.92(1+\cos\theta)$
(iv)	$R + 0.4\times 9.8\cos\theta = 0.4\times\dfrac{v^2}{2.7}$ $R + 3.92\cos\theta = \dfrac{0.4}{2.7}(28.08 - 52.92\cos\theta)$ $R + 3.92\cos\theta = 4.16 - 7.84\cos\theta$ $R = 4.16 - 11.76\cos\theta$	M1 A1 M1 A1 E1 **5**	Radial equation with 3 terms Substituting expression for v^2 *SR* If θ is taken to the downward vertical, maximum marks are: M1A0A0 in (iii) M1A1M1A1E0 in (iv)
(v)	Leaves surface when $R = 0$ $\cos\theta = \dfrac{4.16}{11.76}$ $v^2 = 28.08 - 52.92\times\dfrac{4.16}{11.76}$ $(= 9.36)$ Speed is $3.06\ \mathrm{m\,s^{-1}}$	M1 A1 M1 A1 cao **4**	*Dependent on previous M1* or using $mg\cos\theta = \dfrac{mv^2}{r}$

3 (i)	Tension is $637 \times 0.1 = 63.7$ N Energy is $\frac{1}{2} \times 637 \times 0.1^2$ $= 3.185$ J	B1 M1 A1 **3**	
(ii)	Let θ be angle between RA and vertical $\cos\theta = \frac{5}{13}$ ($\theta = 67.4°$) $T\cos\theta = mg$ $63.7 \times \frac{5}{13} = m \times 9.8$ Mass of ring is 2.5 kg	 B1 M1 A1 E1 **4**	 Resolving vertically
(iii)	 Loss of PE is $2.5 \times 9.8 \times (0.9 - 0.5)$ EE at lowest point is $\frac{1}{2} \times 637 \times 0.3^2$ $(= 28.665)$ By conservation of energy, $2.5 \times 9.8 \times 0.4 + \frac{1}{2} \times 2.5u^2 = \frac{1}{2} \times 637 \times 0.3^2 - 3.185$ $9.8 + 1.25u^2 = 25.48$ $u^2 = 12.544$ $u = 3.54$	M1 A1 M1 A1 M1 F1 A1 cao **7**	Considering PE or PE at start and finish Award M1 if not more than one error Equation involving KE, PE and EE
(iv)	From lowest point to level of A, Loss of EE is 28.665 Gain in PE is $2.5 \times 9.8 \times 0.9 = 22.05$ Since $28.665 > 22.05$, Ring will rise above level of A	 M1 M1 M1 A1 cao **4**	 EE at 'start' and at level of A PE at 'start' and at level of A (For M2 it must be the same 'start') Comparing EE and PE *(or equivalent,* *e.g.* $\frac{1}{2}mu^2 + 3.185 = mg \times 0.5 + \frac{1}{2}mv^2$ *)* Fully correct derivation
			SR If 637 is used as modulus, maximum marks are: (i) B0M1A0 (ii) B1M1A1E0 (iii) M1A1M1A1M1F1A0 (iv) M1M1M1A0

4 (a)	Area is $\int_0^2 x^3\,dx = \left[\tfrac{1}{4}x^4\right]_0^2 = 4$	B1	
	$\int x\,y\,dx = \int_0^2 x^4\,dx$	M1	
	$= \left[\tfrac{1}{5}x^5\right]_0^2 = 6.4$	A1	
	$\bar{x} = \dfrac{6.4}{4} = 1.6$	A1	
	$\int \tfrac{1}{2}y^2\,dx = \int_0^2 \tfrac{1}{2}x^6\,dx$	M1	Condone omission of $\tfrac{1}{2}$
	$= \left[\tfrac{1}{14}x^7\right]_0^2 = \dfrac{64}{7}$	A1	
	$\bar{y} = \dfrac{\int \tfrac{1}{2}y^2\,dx}{\int y\,dx}$	M1	
	$= \dfrac{\frac{64}{7}}{4} = \dfrac{16}{7}$	A1 **8**	Accept 2.3 from correct working
(b)(i)	Volume is $\int \pi y^2\,dx = \int_1^2 \pi(4 - x^2)\,dx$	M1	*π may be omitted throughout*
	$= \pi\left[4x - \tfrac{1}{3}x^3\right]_1^2 = \tfrac{5}{3}\pi$	A1	For $\tfrac{5}{3}$
	$\int \pi x y^2\,dx = \int_1^2 \pi x(4 - x^2)\,dx$	M1	
	$= \pi\left[2x^2 - \tfrac{1}{4}x^4\right]_1^2 = \tfrac{9}{4}\pi$	A1	For $\tfrac{9}{4}$
	$\bar{x} = \dfrac{\int \pi x y^2\,dx}{\int \pi y^2\,dx}$	M1	
	$= \dfrac{\frac{9}{4}\pi}{\frac{5}{3}\pi} = \dfrac{27}{20} = 1.35$	E1 **6**	Must be fully correct
(ii)	Height of solid is $h = 2\sqrt{3}$	B1	
	$T\,h = mg \times 0.35$	M1	Taking moments
	$F = T = 0.101mg,\quad R = mg$	F1	
	Least coefficient of friction is $\dfrac{F}{R} = 0.101$	A1 **4**	Must be fully correct (e.g. A0 if $m = \tfrac{5}{3}\pi$ is used)

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1(a)(i)	[Force] $= \mathrm{M\,L\,T^{-2}}$ [Density] $= \mathrm{M\,L^{-3}}$	B1 B1 **2**	
(ii)	$[E] = \dfrac{[F][l_0]}{[A][l-l_0]} = \dfrac{(\mathrm{M\,L\,T^{-2}})(\mathrm{L})}{(\mathrm{L^2})(\mathrm{L})}$ $= \mathrm{M\,L^{-1}\,T^{-2}}$	B1 M1 A1 **3**	for $[A] = \mathrm{L^2}$ Obtaining the dimensions of *E*
(iii)	$\mathrm{T} = \mathrm{L}^{\alpha}(\mathrm{M\,L^{-3}})^{\beta}(\mathrm{M\,L^{-1}\,T^{-2}})^{\gamma}$ $-2\gamma = 1, \quad \beta + \gamma = 0$ $\gamma = -\frac{1}{2}$ $\beta = \frac{1}{2}$ $\alpha - 3\beta - \gamma = 0$ $\alpha = 1$	B1 cao F1 M1 A1 A1 **5**	Obtaining equation involving α, β, γ
(b)	$AP = 1.7$ m $F = T\cos\theta$ $R + T\sin\theta = 5\times 9.8$ $T\cos\theta = 0.4(49 - T\sin\theta)$ $\frac{8}{17}T = 0.4(49 - \frac{15}{17}T)$ $T = 23.8$ $T = k(1.7 - 1.5)$ Stiffness is $119\ \mathrm{N\,m^{-1}}$	B1 M1 M1 M1 A1 A1 M1 A1 **8**	Resolving in any direction Resolving in another direction *(M1 for resolving requires no force omitted, with attempt to resolve all appropriate forces)* Using $F = 0.4R$ to obtain an equation involving just one force (or *k*) Correct equation *Allow* $T\cos 61.9$ *etc* or $R = 28$ or $F = 11.2$ *May be implied* *Allow M1 for* $T = \dfrac{\lambda}{1.5}\times 0.2$ If $R = 49$ is assumed, max marks are B1M1M0M0A0A0M1A0

2(a)(i)	$0.1 + 0.01 \times 9.8 = 0.01 \times \frac{u^2}{0.55}$ Speed is $3.3\ \mathrm{m\,s^{-1}}$	M1 A1 A1 **3**	Using acceleration $u^2 / 0.55$
(ii)	$\frac{1}{2}m(v^2 - u^2) = mg(2 \times 0.55 - 0.15)$ $\frac{1}{2}(v^2 - 3.3^2) = 9.8 \times 0.95$ $v^2 = 29.51$ $R - mg\cos\theta = m\frac{v^2}{a}$ $R - 0.01 \times 9.8 \times \frac{0.4}{0.55} = 0.01 \times \frac{29.51}{0.55}$ Normal reaction is 0.608 N	M1 A1 M1 A1 A1 **5**	Using conservation of energy *(ft is* $v^2 = u^2 + 18.62$*)* Forces and acceleration towards centre *(ft is* $\frac{u^2 + 22.54}{55}$ *)*
(b)(i)	$T = 0.8 r \omega^2$ $T = \frac{160}{2}(r - 2)$ $\omega^2 = \frac{80(r-2)}{0.8r} = \frac{100(r-2)}{r}$ $\omega^2 = 100 - \frac{200}{r} < 100$, so $\omega < 10$	B1 B1 E1 E1 **4**	
(ii)	$\mathrm{EE} = \frac{1}{2} \times \frac{160}{2} \times (r-2)^2 = 40(r-2)^2$ $\mathrm{KE} = \frac{1}{2}m(r\omega)^2$ $= \frac{1}{2} \times 0.8 \times r^2 \times \frac{100(r-2)}{r}$ $= 40r(r-2)$ Since $r > r - 2$, $40r(r-2) > 40(r-2)^2$ i.e. KE > EE	B1 M1 A1 E1 **4**	Use of $\frac{1}{2}mv^2$ with $v = r\omega$ From fully correct working only
(iii)	When $\omega = 6$, $36 = \frac{100(r-2)}{r}$ $r = 3.125$ $T = 80(r-2) = 80(3.125 - 2)$ Tension is 90 N	M1 M1 A1 cao **3**	Obtaining *r*

3 (i)	$\dfrac{dx}{dt} = A\omega\cos\omega t - B\omega\sin\omega t$ $\dfrac{d^2x}{dt^2} = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$ $= -\omega^2(A\sin\omega t + B\cos\omega t) = -\omega^2 x$	B1 B1 ft E1 **3**	*Must follow from their* $\dot{x}$ Fully correct completion *SR* For $\dot{x} = -A\omega\cos\omega t + B\omega\sin\omega t$ $\ddot{x} = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$ award B0B1E0
(ii)	$B = 2$ $A\omega = -1.44$ $-B\omega^2 = -0.18$ *or* $-0.18 = -\omega^2(2)$ $\omega = 0.3,\quad A = -4.8$	B1 M1 A1 cao M1 A1 cao A1 cao **6**	Using $\dfrac{dx}{dt} = -1.44$ when $t = 0$ $\dfrac{d^2x}{dt^2} = -0.18$ when $t = 0$ (or $x = 2$)
(iii)	Period is $\dfrac{2\pi}{\omega} = \dfrac{2\pi}{0.3} = 20.94 = 20.9$ s (3 sf) Amplitude is $\sqrt{A^2 + B^2} = \sqrt{4.8^2 + 2^2}$ $= 5.2$ m	E1 M1 A1 **3**	or $1.44^2 = 0.3^2(a^2 - 2^2)$
(iv)	$x = -4.8\sin 0.3t + 2\cos 0.3t$ $v = -1.44\cos 0.3t - 0.6\sin 0.3t$ When $t = 12,\ x = 0.3306$ ($v = 1.56$) When $t = 24,\ x = -2.5929$ ($v = -1.35$) Distance travelled is $(5.2 - 0.3306) + 5.2 + 2.5929$ $= 12.7$ m	M1 A1 M1 M1 A1 **5**	Finding *x* when $t = 12$ and $t = 24$ Both displacements correct Considering change of direction Correct method for distance ft from their A, B, ω and amplitude: *Third M1 requires the method to be comparable to the correct one* *A1A1 both require* $\omega \approx 0.3,\ A \neq 0,\ B \neq 0$ *Note* ft from $A = +4.8$ is $x_{12} = -3.92$ ($v < 0$) $x_{24} = 5.03$ ($v > 0$) Distance is $(5.2 - 3.92) + 5.2 + 5.03$ $= 11.5$

4 (i)	$V = \int_1^8 \pi (x^{-\frac{1}{3}})^2 \, dx$ $= \pi \left[3x^{\frac{1}{3}} \right]_1^8 = 3\pi$ $V\bar{x} = \int_1^8 \pi x (x^{-\frac{1}{3}})^2 \, dx$ $= \pi \left[\frac{3}{4} x^{\frac{4}{3}} \right]_1^8 = \frac{45}{4}\pi$ $\bar{x} = \frac{\frac{45}{4}\pi}{3\pi}$ $= \frac{15}{4} = 3.75$	M1 A1 M1 A1 M1 A1 **6**	π *may be omitted throughout* *Dependent on previous M1M1*
(ii)	$A = \int_1^8 x^{-\frac{1}{3}} \, dx$ $= \left[\frac{3}{2} x^{\frac{2}{3}} \right]_1^8 = \frac{9}{2} = 4.5$ $A\bar{x} = \int_1^8 x (x^{-\frac{1}{3}}) \, dx$ $= \left[\frac{3}{5} x^{\frac{5}{3}} \right]_1^8 = \frac{93}{5} = 18.6$ $\bar{x} = \frac{18.6}{4.5} = \frac{62}{15} \quad (\approx 4.13)$ $A\bar{y} = \int_1^8 \frac{1}{2} (x^{-\frac{1}{3}})^2 \, dx$ $= \left[\frac{3}{2} x^{\frac{1}{3}} \right]_1^8 = \frac{3}{2} = 1.5$ $\bar{y} = \frac{1.5}{4.5} = \frac{1}{3}$	M1 A1 M1 A1 A1 M1 A1 A1 **8**	If $\frac{1}{2}$ omitted, award M1A0A0

(iii)	$(1)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}+(3.5)\begin{pmatrix}4.5\\ 0.25\end{pmatrix}=(4.5)\begin{pmatrix}62/15\\ 1/3\end{pmatrix}=\begin{pmatrix}18.6\\ 1.5\end{pmatrix}$ $\bar{x}=2.85$ $\bar{y}=0.625$	M1 M1 A1 A1	**4**	Attempt formula for CM of composite body (one coordinate sufficient) Formulae for both coordinates; signs must now be correct, but areas (1 and 3.5) may be wrong. ft only if $1<\bar{x}<8$ ft only if $0.5<\bar{y}<1$ *Other methods:* M1A1 for $\bar{x}$ M1A1 for $\bar{y}$ *(In each case, M1 requires a complete and correct method leading to a numerical value)*

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1(a)(i)	[Velocity] $= \mathrm{L\,T^{-1}}$ [Acceleration] $= \mathrm{L\,T^{-2}}$ [Force] $= \mathrm{M\,L\,T^{-2}}$	B1 B1 B1 **3**	*(Deduct 1 mark if kg, m, s are consistently used instead of M, L, T)*
(ii)	$[\lambda] = \dfrac{[\text{Force}]}{[v^2]} = \dfrac{\mathrm{M\,L\,T^{-2}}}{(\mathrm{L\,T^{-1}})^2}$ $= \mathrm{M\,L^{-1}}$	M1 A1 cao **2**	
(iii)	$\left[\dfrac{U^2}{2g}\right] = \dfrac{(\mathrm{L\,T^{-1}})^2}{\mathrm{L\,T^{-2}}} = \mathrm{L}$ $\left[\dfrac{\lambda U^4}{4mg^2}\right] = \dfrac{(\mathrm{M\,L^{-1}})(\mathrm{L\,T^{-1}})^4}{\mathrm{M\,(L\,T^{-2})^2}}$ $= \dfrac{\mathrm{M\,L^3\,T^{-4}}}{\mathrm{M\,L^2\,T^{-4}}} = \mathrm{L}$ [H] = L ; all 3 terms have the same dimensions	B1 cao M1 A1 cao E1 **4**	*(Condone constants left in)* *Dependent on B1M1A1*
(iv)	$(\mathrm{M\,L^{-1}})^2\,(\mathrm{L\,T^{-1}})^{\alpha}\,\mathrm{M}^{\beta}\,(\mathrm{L\,T^{-2}})^{\gamma} = \mathrm{L}$ $\beta = -2$ $-2 + \alpha + \gamma = 1$ $-\alpha - 2\gamma = 0$ $\alpha = 6$ $\gamma = -3$	B1 cao M1 A1 A1 cao A1 cao **5**	At least one equation in α, γ One equation correct

(b)	EE is $\frac{1}{2}\times\frac{2060}{24}\times 6^2 \quad (=1545)$ (PE gained) = (EE lost) + (KE lost) $50\times 9.8\times h = 1545 + \frac{1}{2}\times 50\times 12^2$ $490h = 1545 + 3600$ $h = 10.5$ OA $= 30 - h = 19.5$ m	B1 M1 F1 A1 **4**	 Equation involving PE, EE and KE Can be awarded from start to point where string becomes slack *or* any complete method (e.g. SHM) for finding v^2 at natural length If B0, give A1 for $v^2 = 88.2$ correctly obtained *or* $0 = 88.2 - 2\times 9.8\times s \quad (s = 4.5)$ *Notes* $\frac{1}{2}\times\frac{2060}{24}\times 6$ *used as EE can earn B0M1F1A0* $\frac{2060}{24}\times 6$ *used as EE gets B0M0*

2 (i)	$T\cos\alpha = mg$ $3.92\cos\alpha = 0.3\times 9.8$ $\cos\alpha = 0.75$ Angle is $41.4°$ (0.723 rad)	M1 A1 **2**	Resolving vertically *(Condone sin / cos mix for M marks throughout this question)*
(ii)	$T\sin\alpha = m\dfrac{v^2}{r}$ $3.92\sin\alpha = 0.3\times\dfrac{v^2}{4.2\sin\alpha}$ Speed is $4.9\ \text{m s}^{-1}$	M1 B1 A1 A1 **4**	Force and acceleration towards centre (condone $v^2/4.2$ or $4.2\omega^2$) For radius is $4.2\sin\alpha$ $(=2.778)$ Not awarded for equation in ω unless $v=(4.2\sin\alpha)\omega$ also appears
(iii)	$T - mg\cos\theta = m\dfrac{v^2}{a}$ $T - 0.3\times 9.8\times\cos 60° = 0.3\times\dfrac{8.4^2}{4.2}$ Tension is 6.51 N	M1 A1 A1 **3**	Forces and acceleration towards O
(iv)	$\frac{1}{2}mv^2 - mg\times 4.2\cos\theta = \frac{1}{2}m\times 8.4^2 - mg\times 4.2\cos 60°$ $v^2 - 82.32\cos\theta = 70.56 - 41.16$ $v^2 = 29.4 + 82.32\cos\theta$	M1 M1 A1 E1 **4**	For $(-)mg\times 4.2\cos\theta$ in PE Equation involving $\frac{1}{2}mv^2$ and PE
(v)	$(T) - mg\cos\theta = m\dfrac{v^2}{a}$ $(T) - m\times 9.8\cos\theta = m\times\dfrac{29.4+82.32\cos\theta}{4.2}$ String becomes slack when $T=0$ $-9.8\cos\theta = 7 + 19.6\cos\theta$ $\cos\theta = -\dfrac{7}{29.4}$ $\theta = 104°$ (1.81 rad)	M1 M1 A1 M1 A1 **5**	Force and acceleration towards O Substituting for v^2 *Dependent on first M1* *No marks for* $v=0 \Rightarrow \theta=111°$

3 (i)	$T_{PB} = 35(x-3.2)$ $[= 35x - 112]$ $T_{BQ} = 5(6.5 - x - 1.8)$ $= 5(4.7 - x)$ $[= 23.5 - 5x]$	B1 M1 A1 **3**	 Finding extension of BQ
(ii)	$T_{BQ} + mg - T_{PB} = m\frac{d^2x}{dt^2}$ $5(4.7-x) + 2.5\times 9.8 - 35(x-3.2) = 2.5\frac{d^2x}{dt^2}$ $160 - 40x = 2.5\frac{d^2x}{dt^2}$ $\frac{d^2x}{dt^2} = 64 - 16x$	M1 A2 E1 **4**	Equation of motion (condone one missing force) Give A1 for three terms correct
(iii)	At the centre, $\frac{d^2x}{dt^2} = 0$ $x = 4$	M1 A1 **2**	
(iv)	$\omega^2 = 16$ Period is $\frac{2\pi}{\sqrt{16}} = \frac{1}{2}\pi = 1.57$ s	M1 A1 **2**	Seen or implied *(Allow M1 for $\omega = 16$)* Accept $\frac{1}{2}\pi$
(v)	Amplitude $A = 4.4 - 4 = 0.4$ m Maximum speed is $A\omega$ $= 0.4 \times 4 = 1.6 \text{ m s}^{-1}$	B1 ft M1 A1 cao **3**	ft is $\lvert 4.4 - \text{(iii)} \rvert$
(vi)	$x = 4 + 0.4\cos 4t$ $v = (-)\,1.6\sin 4t$ When $v = 0.9$, $\sin 4t = -\frac{0.9}{1.6}$ $4t = \pi + 0.5974$ Time is 0.935 s	M1 A1 M1 A1 cao **4**	For $v = C\sin\omega t$ or $C\cos\omega t$ *This M1A1 can be earned in (v)* Fully correct method for finding the required time e.g. $\frac{1}{4}\text{arcsin}\frac{0.9}{1.6} + \frac{1}{2}\text{period}$
	OR $0.9^2 = 16(0.4^2 - y^2)$ $y = -0.3307$ M1 $y = 0.4\cos 4t$ A1 $\cos 4t = -\frac{0.3307}{0.4}$ $4t = \pi + 0.5974$ M1 Time is 0.935 s A1 cao		Using $v^2 = \omega^2(A^2 - y^2)$ *and* $y = A\cos\omega t$ or $A\sin\omega t$ For $y = (\pm)\,0.331$ *and* $y = 0.4\cos 4t$

4 (a)(i)	$V=\int \pi x^2\,\mathrm{d}y \;=\int_0^8 \pi(4-\tfrac{1}{2}y)\,\mathrm{d}y$ $=\pi\left[4y-\tfrac{1}{4}y^2\right]_0^8=16\pi$ $V\bar{y}=\int \pi y x^2\,\mathrm{d}y$ $=\int_0^8 \pi y(4-\tfrac{1}{2}y)\,\mathrm{d}y$ $=\pi\left[2y^2-\tfrac{1}{6}y^3\right]_0^8=\dfrac{128}{3}\pi$ $\bar{y}=\dfrac{\frac{128}{3}\pi}{16\pi}$ $=\dfrac{8}{3}\quad(\approx 2.67)$	M1 A1 M1 A1 A1 M1 A1 **7**	*π may be omitted throughout* Limits not required for M marks throughout this question *Dependent on M1M1*
(ii)	CM is vertically above lower corner $\tan\theta=\dfrac{2}{\bar{y}}=\dfrac{2}{8/3}\quad(=\dfrac{3}{4})$ $\theta=36.9°\quad(=0.6435\text{ rad})$	M1 M1 A1 A1 **4**	Trig in a triangle including θ *Dependent on previous M1* Correct expression for $\tan\theta$ or $\tan(90-\theta)$ *Notes* $\tan\theta=\dfrac{2}{\text{cand's }\bar{y}}$ *implies M1M1A1* $\tan\theta=\dfrac{\text{cand's }\bar{y}}{2}$ *implies M1M1* $\tan\theta=\dfrac{1}{\text{cand's }\bar{y}}$ *without further evidence is M0M0*

(b)	$A=\int_{-2}^{2}(8-2x^2)\,dx$	M1	*May use* $0\le x\le 2$ *throughout* or $(2)\int_0^8\sqrt{4-\frac{1}{2}y}\,dy$
	$=\left[\,8x-\frac{2}{3}x^3\,\right]_{-2}^{2}=\dfrac{64}{3}$	A1	
	$A\bar{y}=\int_{-2}^{2}\frac{1}{2}(8-2x^2)^2\,dx$	M1	or $(2)\int_0^8 y\sqrt{4-\frac{1}{2}y}\,dy$ *(M0 if ½ is omitted)*
	$=\left[\,32x-\frac{16}{3}x^3+\frac{2}{5}x^5\,\right]_{-2}^{2}$	M1	For $32x-\frac{16}{3}x^3+\frac{2}{5}x^5$ *Allow one error* or $-\frac{8}{3}y(4-\frac{1}{2}y)^{\frac{3}{2}}-\frac{32}{15}(4-\frac{1}{2}y)^{\frac{5}{2}}$ or $-\frac{64}{3}(4-\frac{1}{2}y)^{\frac{3}{2}}+\frac{16}{5}(4-\frac{1}{2}y)^{\frac{5}{2}}$
	$=\dfrac{1024}{15}$	A1	
	$\bar{y}=\dfrac{1024/15}{64/3}$		
	$=\dfrac{16}{5}=3.2$	M1	*Dependent on first two M1's*
		A1 **7**	

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1 (i)	[Force] $= \mathrm{M\,L\,T^{-2}}$ [Density] $= \mathrm{M\,L^{-3}}$	B1 B1 **2**	
(ii)	$[\eta] = \dfrac{[F][d]}{[A][v_2 - v_1]} = \dfrac{(\mathrm{M\,L\,T^{-2}})(\mathrm{L})}{(\mathrm{L^2})(\mathrm{L\,T^{-1}})}$ $= \mathrm{M\,L^{-1}\,T^{-1}}$	B1 M1 A1 **3**	for $[A] = \mathrm{L^2}$ and $[v] = \mathrm{L\,T^{-1}}$ Obtaining the dimensions of η
(iii)	$\left[\dfrac{2a^2\rho g}{9\eta}\right] = \dfrac{\mathrm{L^2(M\,L^{-3})(L\,T^{-2})}}{\mathrm{M\,L^{-1}\,T^{-1}}} = \mathrm{L\,T^{-1}}$ which is same as the dimensions of v	B1 M1 E1 **3**	For $[\,g\,] = \mathrm{L\,T^{-2}}$ Simplifying dimensions of RHS Correctly shown
(iv)	$(\mathrm{M\,L^{-3}})\mathrm{L}^{\alpha}(\mathrm{L\,T^{-1}})^{\beta}(\mathrm{M\,L^{-1}\,T^{-1}})^{\gamma}$ is dimensionless $\gamma = -1$ $-\beta - \gamma = 0$ $-3 + \alpha + \beta - \gamma = 0$ $\alpha = 1, \quad \beta = 1$	B1 cao M1 M1A1 A1 cao **5**	
(v)	$R = \dfrac{\rho w v}{\eta} = \dfrac{0.4 \times 25 \times 150}{1.6 \times 10^{-5}} \quad (= 9.375 \times 10^7)$ $= \dfrac{1.3 \times 5v}{1.8 \times 10^{-5}}$ Required velocity is $260\ \mathrm{m\,s^{-1}}$	M1 A1 A1 cao **3**	Evaluating R Equation for v

2 (a)(i)	$T\cos\alpha = T\cos\beta + 0.27\times 9.8$ $\sin\alpha = \frac{1.2}{2.0} = \frac{3}{5},\ \cos\alpha = \frac{4}{5}\quad (\alpha = 36.87°)$ $\sin\beta = \frac{1.2}{1.3} = \frac{12}{13},\ \cos\beta = \frac{5}{13}\quad (\beta = 67.38°)$ $\frac{27}{65}T = 2.646$ Tension is 6.37 N	M1 A1 B1 M1 E1 **5**	Resolving vertically (weight and at least one resolved tension) Allow T_1 and T_2 For $\cos\alpha$ and $\cos\beta$ [or α and β] Obtaining numerical equation for T e.g. $T(\cos 36.9 - \cos 67.4) = 0.27\times 9.8$ *(Condone 6.36 to 6.38)*
(ii)	$T\sin\alpha + T\sin\beta = 0.27\times\frac{v^2}{1.2}$ $6.37\times\frac{3}{5} + 6.37\times\frac{12}{13} = 0.27\times\frac{v^2}{1.2}$ $v^2 = 43.12$ Speed is $6.57\ \mathrm{m\,s^{-1}}$	M1 A1 M1 A1 **4**	Using $v^2/1.2$ Allow T_1 and T_2 Obtaining numerical equation for v^2
(b)(i)	$0.2\times 9.8 = 0.2\times\frac{u^2}{1.25}$ $u^2 = 9.8\times 1.25 = 12.25$ Speed is $3.5\ \mathrm{m\,s^{-1}}$	M1 E1 **2**	Using acceleration $u^2/1.25$
(ii)	$\frac{1}{2}m(v^2 - 3.5^2) = mg(1.25 - 1.25\cos 60)$ $v^2 = 24.5$ Radial component is $\frac{24.5}{1.25}$ $= 19.6\ \mathrm{m\,s^{-2}}$ Tangential component is $g\sin 60$ $= 8.49\ \mathrm{m\,s^{-2}}$	M1 A1 M1 A1 M1 A1 **6**	Using conservation of energy With numerical value obtained by using energy (M0 if mass, or another term, included) For sight of $(m)g\sin 60°$ with no other terms
(iii)	$T + 0.2\times 9.8\cos 60 = 0.2\times 19.6$ Tension is 2.94 N	M1 A1 cao **2**	Radial equation (3 terms) *This M1 can be awarded in (ii)*

3 (i)	$\frac{980}{25}y = 5\times 9.8$ Extension is 1.25 m	M1 A1 **2**	Using $\frac{\lambda y}{l_0}$ (Allow M1 for $980y = mg$)
(ii)	$T = \frac{980}{25}(1.25+x)$ $5\times 9.8 - 39.2(1.25+x) = 5\frac{d^2x}{dt^2}$ $-39.2x = 5\frac{d^2x}{dt^2}$ $\frac{d^2x}{dt^2} = -7.84x$	B1 (ft) M1 F1 E1 **4**	*(ft) indicates ft from previous parts as for A marks* Equation of motion with three terms Must have $\ddot{x}$ In terms of x only
(iii)	$8.4^2 = 7.84(A^2 - 1.25^2)$ Amplitude is 3.25 m	M2 A1 A1 **4**	Using $v^2 = \omega^2(A^2 - x^2)$
	OR M2 $\frac{980}{2\times 25}y^2 = 5\times 9.8y + \frac{1}{2}\times 5\times 8.4^2$ A1 $y = 4.5$ Amplitude is $4.5 - 1.25 = 3.25$ m A1		Equation involving EE, PE and KE
	OR $x = A\sin 2.8t + B\cos 2.8t$ $x = -1.25,\ v = 8.4$ when $t = 0$ M2 $\Rightarrow A = 3,\ B = -1.25$ A1 Amplitude is $\sqrt{A^2 + B^2} = 3.25$ A1		Obtaining A and B Both correct
(iv)	Maximum speed is $A\omega = 3.25\times 2.8$ $= 9.1\ \text{m s}^{-1}$	M1 A1 **2**	or equation involving EE, PE and KE ft only if answer is greater than 8.4
(v)	$x = 3.25\cos 2.8t$ $-1.25 = 3.25\cos 2.8t$ Time is 0.702 s	B1 (ft) M1 M1 A1 cao **4**	or $x = 3.25\sin 2.8t$ or $v = 9.1\cos 2.8t$ or $v = 9.1\sin 2.8t$ or $x = 3.25\sin(2.8t+\varepsilon)$ etc or $x = \pm 3\sin 2.8t \pm 1.25\cos 2.8t$ Obtaining equation for t or ε by setting $x = (\pm)1.25$ or $v = (\pm)8.4$ or solving $\pm 3\sin 2.8t \pm 1.25\cos 2.8t = 3.25$ Strategy for finding the required time e.g. $\frac{1}{2.8}\sin^{-1}\frac{1.25}{3.25} + \frac{1}{4}\times\frac{2\pi}{2.8}$ $2.8t - 0.3948 = \frac{1}{2}\pi$ or $2.8t - 1.966 = 0$

(vi)	e.g. Rope is light Rock is a particle No air resistance / friction / external forces Rope obeys Hooke's law / Perfectly elastic / Within elastic limit / No energy loss in rope	B1B1B1 **3**	Three modelling assumptions

4 (a)	$\int \frac{1}{2}y^2\,dx = \int_{-a}^{a} \frac{1}{2}(a^2-x^2)\,dx$	M1	For integral of (a^2-x^2)
	$= \left[\frac{1}{2}(a^2x - \frac{1}{3}x^3) \right]_{-a}^{a}$		
	$= \frac{2}{3}a^3$	A1	
	$\bar{y} = \dfrac{\frac{2}{3}a^3}{\frac{1}{2}\pi a^2}$	M1	*Dependent on previous M1*
	$= \dfrac{4a}{3\pi}$	E1 **4**	
(b)(i)			*π may be omitted throughout*
	$V = \int \pi y^2\,dx = \int_0^h \pi(mx)^2\,dx$	M1	For integral of x^2 or use of $V = \frac{1}{3}\pi r^2 h$ and $r = mh$
	$= \left[\frac{1}{3}\pi m^2 x^3 \right]_0^h = \frac{1}{3}\pi m^2 h^3$	A1	
	$\int \pi x y^2\,dx = \int_0^h \pi x(mx)^2\,dx$	M1	For integral of x^3
	$= \left[\frac{1}{4}\pi m^2 x^4 \right]_0^h = \frac{1}{4}\pi m^2 h^4$	A1	
	$\bar{x} = \dfrac{\frac{1}{4}\pi m^2 h^4}{\frac{1}{3}\pi m^2 h^3}$	M1	*Dependent on M1 for integral of x^3*
	$= \frac{3}{4}h$	E1 **6**	
(ii)	$m_1 = \frac{1}{3}\pi \times 0.7^2 \times 2.4\rho = \frac{1}{3}\pi\rho \times 1.176$ $\mathrm{VG}_1 = 1.8$ $m_2 = \frac{1}{3}\pi \times 0.4^2 \times 1.1\rho = \frac{1}{3}\pi\rho \times 0.176$	B1	For m_1 and m_2 (or volumes)
	$\mathrm{VG}_2 = 1.3 + \frac{3}{4}\times 1.1 = 2.125$	B1	or $\frac{1}{4}\times 1.1$ from base
	$(m_1 - m_2)(\mathrm{VG}) + m_2(\mathrm{VG}_2) = m_1(\mathrm{VG}_1)$	M1	Attempt formula for composite body
	$(\mathrm{VG}) + 0.176 \times 2.125 = 1.176 \times 1.8$	F1	
	Distance (VG) is 1.74 m	A1 **5**	
(iii)	VQG is a right-angle	M1	
	$\mathrm{VQ} = \mathrm{VG}\cos\theta$ where $\tan\theta = \dfrac{0.7}{2.4}$ $(\theta = 16.26°)$	M1	
	$\mathrm{VQ} = 1.7428 \times \dfrac{24}{25}$		
	$= 1.67$ m	A1 **3**	ft is $\mathrm{VG}\times 0.96$

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1 (i)	$\frac{1}{2}m(v^2-1.4^2)=m\times 9.8(2.6-2.6\cos\theta)$ $v^2-1.96=50.96-50.96\cos\theta$ $v^2=52.92-50.96\cos\theta$	M1 A1 E1 **3**	Equation involving KE and PE
(ii)	$0.65\times 9.8\cos\theta-R=0.65\times\frac{v^2}{2.6}$ $6.37\cos\theta-R=0.25(52.92-50.96\cos\theta)$ $6.37\cos\theta-R=13.23-12.74\cos\theta$ $R=19.11\cos\theta-13.23$	M1 A1 M1 A1 **4**	Radial equation involving v^2/r Substituting for v^2 *Dependent on previous M1* *Special case:* $R=13.23-19.11\cos\theta$ *earns* M1A0M1SC1
(iii)	Leaves surface when $R=0$ $\cos\theta=\frac{13.23}{19.11}$ $(=\frac{9}{13})$ $(\theta=46.19°)$ $v^2=52.92-50.96\times\frac{9}{13}$ Speed is $4.2\ \mathrm{m\,s^{-1}}$	M1 A1 M1 A1 **4**	(ft if $R=a+b\cos\theta$ and $0<-\frac{a}{b}<1$) *Dependent on previous M1*
(iv)	$T\sin\alpha+R\cos\alpha=0.65\times 9.8$ $T\cos\alpha-R\sin\alpha=0.65\times\frac{1.2^2}{2.4}$	M1 A1 M1 A1	Resolving vertically (3 terms) Horiz eqn involving v^2/r or $r\omega^2$
	OR $T-mg\sin\alpha=m\left(\frac{1.2^2}{2.4}\right)\cos\alpha$ M1A1 $mg\cos\alpha-R=m\left(\frac{1.2^2}{2.4}\right)\sin\alpha$ M1A1		
	$\sin\alpha=\frac{2.4}{2.6}=\frac{12}{13}$, $\cos\alpha=\frac{5}{13}$ $(\alpha=67.38°)$ Tension is 6.03 N Normal reaction is 2.09 N	M1 M1 A1 A1 **8**	Solving to obtain a value of *T* or *R* *Dependent on necessary M1s* *(Accept 6 , 2.1)* *Treat* $\omega=1.2$ *as a misread, leading to* $T=6.744$, $R=0.3764$ *for 7 / 8*

2 (i)	$\frac{1}{2}\times 5000x^2 = \frac{1}{2}\times 400\times 3^2$ Compression is 0.849 m	M1 A1 A1 **3**	Equation involving EE and KE Accept $\frac{3\sqrt{2}}{5}$
(ii)	Change in PE is $400\times 9.8\times(7.35+1.4)\sin\theta$ $= 400\times 9.8\times 8.75\times\frac{1}{7}$ $= 4900\ \mathrm{J}$ Change in EE is $\frac{1}{2}\times 5000\times 1.4^2$ $= 4900\ \mathrm{J}$ Since Loss of PE = Gain of EE, car will be at rest	M1 A1 M1 E1 **4**	Or $400\times 9.8\times 1.4\sin\theta$ *and* $\frac{1}{2}\times 400\times 4.54^2$ Or $784+4116$ M1M1A1 can also be given for a correct equation in *x* (compression): $2500x^2-560x-4116=0$ Conclusion required, or solving equation to obtain $x=1.4$
(iii)	WD against resistance is $7560(24+x)$ Change in EE is $\frac{1}{2}\times 5000x^2$ Change in KE is $\frac{1}{2}\times 400\times 30^2$ Change in PE is $400\times 9.8\times(24+x)\times\frac{1}{7}$	B1 B1 B1 B1	($=181440+7560x$) ($=2500x^2$) ($=180000$) ($=13440+560x$)
	OR Speed $7.75\ \mathrm{m\,s^{-1}}$ when it hits buffer, then WD against resistance is $7560x$ B1 Change in EE is $\frac{1}{2}\times 5000x^2$ B1 Change in KE is $\frac{1}{2}\times 400\times 7.75^2$ B1 Change in PE is $400\times 9.8\times x\times\frac{1}{7}$ B1		 ($=2500x^2$) ($=12000$) ($=560x$)
	$-7560(24+x)=\frac{1}{2}\times 5000x^2-\frac{1}{2}\times 400\times 30^2$ $-400\times 9.8\times(24+x)\times\frac{1}{7}$ $-7560(24+x)=2500x^2-180\,000-560(24+x)$ $-3.024(24+x)=x^2-72-0.224(24+x)$ $x^2+2.8x-4.8=0$ $x=\frac{-2.8+\sqrt{2.8^2+19.2}}{2}$ $=1.2$	M1 F1 M1 A1 M1 A1 **10**	Equation involving WD, EE, KE, PE Simplification to three term quadratic

3(a)(i)	[Velocity] $= \mathrm{L\,T^{-1}}$ [Force] $= \mathrm{M\,L\,T^{-2}}$ [Density] $= \mathrm{M\,L^{-3}}$	B1 B1 B1 **3**	*Deduct 1 mark for* $\mathrm{m\,s^{-1}}$ *etc*
(ii)	$\mathrm{M\,L\,T^{-2}} = (\mathrm{M\,L^{-3}})^{\alpha}(\mathrm{L\,T^{-1}})^{\beta}(\mathrm{L^2})^{\gamma}$ $\alpha = 1$ $\beta = 2$ $-3\alpha + \beta + 2\gamma = 1$ $\gamma = 1$	B1 B1 M1A1 A1 **5**	(ft if equation involves α, β and γ)
(b)(i)	$\frac{2\pi}{\omega} = 4.3$ $\omega = \frac{2\pi}{4.3}$ $(= 1.4612)$	M1 A1	
	$\dot{\theta}^2 = 1.4612^2(0.08^2 - 0.05^2)$ Angular speed is $0.0913\ \mathrm{rad\,s^{-1}}$	M1 F1 A1 **5**	Using $\omega^2(A^2 - \theta^2)$ For RHS (b.o.d. for $v = 0.0913\ \mathrm{m\,s^{-1}}$)
	OR $\dot{\theta} = 0.08\omega\cos\omega t$ M1 $= 0.08 \times 1.4612\cos 0.6751$ F1 $= 0.0913$ A1		Or $\dot{\theta} = (-)\ 0.08\omega\sin\omega t$ $= (-)\ 0.08 \times 1.4612\sin 0.8957$
(ii)	$\theta = 0.08\sin\omega t$ When $\theta = 0.05$, $0.08\sin\omega t = 0.05$ $\omega t = 0.6751$ $t = 0.462$ Time taken is 2×0.462 $= 0.924\ \mathrm{s}$	B1 M1 A1 cao M1 A1 cao **5**	or $\theta = 0.08\cos\omega t$ Using $\theta = (\pm)0.05$ to obtain an equation for *t* *B1M1 above can be earned in (i)* or $t = 0.613$ from $\theta = 0.08\cos\omega t$ or $t = 1.537$ from $\theta = 0.08\cos\omega t$ Strategy for finding the required time (2×0.462 or $\frac{1}{2} \times 4.3 - 2 \times 0.613$ or $1.537 - 0.613$) *Dep on first M1* *For* $\theta = 0.05\sin\omega t$ *, max B0M1A0M0* *(for* $0.05 = 0.05\sin\omega t$ *)*

4 (a)	Area is $\int_0^{\ln 3} e^x\,dx = \left[e^x \right]_0^{\ln 3}$	M1	
	$= 2$	A1	
	$\int xy\,dx = \int_0^{\ln 3} x e^x\,dx$	M1	
	$= \left[x e^x - e^x \right]_0^{\ln 3}$	M1 A1	Integration by parts For $x e^x - e^x$
	$= 3\ln 3 - 2$		
	$\bar{x} = \dfrac{3\ln 3 - 2}{2} = \frac{3}{2}\ln 3 - 1$	A1	ww full marks (B4) Give B3 for 0.65
	$\int \frac{1}{2} y^2\,dx = \int_0^{\ln 3} \frac{1}{2}(e^x)^2\,dx$	M1	For integral of $(e^x)^2$
	$= \left[\frac{1}{4} e^{2x} \right]_0^{\ln 3}$	A1	For $\frac{1}{4}e^{2x}$
	$= 2$		
	$\bar{y} = \dfrac{2}{2} = 1$	A1	If area wrong, SC1 for $\bar{x} = \dfrac{3\ln 3 - 2}{area}$ and $\bar{y} = \dfrac{2}{area}$
		9	
(b)(i)	Volume is $\int \pi y^2\,dx = \int_2^a \pi \dfrac{36}{x^4}\,dx$	M1	*π may be omitted throughout*
	$= \pi\left[-\dfrac{12}{x^3} \right]_2^a = \pi\left(\dfrac{3}{2} - \dfrac{12}{a^3} \right)$	A1	
	$\int \pi x y^2\,dx = \int_2^a \pi \dfrac{36}{x^3}\,dx$	M1	
	$= \pi\left[-\dfrac{18}{x^2} \right]_2^a = \pi\left(\dfrac{9}{2} - \dfrac{18}{a^2} \right)$	A1	
	$\bar{x} = \dfrac{\int \pi x y^2\,dx}{\int \pi y^2\,dx}$	M1	
	$= \dfrac{\pi\left(\frac{9}{2} - \frac{18}{a^2} \right)}{\pi\left(\frac{3}{2} - \frac{12}{a^3} \right)} = \dfrac{3(a^3 - 4a)}{a^3 - 8}$	E1	
		6	
(ii)	Since $a > 2$, $4a > 8$	M1	*Condone $\geq$ instead of $>$ throughout*
	so $a^3 - 4a < a^3 - 8$	A1	
	Hence $\bar{x} = \dfrac{3(a^3 - 4a)}{a^3 - 8} < 3$ i.e. CM is less than 3 units from O	E1 **3**	Fully acceptable explanation *Dependent on M1A1*
	OR As $a \to \infty$, $\bar{x} = \dfrac{3(1 - 4a^{-2})}{1 - 8a^{-3}} \to 3$ M1A1 Since $\bar{x}$ increases as *a* increases, $\bar{x}$ is less than 3 E1		Accept $\bar{x} \approx \dfrac{3a^3}{a^3} \to 3$, etc (M1 for $\bar{x} \to 3$ stated, but A1 requires correct justification)

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1(a) (i)	[Density] $= \mathrm{M\,L^{-3}}$ [Kinetic Energy] $= \mathrm{M\,L^2\,T^{-2}}$ [Power] $= \mathrm{M\,L^2\,T^{-3}}$	B1 B1 B1	*(Deduct B1 for* $\mathrm{kg\,m^{-3}}$ *etc)*	3
(ii)	$\mathrm{M\,L^2\,T^{-3}} = [\eta]\mathrm{L}(\mathrm{L\,T^{-1}})^2$ $[\eta] = \mathrm{M\,L^{-1}\,T^{-1}}$	B1 M1 A1	For $[v] = \mathrm{L\,T^{-1}}$ *Can be earned in (iii)* Obtaining the dimensions of η	3
(iii)	$\mathrm{M\,L^2\,T^{-3}} = (\mathrm{M\,L^{-3}})^{\alpha}\,\mathrm{L}^{\beta}\,(\mathrm{L\,T^{-1}})^{\gamma}$ $\alpha = 1$ $-3 = -\gamma$ $\gamma = 3$ $2 = -3\alpha + \beta + \gamma$ $\beta = 2$	B1 cao M1 A1 M1 A1 A1	Considering powers of T *(No ft if* $\gamma = 0$ *)* Considering powers of L Correct equation *(ft requires 4 terms)* *(No ft if* $\beta = 0$ *)*	6
(b)	EE at start is $\frac{1}{2}k \times 0.8^2$ EE at end is $\frac{1}{2}k \times 0.3^2$ $\frac{1}{2}k \times 0.8^2 = \frac{1}{2}k \times 0.3^2 + 5.5 \times 9.8 \times 3.5$ Stiffness is $686\ \mathrm{N\,m^{-1}}$	M1 A1 A1 M1 F1 A1	Calculating elastic energy k may be $\dfrac{\lambda}{l}$ or $\dfrac{\lambda}{1.2}$ Equation involving EE and PE *(must have three terms)* (A0 for $\lambda = 823.2$)	6
				[18]

2 (a)	$\int \pi x y^2 \, dx \;= \int_0^a \pi x(a^2 - x^2)\, dx$ $= \pi\left[\ \tfrac{1}{2}a^2x^2 - \tfrac{1}{4}x^4\ \right]_0^a$ $= \tfrac{1}{4}\pi a^4$ $\bar{x} = \dfrac{\tfrac{1}{4}\pi a^4}{\tfrac{2}{3}\pi a^3}$ $= \tfrac{3}{8}a$	M1 A1 A1 M1 E1	*Limits not required* For $\tfrac{1}{2}a^2x^2 - \tfrac{1}{4}x^4$	5
(b) (i)	Area is $\int_1^4 (2-\sqrt{x})\, dx$ $= \left[\ 2x - \tfrac{2}{3}x^{3/2}\ \right]_1^4 \quad (= \dfrac{4}{3})$ $\int x y \, dx \;= \int_1^4 x(2-\sqrt{x})\, dx$ $= \left[\ x^2 - \tfrac{2}{5}x^{\frac{5}{2}}\ \right]_1^4 \quad (= \dfrac{13}{5})$ $\bar{x} = \dfrac{13/5}{4/3} = \dfrac{39}{20} = 1.95$ $\int \tfrac{1}{2}y^2 \, dx \;= \int_1^4 \tfrac{1}{2}(2-\sqrt{x})^2 \, dx$ $= \left[\ 2x - \tfrac{4}{3}x^{3/2} + \tfrac{1}{4}x^2\ \right]_1^4 \quad (= \dfrac{5}{12})$ $\bar{y} = \dfrac{5/12}{4/3} = \dfrac{5}{16} = 0.3125$	M1 A1 M1 A1 A1 M1 A2 A1	*Limits not required* For $2x - \tfrac{2}{3}x^{\frac{3}{2}}$ *Limits not required* For $x^2 - \tfrac{2}{5}x^{\frac{5}{2}}$ $\int (2-\sqrt{x})^2 \, dx$ *or* $\int \left((2-y)^2 - 1\right) y \, dy$ For $2x - \tfrac{4}{3}x^{\frac{3}{2}} + \tfrac{1}{4}x^2$ *or* $\tfrac{3}{2}y^2 - \tfrac{4}{3}y^3 + \tfrac{1}{4}y^4$ Give A1 for two terms correct, or all correct with ½ omitted	9
(ii)	Taking moments about A $T_C \times 3 - W \times 0.95 = 0$ $T_A + T_C = W$ $T_A = \dfrac{41}{60}W, \quad T_C = \dfrac{19}{60}W$	M1 A1 M1 A1	Moments equation (no force omitted) Any correct moments equation (May involve both T_A and T_C) Accept Wg or $W = \tfrac{4}{3}, \tfrac{4}{3}g$ here Resolving vertically (or a second moments equation) *Accept* $0.68W$, $0.32W$	4
				[18]

3 (i)	By conservation of energy, $\frac{1}{2}\times 0.6\times 6^2 - \frac{1}{2}\times 0.6 v^2 = 0.6\times 9.8(1.25 - 1.25\cos\theta)$ $36 - v^2 = 24.5 - 24.5\cos\theta$ $v^2 = 11.5 + 24.5\cos\theta$	M1 A1 E1	Equation involving KE and PE	3
(ii)	$T - 0.6\times 9.8\cos\theta = 0.6\times\frac{v^2}{1.25}$ $T - 5.88\cos\theta = 0.48(11.5 + 24.5\cos\theta)$ $T = 5.52 + 17.64\cos\theta$	M1 A1 M1 A1	For acceleration $\frac{v^2}{r}$ Substituting for v^2	4
(iii)	String becomes slack when $T = 0$ $\cos\theta = -\frac{5.52}{17.64}$ ($\theta = 108.2°$ or 1.889 rad) $v^2 = 11.5 - 24.5\times\frac{5.52}{17.64}$ Speed is $1.96\ \mathrm{m\,s^{-1}}$ (3 sf)	M1 A1 M1 A1 cao	 *May be implied* or $0.6\times 9.8\times\frac{5.52}{17.64} = 0.6\times\frac{v^2}{1.25}$ or $-0.6\times 9.8\times\frac{v^2 - 11.5}{24.5} = 0.6\times\frac{v^2}{1.25}$	4
(iv)	$T_1\cos\theta = mg$ $T_1\times\frac{1.2}{1.25} = 0.6\times 9.8$ (where θ is angle COP) Tension in OP is 6.125 N $T_1\sin\theta + T_2 = \frac{mv^2}{0.35}$ $6.125\times\frac{0.35}{1.25} + T_2 = \frac{0.6\times 1.4^2}{0.35}$ Tension in CP is 1.645 N	M1 A1 A1 M1 F1B1 A1	Resolving vertically Horizontal equation (three terms) For LHS and RHS	7
				[18]

4(i)	$T_{AP} = \frac{7.35}{1.5}\times 0.05 \quad (=0.245)$ $T_{BP} = \frac{7.35}{2.5}\times 0.5 \quad (=1.47)$ Resultant force up the plane is $T_{BP} - T_{AP} - mg\sin 30°$ $= 1.47 - 0.245 - 0.25\times 9.8\sin 30°$ $= 1.47 - 0.245 - 1.225$ $= 0$ Hence there is no acceleration	M1 A1 A1 M1 E1	Using Hooke's law or $\frac{7.35}{1.5}(\mathrm{AP}-1.5)$ or $\frac{7.35}{2.5}(2.05-\mathrm{AP})$ Correctly shown	5
(ii)	$T_{AP} = \frac{7.35}{1.5}(0.05+x) \quad (=0.245+4.9x)$ $T_{BP} = \frac{7.35}{2.5}(4.55-1.55-x-2.5)$ $= 2.94(0.5-x)$ $= 1.47 - 2.94x$	B1 M1 E1		3
(iii)	$T_{BP} - T_{AP} - mg\sin 30° = m\frac{d^2x}{dt^2}$ $(1.47-2.94x)-(0.245+4.9x)-1.225 = 0.25\frac{d^2x}{dt^2}$ $\frac{d^2x}{dt^2} = -31.36x$ Hence the motion is simple harmonic Period is $\frac{2\pi}{\sqrt{31.36}} = \frac{2\pi}{5.6}$ Period is 1.12 s (3 sf)	M1 A2 E1 B1 cao	Equation of motion parallel to plane Give A1 for an equation which is correct apart from sign errors Must state conclusion. Working must be fully correct (cao) *If a is used for accn down plane, then $a = 31.36x$ can earn M1A2; but E1 requires comment about directions* *Accept* $\frac{5\pi}{14}$	5
(iv)	$x = -0.05\cos 5.6t$ $v = 0.28\sin 5.6t$ $-0.2 = 0.28\sin 5.6t$ OR $0.2^2 = 31.36(0.05^2 - x^2)$ $x = (\pm)\ 0.035$ $0.035 = -0.05\cos 5.6t$ M1 $5.6t = \pi + 0.7956$ Time is 0.703 s (3 sf)	M1 A1 M1 M1 A1cao	For $A\sin\omega t$ or $A\cos\omega t$ Allow $\pm 0.05\sin/\cos 5.6t$ *Implied by* $v = \pm 0.28\sin/\cos 5.6t$ Using $v = \pm 0.2$ to obtain an equation for t Fully correct strategy for finding the required time	5
				[18]

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1(a)(i)	$\text{AP} = \sqrt{2.4^2 + 0.7^2} = 2.5$ Tension $T = 70 \times 0.35 \quad (= 24.5)$ Resultant vertical force on P is $2T\cos\theta - mg$ $= 2 \times 24.5 \times \frac{2.4}{2.5} - 4.8 \times 9.8$ $= 47.04 - 47.04 = 0$ Hence P is in equilibrium	M1 A1 M1 B1 B1 E1 **6**	 Attempting to resolve vertically For $T \times \frac{2.4}{2.5}$ (or $T\cos 16.3°$ *etc*) For 4.8×9.8 Correctly shown
(ii)	$\text{EE} = \frac{1}{2} \times 70 \times 0.35^2$ Elastic energy is 4.2875 J	M1 A1 **2**	(M0 for $\frac{1}{2} \times 70 \times 0.35$) *Note* If 70 is used as modulus instead of stiffness: (i) M1A0M1B1B1E0 (ii) M1 A1 for 1.99
(iii)	Initial $\text{KE} = \frac{1}{2} \times 4.8 \times 3.5^2$ By conservation of energy $4.8 \times 9.8h = 2 \times 4.2875 + \frac{1}{2} \times 4.8 \times 3.5^2$ $47.04h = 8.575 + 29.4$ Height is 0.807 m (3 sf)	B1 M1 F1 A1 **4**	 Equation involving EE, KE and PE (A0 for 0.8) ft is $\frac{2 \times (\text{ii}) + 29.4}{47.04}$
(b)(i)	$[\text{ Force }] = \text{M L T}^{-2}$ $[\text{ Stiffness }] = \text{M T}^{-2}$	B1 B1 **2**	*Deduct 1 mark if units are used*
(ii)	$\text{L T}^{-1} = \text{M}^{\alpha} (\text{M T}^{-2})^{\beta} \text{L}^{\gamma}$ $\gamma = 1$ $\beta = \frac{1}{2}$ $0 = \alpha + \beta$ $\alpha = -\frac{1}{2}$	 B1 B1 M1 A1 **4**	 Considering powers of M *When [Stiffness] is wrong in (i), allow all marks ft provided the work is comparable and answers are non-zero*

2 (i)	$R\cos\theta = mg$ [θ is angle between OB and vertical] $R\times 0.8 = 0.4\times 9.8$ Normal reaction is 4.9 N	M1 A1 A1 **3**	Resolving vertically
(ii)	$R\sin\theta = m\dfrac{v^2}{r}$ $4.9\times 0.6 = 0.4\times\dfrac{v^2}{1.5}$ $v^2 = 11.025$ Speed is $3.32\ \mathrm{m\,s^{-1}}$ (3 sf)	M1 A1 A1 **3**	For acceleration $\dfrac{v^2}{r}$ or $r\omega^2$ or $4.9\times 0.6 = 0.4\times 1.5\omega^2$ ft is $1.5\sqrt{R}$
(iii)	By conservation of energy $\frac{1}{2}mu^2 = mg\times 2.5$ $u^2 = 5g$ ($u = 7$) $R - mg = m\times\dfrac{u^2}{2.5}$ $R - mg = 2mg$ $R = 3mg$	M1 A1 M1 E1 **4**	Equation involving KE and PE Vertical equation of motion (must have three terms) Correctly shown or $R = 11.76$ and $3\times 0.4\times 9.8 = 11.76$
(iv) **(v)**	$\frac{1}{2}mv^2 = mg\times 2.5\cos\theta$ $v^2 = 5g\cos\theta$ $R - mg\cos\theta = m\times\dfrac{v^2}{2.5}$ When $R = 2mg$ ($= 7.84$), $2mg - mg\cos\theta = \dfrac{mv^2}{2.5}$ $2mg - \dfrac{mv^2}{5} = \dfrac{mv^2}{2.5}$ $7.84 - 0.08v^2 = 0.16v^2$ $v^2 = \dfrac{98}{3}$ Speed is $5.72\ \mathrm{m\,s^{-1}}$ (3 sf) $\cos\theta = \dfrac{v^2}{5g} = \dfrac{2}{3}$ ($\theta = 48.2°$ or 0.841 rad) Tangential acceleration is $g\sin\theta$ Tangential acceleration is $7.30\ \mathrm{m\,s^{-2}}$ (3 sf)	M1 A1 M1 M1 M1 A1 M1 A1 **8**	*Mark (iv) and (v) as one part* Equation involving KE, PE and an angle (θ is angle with vertical) [$\frac{1}{2}mv^2 = mgh$ can earn M1A1, but only if $\cos\theta = h/2.5$ appears somewhere] Equation of motion towards O (must have three terms, and the weight must be resolved) Obtaining an equation for v Obtaining an equation for θ *These two marks are each dependent on M1M1 above* [$g\sin\theta$ in isolation only earns M1 if the angle θ is clearly indicated]

3 (i)	Volume is $\int_1^5 \pi\left(\frac{1}{x}\right)^2 \mathrm{d}x$	M1	π may be omitted throughout *Limits not required*
	$= \pi\left[-\frac{1}{x}\right]_1^5 \quad (=\frac{4}{5}\pi)$	A1	For $-\frac{1}{x}$
	$\int \pi x y^2 \mathrm{d}x = \int_1^5 \pi x\left(\frac{1}{x}\right)^2 \mathrm{d}x$	M1	*Limits not required*
	$= \pi\left[\ln x\right]_1^5 \quad (=\pi\ln 5)$	A1	For $\ln x$
	$\bar{x} = \frac{\pi \ln 5}{\frac{4}{5}\pi} = \frac{5\ln 5}{4}$ *(2.012)*	A1 **5**	*SR* If exact answers are not seen, deduct only the first A1 affected
(ii)	Area is $\int_1^5 \frac{1}{x}\mathrm{d}x$	M1	*Limits not required*
	$= \left[\ln x\right]_1^5 \quad (=\ln 5)$	A1	For $\ln x$
	$\int x y \,\mathrm{d}x = \int_1^5 x\left(\frac{1}{x}\right)\mathrm{d}x \quad \left(=\left[x\right]_1^5 = 4\right)$	M1	*Limits not required*
	$\bar{x} = \frac{4}{\ln 5}$ *(2.485)*	A1	
	$\int \frac{1}{2}y^2 \mathrm{d}x = \int_1^5 \frac{1}{2}\left(\frac{1}{x}\right)^2 \mathrm{d}x$	M1	For $\int \left(\frac{1}{x}\right)^2 \mathrm{d}x$
	$= \left[-\frac{1}{2x}\right]_1^5 \quad (=\frac{2}{5})$	A1	For $-\frac{1}{2x}$
	$\bar{y} = \frac{2/5}{\ln 5} = \frac{2}{5\ln 5}$ *(0.2485)*	A1 **7**	
(iii)	CM of R_2 is $\left(\frac{2}{5\ln 5}, \frac{4}{\ln 5}\right)$	B1B1 ft **2**	*Do not penalise inexact answers in this part*
(iv)		B1	For CM of R_3 is $(\frac{1}{2}, \frac{1}{2})$ (one coordinate is sufficient)
		M1	Using $\sum mx$ with three terms
	$\bar{x} = \frac{(\ln 5)\left(\frac{4}{\ln 5}\right) + (\ln 5)\left(\frac{2}{5\ln 5}\right) + (1)\left(\frac{1}{2}\right)}{\ln 5 + \ln 5 + 1}$	M1	Using $\frac{\sum mx}{\sum m}$ with at least two terms in each sum
	CM is $\left(\frac{4.9}{2\ln 5 + 1}, \frac{4.9}{2\ln 5 + 1}\right)$ *(1.161 , 1.161)*	A1 cao **4**	

4 (i)	$v = \frac{dx}{dt} = A\omega\cos\omega t - B\omega\sin\omega t$	B1	
	$a = \frac{d^2x}{dt^2} = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$	M1	Finding the second derivative
	$= -\omega^2(A\sin\omega t + B\cos\omega t) = -\omega^2 x$	E1 **3**	Correctly shown
(ii)	$B = -16$	B1	
	$\omega = 0.25$	B1	
	$A = 30$	B2 **4**	When A is wrong, give B1 for a correct equation involving A [e.g. $A\omega = 7.5$ or $7.5^2 = \omega^2(A^2 + B^2 - 16^2)$] or for $A = -30$
(iii)	Maximum displacement is $(\pm)\sqrt{A^2 + B^2}$	M1	Or $7.5^2 = \omega^2(\text{amp}^2 - 16^2)$ Or finding t when $v = 0$ and substituting to find x
	Maximum displacement is 34 m	A1	
	Maximum speed is $(\pm)\ 34\omega$ Maximum acceleration is $(\pm)\ 34\omega^2$	M1	For either (any valid method)
	Maximum speed is $8.5\ \text{m s}^{-1}$	F1	Only ft from $\omega \times \text{amp}$
	Maximum acceleration is $2.125\ \text{m s}^{-2}$	F1 **5**	Only ft from $\omega^2 \times \text{amp}$
(iv)	$v = 7.5\cos 0.25t + 4\sin 0.25t$ When $t = 15$, $v = 7.5\cos 3.75 + 4\sin 3.75$ $= -8.44$	M1	
	Speed is $8.44\ \text{m s}^{-1}$ (3 sf); downwards	A1 **2**	
(v)	Period $\frac{2\pi}{\omega} \approx 25$ s, so $t = 0$ to $t = 15$ is less than one period		
	When $t = 15$, $x = 30\sin 3.75 - 16\cos 3.75$ $= -4.02$	M1	
		M1	Take account of change of direction
	Distance travelled is $16 + 34 + 34 + 4.02$	M1	Fully correct strategy for distance
	Distance travelled is 88.0 m (3 sf)	A1 cao **4**	

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1(a)(i)	[Force] $= \mathrm{M\,L\,T^{-2}}$ [Density] $= \mathrm{M\,L^{-3}}$ [Angular speed] $= \mathrm{T^{-1}}$	B1 B1 B1 **3**	*Deduct one mark if given as* $\mathrm{kg\,m\,s^{-2}}$ *etc*
(ii)	[Breaking stress] $= \dfrac{\mathrm{M\,L\,T^{-2}}}{\mathrm{L^2}}$ $= \mathrm{M\,L^{-1}\,T^{-2}}$	M1 E1 **2**	For [Force] $\div \mathrm{L^2}$
(iii)	$1.2\times10^9 \times \dfrac{1}{0.454} \times 0.0254 \times 0.001^2$ $= 67.1\ \mathrm{lb\,in^{-1}\,ms^{-2}}$ (3 sf)	M1 M1 A1 **3**	For $\times 0.001^2$ or $\times \dfrac{1}{0.001^2}$ For $\times \dfrac{0.0254}{0.454}$ 5.6×10^{-8} implies M1M1A0 2.15×10^{16} implies M1M0A0
(iv)	$\mathrm{T^{-1}} = (\mathrm{M\,L^{-1}\,T^{-2}})^{\alpha}(\mathrm{M\,L^{-3}})^{\beta}\,\mathrm{L}^{\gamma}$ $\alpha = \frac{1}{2}$ $\beta = -\frac{1}{2}$ $0 = -\alpha - 3\beta + \gamma$ $\gamma = -1$	 B1 B1 M1 A1 **4**	*All marks ft provided work is comparable* Considering powers of L
(v)	$3140 = k(1.2\times10^9)^{\frac{1}{2}}(7800)^{-\frac{1}{2}}(0.5)^{-1}$ $k = 4.00$ (3 sf) $S = \dfrac{\omega^2\rho r^2}{k^2} = \dfrac{8120^2\times2700\times0.2^2}{4^2}$ $= 4.44\times10^8$ Pa (3 sf)	M1 M1 M1 A1 cao **4**	Obtaining equation for k Obtaining numerical value for k Obtaining equation for S
	OR $S = 1.2\times10^9 \times \left(\dfrac{8120}{3140}\right)^2 \times \dfrac{2700}{7800} \times \left(\dfrac{0.2}{0.5}\right)^2$ M1M1M1 $= 4.44\times10^8$ A1		
(vi)	$\omega = 4(630)^{\frac{1}{2}}(70)^{-\frac{1}{2}}(15)^{-1}$ $= 0.8$	M1 A1 cao **2**	Obtaining equation for ω

2(a)(i)	$T\cos\alpha = m\dfrac{V^2}{r}$ (α is angle APC) $T\times\dfrac{8.4}{30} = 48\times\dfrac{3.5^2}{8.4}$ Tension is 250 N $T\sin\alpha + R = mg$ $250\times 0.96 + R = 48\times 9.8$ Normal reaction is 230.4 N	M1 A1 A1 M1 A1 **5**	Equation of motion including $\dfrac{V^2}{r}$ Or $T\cos 73.7 = ...$ or $T\sin 16.3 = ...$ Resolving vertically (three terms)
(ii)	$T\sin\alpha = mg$ $T\times 0.96 = 48\times 9.8$ $T = 490$ $T\cos\alpha = m\dfrac{V^2}{r}$ $490\times 0.28 = 48\times\dfrac{V^2}{8.4}$ $V = 4.9$	M1 A1 M1 A1 **4**	Vertical equation with $R = 0$ Or $T\sin 73.7 = ...$ or $T\cos 16.3 = ...$ Obtaining equation for V *Allow T=490 obtained in (i) and used correctly in (ii) for full marks*
(b)(i)	$\tfrac{1}{2}m(v^2 - u^2) = m\times 9.8(2.5 - 2.5\cos\theta)$ $v^2 - u^2 = 49(1-\cos\theta)$ $v^2 = u^2 + 49 - 49\cos\theta$	M1 A1 E1 **3**	Equation involving KE and PE
(ii)	$mg\cos\theta - R = m\dfrac{v^2}{r}$ $48\times 9.8\left(\dfrac{u^2+49-v^2}{49}\right) - R = \dfrac{48v^2}{2.5}$ $9.6u^2 + 470.4 - 9.6v^2 - R = 19.2v^2$ $R = 470.4 + 9.6u^2 - 28.8v^2$	M1 A1 M1 A1 **4**	Radial equation (three terms) Obtaining equation in *R*, *u*, *v*
(iii)	$470.4 + 9.6u^2 - 28.8\times 4.15^2 = 0$ $u = 1.63$ (3 sf)	M1 A1 **2**	Substituting $R = 0$ and $v = 4.15$ *or other complete method leading to an equation for u* (ft requires $0 < u < 4.15$)

3 (i)	Tension is $180(10-7)$ $= 540$ N	M1 A1 **2**	Using $T = k \times \text{extension}$
(ii)	$4\times540 = T + 200\times9.8$ $T = 200$ Extension is $\dfrac{200}{80}$ $(= 2.5)$ Natural length is 5.5 m	M1 A1 M1 A1 cao **4**	Resolving vertically
(iii)	$80(2.5-x)+200\times9.8-4\times180(3+x) = 200\dfrac{d^2x}{dt^2}$ $200-80x+1960-2160-720x = 200\dfrac{d^2x}{dt^2}$ $\dfrac{d^2x}{dt^2} = -4x$	B1 ft M1 A1 E1 **4**	For $180(3+x)$ or $80(2.5-x)$ Equation of motion (condone one missing force)
(iv)	Maximum acceleration is $\omega^2 A$ $= 4\times2.2 = 8.8 \text{ m s}^{-2}$	M1 A1 **2**	*Condone* -8.8
(v)	When $x = -1.6$, $v^2 = \omega^2(A^2 - x^2)$ $= 4(2.2^2 - 1.6^2)$ Speed is 3.02 m s^{-1} (3 sf)	M1 A1 **2**	Using $v^2 = \omega^2(A^2 - x^2)$ (or other complete method) *(Allow M1 if $\omega^2 = 2$ or 16 used but M0 if $x = 3.8$ is used)* *Condone* -3.02
(vi)	$x = 2.2\cos 2t$ When $t = 5$, $x = -1.846$ Period is $\dfrac{2\pi}{\omega} = \pi$, 5 s is $\dfrac{5}{\pi} \approx 1.6$ periods Distance travelled is $6\times2.2 + (2.2 - 1.846)$ $= 13.6$ m (3 sf)	B1 M1 M1 A1 **4**	*Condone* $x = -2.2\cos 2t$ *This B1 can be earned in (v)* Obtaining x when $t = 5$ (from $x = A\cos\omega t$ or $x = A\sin\omega t$) Correct strategy for finding distance

4 (a)	Volume is $\int \pi y^2 \mathrm{d}x = \int_k^{4k} \pi(x^2 - k^2)\,\mathrm{d}x$	M1	For $\int (x^2 - k^2)\,\mathrm{d}x$
	$= \pi\left[\ \tfrac{1}{3}x^3 - k^2 x\ \right]_k^{4k} \quad (= 18\pi k^3)$	A1	For $\tfrac{1}{3}x^3 - k^2 x$
	$\int \pi x y^2 \mathrm{d}x = \int_k^{4k} \pi(x^3 - k^2 x)\,\mathrm{d}x$	M1	For $\int xy^2 \mathrm{d}x$
	$= \pi\left[\ \tfrac{1}{4}x^4 - \tfrac{1}{2}k^2 x^2\ \right]_k^{4k} \quad \left(= \dfrac{225\pi k^4}{4}\right)$	A1A1	For $\tfrac{1}{4}x^4$ and $-\tfrac{1}{2}k^2 x^2$
	$\bar{x} = \dfrac{\frac{225}{4}\pi k^4}{18\pi k^3}$	M1	*Dependent on previous M1M1*
	$= \dfrac{25k}{8} = 3.125k$	A1 **7**	
(b)(i)	Area is $\int_0^{2a} \dfrac{x^3}{a^2}\,\mathrm{d}x$	M1	For $\int \dfrac{x^3}{a^2}\,\mathrm{d}x$
	$= \left[\ \dfrac{x^4}{4a^2}\ \right]_0^{2a} \quad (= 4a^2)$	A1	For $\dfrac{x^4}{4a^2}$
	$\int xy\,\mathrm{d}x = \int_0^{2a} \dfrac{x^4}{a^2}\,\mathrm{d}x$	M1	For $\int xy\,\mathrm{d}x$
	$= \left[\ \dfrac{x^5}{5a^2}\ \right]_0^{2a} \quad \left(= \dfrac{32a^3}{5}\right)$	A1	For $\dfrac{x^5}{5a^2}$
	$\bar{x} = \dfrac{\frac{32}{5}a^3}{4a^2} = \dfrac{8a}{5} = 1.6a$	A1	
	$\int \tfrac{1}{2}y^2\,\mathrm{d}x = \int_0^{2a} \dfrac{x^6}{2a^4}\,\mathrm{d}x$	M1	For $\int y^2\,\mathrm{d}x$ or $\int (2a - x)y\,\mathrm{d}y$
	$= \left[\ \dfrac{x^7}{14a^4}\ \right]_0^{2a} \quad \left(= \dfrac{64a^3}{7}\right)$	A1	For $\dfrac{x^7}{14a^4}$ or $ay^2 - \tfrac{3}{7}a^{2/3}y^{7/3}$
	$\bar{y} = \dfrac{\frac{64}{7}a^3}{4a^2} = \dfrac{16a}{7}$	A1 **8**	
(ii)	Centre of mass is vertically below A	M1	*May be implied*
	$\tan\theta = \dfrac{2a - \bar{x}}{8a - \bar{y}} = \dfrac{\frac{2}{5}a}{\frac{40}{7}a} \quad (= 0.07)$	M1	Condone reciprocal
	Angle is $4.00°$ (3 sf)	A1 **3**	

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Mark Scheme for June 2011

Oxford Cambridge and RSA Examinations

1 (i)	$\dfrac{dx}{dt} = A\omega\cos\omega t$ $\dfrac{d^2x}{dt^2} = -A\omega^2\sin\omega t$ $= -\omega^2(A\sin\omega t) = -\omega^2 x$ $\left(\dfrac{dx}{dt}\right)^2 = A^2\omega^2\cos^2\omega t = A^2\omega^2(1-\sin^2\omega t)$ $= \omega^2(A^2 - A^2\sin^2\omega t) = \omega^2(A^2 - x^2)$	B1 M1 E1 M1 E1 **5**	 Obtaining second derivative Using $\cos^2\omega t = 1-\sin^2\omega t$
(ii)	$1.2^2 = \omega^2(A^2 - 0.7^2)$ $0.75^2 = \omega^2(A^2 - 2^2)$ $\dfrac{A^2 - 0.49}{A^2 - 4} = \dfrac{1.2^2}{0.75^2}$ $A^2 - 0.49 = 2.56(A^2 - 4)$ $9.75 = 1.56A^2$ $A^2 = 6.25$ Amplitude is 2.5 m $1.44 = \omega^2(2.5^2 - 0.7^2)$ $\omega = 0.5$ Period is $\dfrac{2\pi}{\omega} = \dfrac{2\pi}{0.5}$ $= 4\pi = 12.6$ s (3 sf)	M1 A1 M1 E1 M1 A1 **6**	Using $v^2 = \omega^2(A^2 - x^2)$ Two correct equations (M0 if $x = 7.3$ used, etc) Eliminating ω or Eliminating A or Substituting $A = 2.5$ into both equations Correctly shown Using $\dfrac{2\pi}{\omega}$
(iii)	Maximum speed is $A\omega = 1.25\text{ m s}^{-1}$	B1 **1**	ft only if greater than 1.2
(iv)	Magnitude is $0.5^2 \times 1.6$ $= 0.4\text{ m s}^{-2}$ Direction is upwards	M1 A1 B1 **3**	 Accept -0.4 B0 for just 'towards centre'
(v)	$x = 2.5\sin(0.5t)$ When $x = -2$, $t = -1.855$ (or 10.71) When $x = 1$, $t = 0.823$ (or 13.39) Time taken is $0.823 - (-1.855)$ $= 2.68$ s (3 sf)	B1 M1 A1 **3**	or $x = 2.5\cos(0.5t)$ or $t = (\pm)\ 4.996$ or $t = (\pm)\ 2.319$ Correct strategy for finding time (must use radians) (ft is $1.3388/\omega$)

2(a)(i)	$0.6+0.2\times 9.8=0.2\times\dfrac{u^2}{3.2}$ Speed is $6.4\ \text{m s}^{-1}$	M1 A1 A1 **3**	For acceleration $\dfrac{u^2}{3.2}$
(ii)	(*A*) $\frac{1}{2}m(v^2-u^2)=m\times 9.8(3.2+3.2\cos 60°)$ $v^2=135.04$ Radial component is $\dfrac{v^2}{3.2}=42.2\ \text{m s}^{-2}$ Tangential component is $g\sin 60°$ $=8.49\ \text{m s}^{-2}$ (3 sf)	M1 A1 A1 M1 A1 **5**	Equation involving KE and PE (ft is $29.4+\dfrac{u^2}{3.2}$) M1A0 for $g\cos 60°$ M0 for $mg\sin 60°$ *If radial and tangential components are interchanged, withhold first A1*
	(*B*) $T-mg\cos 60°=ma$ $T-0.2\times 9.8\cos 60°=0.2\times 42.2$ Tension is 9.42 N	M1 A1 A1 cao **3**	Radial equation (three terms) (Allow M1 for $T-mg=ma$) *This M1 can be awarded in (A)* *ft dependent on M1 for energy in (A)* *SC* If $60°$ with upward vertical, *(A)* M1A0A0 M1A1 (8.49) *(B)* M1A1A1 (3.54)
(b)(i)	$T\cos 36°+0.75\sin 36°=0.2\times 9.8$ Tension is 1.88 N (3 sf)	M1 A1 **2**	Resolving vertically (three terms) *Allow sin/cos confusion, but both T and R must be resolved*
(ii)	Angular speed $\omega=\dfrac{2\pi}{1.8}$ ($=3.491$) $T\sin 36°-0.75\cos 36°=0.2r\left(\dfrac{2\pi}{1.8}\right)^2$ $r=0.204$ Length of string is $\dfrac{r}{\sin 36°}$ $=0.347$ m (3 sf)	B1 M1 A1 M1 A1 cao **5**	Or $v=\dfrac{2\pi r}{1.8}$ Horiz eqn involving $r\omega^2$ or v^2/r Equation for r (or l) *Dependent on previous M1*

3 (i)	Elastic energy is $\frac{1}{2}\times\frac{573.3}{3.9}\times 0.9^2$ $= 59.535$ J	M1 A1 **2**	Allow one error *(Allow 60 A0 for 59)*
(ii)	Length of string at bottom is $2\sqrt{1.8^2+2.4^2}$ $(=6)$ $\frac{1}{2}\times\frac{573.3}{3.9}\times(2.1^2-0.9^2)=m\times 9.8\times 1.8$ $324.135-59.535=17.64m$ Mass is 15 kg	M1 M1 B1B1 E1 **5**	Finding length of string (or half-string) Equation involving EE and PE For change in EE and change in PE
(iii)	Length of string is $2\sqrt{1.0^2+2.4^2}=5.2$ Tension $T=\frac{573.3}{3.9}\times 1.3$ $(=191.1)$ $2T\sin\alpha-mg=2\times 191.1\times\frac{1.0}{2.6}-15\times 9.8$ $=147-147$ $=0$, hence it is in equilibrium	M1 A1 M1 E1 **4**	Finding tension (via Hooke's law) Finding vertical component of tension Give A1 for $T=191.1$ obtained from resolving vertically *SC* If 573.3 is used as stiffness: (i) M1A0 (ii) M1M1B0B1E0 (iii) M1A1 (745.29) M1E0
(iv)	$[\,8\pi^2h^3\,]=\mathrm{L}^3$, $[\,8h^3-ad^2\,]=\mathrm{L}^3$ So $\frac{8\pi^2h^3}{8h^3-ad^2}$ is dimensionless	E1 **1**	*Condone* '$\mathrm{L}^3/\mathrm{L}^3=0$, *dimensionless'* *But E0 for* $\frac{\mathrm{L}^3}{\mathrm{L}^3-\mathrm{L}^3}=\frac{\mathrm{L}^3}{0}$
(v)	$\mathrm{T}=\mathrm{M}^\alpha\mathrm{L}^\beta(\mathrm{M\,L\,T^{-2}})^\gamma$ $\gamma=-\frac{1}{2}$ $\alpha+\gamma=0$, so $\alpha=\frac{1}{2}$ $\beta+\gamma=0$, so $\beta=\frac{1}{2}$	B1 B1 B1 B1 **4**	For $[\,\lambda\,]=\mathrm{M\,L\,T^{-2}}$ If γ is wrong but non-zero, give B1 ft for $\alpha=\beta=-\gamma$
(vi)	$a=3.9$, $\lambda=573.3$, $d=4.8$, $h=2.6$, $m=15$ Period is $\sqrt{\frac{8\pi^2h^3}{8h^3-ad^2}}\,m^{1/2}a^{1/2}\lambda^{-1/2}=1.67$ s (3 sf)	M1 A1 cao **2**	Using formula with numerical α,β,γ (must use the complete formula)

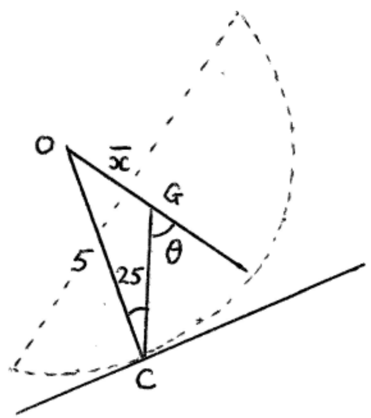
4 (i)	Area is $\int_0^3 (x^2+5)\,dx$	M1	For $\int (x^2+5)\,dx$
	$=\left[\frac{1}{3}x^3+5x\right]_0^3 \quad (=24)$	A1	For $\frac{1}{3}x^3+5x$
	$\int xy\,dx=\int_0^3 (x^3+5x)\,dx$	M1	For $\int xy\,dx$
	$=\left[\frac{1}{4}x^4+\frac{5}{2}x^2\right]_0^3 \quad (=\frac{171}{4})$	A1	For $\frac{1}{4}x^4+\frac{5}{2}x^2$
	$\bar{x}=\frac{42.75}{24}=\frac{57}{32}=1.78125$	A1	
	$\int \frac{1}{2}y^2\,dx=\int_0^3 \frac{1}{2}(x^4+10x^2+25)\,dx$	M1	For $\int y^2\,dx$
	$=\left[\frac{1}{10}x^5+\frac{5}{3}x^3+\frac{25}{2}x\right]_0^3 \quad (=106.8)$	A2	For $\frac{1}{10}x^5+\frac{5}{3}x^3+\frac{25}{2}x$ Give A1 for two terms correct
	$\bar{y}=\frac{106.8}{24}=\frac{89}{20}=4.45$	A1 **9**	
(ii)	Volume is $\int \pi x^2 dy=\int_5^{14}\pi(y-5)\,dy$	M1	For $\int (y-5)\,dy$
	$=\pi\left[\frac{1}{2}y^2-5y\right]_5^{14} \quad (=40.5\pi)$	A1	For $\left[\frac{1}{2}y^2-5y\right]_5^{14}$
	$\int \pi x^2 y\,dy=\int_5^{14}\pi(y^2-5y)\,dy$	M1	For $\int x^2 y\,dx$
	$=\pi\left[\frac{1}{3}y^3-\frac{5}{2}y^2\right]_5^{14} \quad (=445.5\pi)$	A1	For $\frac{1}{3}y^3-\frac{5}{2}y^2$
	$\bar{y}=\frac{445.5\pi}{40.5\pi}$	M1	*Dependent on previous M1M1*
	$=11$	A1 **6**	
(iii)	Volume of whole cylinder is $\pi\times 3^2\times 14=126\pi$		
	$126\pi\times 7=40.5\pi\times 11+(126\pi-40.5\pi)\times \bar{y}_A$	M1 A1	Using formula for composite body
	$\bar{y}_A=\frac{126\pi\times 7-40.5\pi\times 11}{126\pi-40.5\pi}$		
	$=\frac{97}{19}=5.105 \quad$ (4 sf)	A1 cao **3**	

Question			Answer	Marks	Guidance	
1	(iv)		$\mathrm{T} = \mathrm{M}^{\alpha}\,\mathrm{L}^{\beta}\,(\mathrm{M\,L\,T}^{-2})^{\gamma}$			
			$\gamma = -\frac{1}{2}$	B1	CAO	
			$\alpha + \gamma = 0,\quad \beta + \gamma = 0$	M1	Considering powers of M or L	
			$\alpha = \frac{1}{2},\quad \beta = \frac{1}{2}$	A1A1	FT $\alpha = -\gamma,\ \beta = -\gamma$ (provided non-zero)	
				[4]		
1	(v)		$0.718 = k(8)^{\frac{1}{2}}(0.4)^{\frac{1}{2}}(125)^{-\frac{1}{2}}$	M1	Obtaining equation for k	Or using ratio and powers
			$k = 4.4875$			
			$t = (4.4875)(75)^{\frac{1}{2}}(3)^{\frac{1}{2}}(20)^{-\frac{1}{2}}$	M1	Obtaining expression for new time	Or $\times\left(\frac{75}{8}\right)^{\frac{1}{2}}\times\left(\frac{3}{0.4}\right)^{\frac{1}{2}}\times\left(\frac{20}{125}\right)^{-\frac{1}{2}}$
			New time is 15.1 s (3 sf)	A1	CAO *No penalty for using* $b = 1.2$ *and* $b = 9$	
				[3]		
2	(a)	(i)	$R\cos 18° = 800 \times 9.8$ ($R = 8243$)	M1	Resolving vertically	Might also include F
				M1	Horizontal equation of motion	Might also include F
			$R\sin 18° = 800 \times \frac{v^2}{45}$	A1		
			$\tan 18° = \frac{v^2}{45 \times 9.8}$			
			Speed is $12.0\ \mathrm{m\,s^{-1}}$ (3 sf)	A1		
				[4]		
2	(a)	(ii)		M1	Resolving vertically (three terms)	
			$R\cos 18° = F\sin 18° + 800 \times 9.8$	A1		
				M1	Horizontal equation (three terms)	
			$R\sin 18° + F\cos 18° = 800 \times \frac{15^2}{45}$	A1		
				M1	Obtaining a value for F or R	*Dependent on previous M1M1*
			Frictional force is 1380 N (3 sf)	A1		
			Normal reaction is 8690 N (3 sf)	A1		
				[7]		

Question			Answer	Marks	Guidance	
2	(b)			M1	Equation involving KE and PE	$h=2.1$ implies M1 a is length of the string
			$\frac{1}{2}m(7^2-2.8^2)=mg(a+a\cos\theta)$	A1	Correct equation involving a and θ	(Might use angle with downward vertical or horizontal)
			$a(1+\cos\theta)=2.1$			
				M1	Radial equation of motion	Might also involve T
			$mg\cos\theta=m\times\frac{2.8^2}{a}$	A1	Correct equation involving a and θ	
			$a\cos\theta=0.8$			
				M1	Eliminating θ or a	*Dependent on previous M1M1*
			Length of string is 1.3 m	A1		
			Angle with upward vertical is 52.0° (3 sf)	A1		A0 for 128° or 38°
				[7]		
3	(i)		$\dot{x}=-A\omega\sin(\omega t-\phi)$	B1		
			$\ddot{x}=-A\omega^2\cos(\omega t-\phi)$	M1	Obtaining second derivative	*Allow one error*
			$\ddot{x}=-\omega^2(x-c)$	E1	Correctly shown	
				[3]		
3	(ii)		$c=10$	B1		
			$A=6$	B1	Accept $A=-6$	
			$\frac{2\pi}{\omega}=10$	M1	Using $\frac{2\pi}{\omega}$	*Or other complete method for finding ω*
			$\omega=\frac{\pi}{5}$	A1	Accept $\omega=-\frac{\pi}{5}$	*Allow* $\frac{2\pi}{10}$ *etc*
			$x=16$ when $t=3\Rightarrow 3\omega-\phi=0$	M1	Obtaining simple relationship between ϕ and ω. *NB $\phi=3$ is M0*	Or $x=10+6\cos\{\frac{\pi}{5}(t-3)\}$
			$\phi=\frac{3\pi}{5}$	A1	NB other values possible *If exact values not seen, give A0A1 for both $\omega=0.63$ and $\phi=1.9$*	e.g. $\phi=-\frac{7\pi}{5}$, $\phi=\frac{13\pi}{5}$, $x=10-6\cos(\frac{\pi}{5}t-\frac{8\pi}{5})$ etc
				[6]	*Max 5/6 if values are not consistent*	

Question			Answer	Marks	Guidance	
3	(iii)		Maximum speed is $A\omega$	M1	Or e.g. evaluating $\dot{x}$ when $t = 5.5$	
			Maximum speed is $\frac{6\pi}{5}$ or $3.77\ \mathrm{m\,s^{-1}}$ (3 sf)	A1	FT is $\lvert A\omega \rvert$ (must be positive)	
				[2]		
3	(iv)		When $t = 0$, height is 8.15 m (3 sf)	B1	FT is $c + A\cos\phi$ (provided $4 < x < 16$)	Must use radians
			$v = -\frac{6\pi}{5}\sin(\frac{\pi t}{5} - \frac{3\pi}{5})$	M1	Or $v^2 = \left(\frac{\pi}{5}\right)^2 (6^2 - 1.854^2)$	*Allow one error in differentiation*
			When $t = 0$, velocity is $3.59\ \mathrm{m\,s^{-1}}$ (3 sf)	A1	FT is $A\omega\sin\phi$ (must be positive)	($\phi = 3$ gives $x = 4.06$, $v = 0.532$)
				[3]		
3	(v)		When $t = 0$, $x = 8.146$			
			When $t = 14$, $x = 14.854$	M1	Finding x when $t = 14$	Correct (FT) value, or evidence of substitution, required ($\phi = 3$ gives $x = 15.3$)
				M1	$(16 - 14.854)$ used	Requires $4 < x(14) < 16$
			$(16 - 8.146) + 12 + 12 + (16 - 14.854)$	M1	Fully correct strategy	Also requires $4 < x(0) < 16$
			Distance is 33 m	A1	CAO	
				[4]		

Question			Answer	Marks	Guidance	
4	(a)		$A=\int_0^9 (3-\sqrt{x})\,\mathrm{d}x$	M1		
			$=\left[\ 3x-\frac{2}{3}x^{\frac{3}{2}}\ \right]_0^9 \quad (=9)$	A1	For $3x-\frac{2}{3}x^{\frac{3}{2}}$	
			$A\bar{x}=\int xy\,\mathrm{d}x=\int_0^9 x(3-\sqrt{x})\,\mathrm{d}x$	M1	For $\int xy\,\mathrm{d}x$	
			$=\left[\ \frac{3}{2}x^2-\frac{2}{5}x^{\frac{5}{2}}\ \right]_0^9 \quad (=24.3)$	A1	For $\frac{3}{2}x^2-\frac{2}{5}x^{\frac{5}{2}}$	
			$\bar{x}=\dfrac{24.3}{9}=2.7$	A1		
			$A\bar{y}=\int \frac{1}{2}y^2\,\mathrm{d}x=\int_0^9 \frac{1}{2}(3-\sqrt{x})^2\,\mathrm{d}x$	M1	For $\int \ldots y^2\,\mathrm{d}x$	Or $\int_{(0)}^{(3)} (3-y)^2\,y\,\mathrm{d}y$
				M1	Expanding (three terms) and integrating *(allow one error)*	
			$=\left[\ \frac{9}{2}x-2x^{\frac{3}{2}}+\frac{1}{4}x^2\ \right]_0^9 \quad (=6.75)$	A1	For $\frac{9}{2}x-2x^{\frac{3}{2}}+\frac{1}{4}x^2$	Or $\frac{9}{2}y^2-2y^3+\frac{1}{4}y^4$
			$\bar{y}=\dfrac{6.75}{9}=0.75$	A1		
				[9]		

Question			Answer	Marks	Guidance	
4	(b)	(i)	$V=\int_2^5 \pi(25-x^2)\,\mathrm{d}x$	M1	For $\int \ldots (25-x^2)\,\mathrm{d}x$	
			$=\pi\left[\,25x-\frac{1}{3}x^3\,\right]_2^5 \quad (=36\pi\,)$	A1	For $25x-\frac{1}{3}x^3$	
			$V\bar{x}=\int \pi xy^2\,\mathrm{d}x=\int_2^5 \pi x(25-x^2)\,\mathrm{d}x$	M1	For $\int xy^2\,\mathrm{d}x$	
			$=\pi\left[\,\frac{25}{2}x^2-\frac{1}{4}x^4\,\right]_2^5 \quad \left(=\frac{441\pi}{4}\,\right)$	A1	For $\frac{25}{2}x^2-\frac{1}{4}x^4$	
			$\bar{x}=\dfrac{\frac{441}{4}\pi}{36\pi}=\dfrac{49}{16} \quad (=3.0625)$	A1	Accept 3.1 from correct working	
				[5]		
4	(b)	(ii)				
				M1	CG is vertical *(may be implied)*	Lenient, if CG drawn. Needs to be quite accurate if CG not drawn
				M1	Using triangle OGC *or equivalent*	
			$\dfrac{\sin\theta}{5}=\dfrac{\sin 25°}{\bar{x}}$	M1		
			$\theta=43.6°$	A1	Accept art 43° or 44° from correct work FT is $\sin^{-1}\left(\dfrac{2.113}{\bar{x}}\right)$	Provided $2.113<\bar{x}<5$
				[4]		

Question			Answer	Marks	Guidance	
1	(a)		$A\omega = 5.1$	B1		
				M1	Using $v^2 = \omega^2(A^2 - x^2)$	
			$4.5^2 = \omega^2(A^2 - 6^2)$	A1		
			$4.5^2 = 5.1^2 - 36\omega^2$ $\omega = 0.4$	M1	Eliminating A or ω	
			Period ($\frac{2\pi}{\omega}$) is $5\pi = 15.7$ s (3 sf)	A1		Allow 5π
			Amplitude (A) is 12.75 m	A1 **[6]**		
1	(b)	(i)	$[\,F\,] = \mathrm{M\,L\,T^{-2}}$	B1		
			$[\,G\,] = \left[\dfrac{Fd^2}{m_1 m_2}\right] = \dfrac{\mathrm{M\,L\,T^{-2}\,L^2}}{\mathrm{M^2}}$	M1	Obtaining dimensions of G	
			$= \mathrm{M^{-1}\,L^3\,T^{-2}}$	A1 **[3]**		
1	(b)	(ii)	$\mathrm{T^{-1}} = (\mathrm{M^{-1}\,L^3\,T^{-2}})^\alpha\,\mathrm{M}^\beta\,\mathrm{L}^\gamma$			
			$\alpha = \frac{1}{2}$	B1		
			$-\alpha + \beta = 0$	M1	Considering powers of M	
			$\beta = \frac{1}{2}$	A1		
			$3\alpha + \gamma = 0$	M1	Considering powers of L	
			$\gamma = -\frac{3}{2}$	A1 **[5]**		*All marks FT from wrong [G] if comparable. No FT within part **(ii)**.*

Question			Answer	Marks	Guidance	
1	(b)	(iii)		M1M1	For $\left(\frac{4.86\times10^{14}}{2500}\right)^{\pm\frac{1}{2}}$ and $\left(\frac{30\,000}{50}\right)^{\pm\frac{3}{2}}$	*Requires* $\beta\neq0,\ \gamma\neq0$
			$\omega=2.0\times10^{-6}\times\left(\frac{4.86\times10^{14}}{2500}\right)^{\frac{1}{2}}\times\left(\frac{30\,000}{50}\right)^{-\frac{3}{2}}$	A1	Correct equation for ω	*FT if comparable*
		OR	$2.0\times10^{-6}=k\times G^{\frac{1}{2}}\times2500^{\frac{1}{2}}\times50^{-\frac{3}{2}}$		M1 *Requires* $\beta\neq0$ *or* $\gamma\neq0$	
			$kG^{\frac{1}{2}}=1.414\times10^{-5}$		M1 *Requires* $\beta\neq0$ *and* $\gamma\neq0$	Condone the use of any value for G (including $G=1$)
			$\omega=1.414\times10^{-5}\times(4.86\times10^{14})^{\frac{1}{2}}\times30\,000^{-\frac{3}{2}}$		A1 Correct equation for ω	*FT if comparable*
			Angular speed is 6.0×10^{-5} rad s^{-1}	A1 **[4]**	CAO	
2	(a)	(i)		M1	Equation involving initial KE, final KE and attempt at PE	
			$\frac{1}{2}mv^2-\frac{1}{2}m(1.2)^2=mg(0.8-0.8\cos\frac{1}{6}\pi)$	A1		
			$v^2=3.5407$			
			Radial component $\left(\frac{v^2}{0.8}\right)$ is 4.43 m s^{-2} (3 sf)	A1		
			$(\pm)mg\sin\frac{1}{6}\pi=ma_T$	M1	Allow M1 for $\cos\frac{1}{6}\pi$ used instead of $\sin\frac{1}{6}\pi$; but M0 for $a_T=mg\sin\frac{1}{6}\pi$	
			Tangential component is 4.9 m s^{-2}	A1 **[5]**	Allow $\frac{1}{2}g$	

Question			Answer	Marks	Guidance	
2	(a)	(ii)	$\frac{1}{2}mv^2 - \frac{1}{2}m(1.2)^2 = mg(0.8 - 0.8\cos\theta)$	M1	Equation involving initial KE, final KE and attempt at PE in general position	θ between OP and upward vertical Allow *mgh* for PE if *h* is linked to θ in later work
				M1	Equation involving resolved component of weight and v^2 / r	
			$mg\cos\theta - R = \dfrac{mv^2}{0.8}$	A1	*R* may be omitted	
			Leaves surface when $R = 0$	M1	May be implied	e.g. Implied by $mg\cos\theta = \dfrac{mv^2}{0.8}$
			$v^2 - 1.44 = 2\times 9.8\times 0.8(1 - \dfrac{v^2}{7.84})$	M1	Obtaining equation in *v* or θ *Dependent on previous M1M1M1*	$\cos\theta = \dfrac{107}{147} = 0.728$ $\theta = 0.756$ rad or $43.3°$
			Speed is 2.39 m s^{-1}	A1		
				[6]		
2	(b)		$T_R \sin\alpha + T_S \sin\beta = mg$	M1	Resolving vertically (three terms)	$\alpha = R\hat{Q}C = 53.1°$, $\beta = S\hat{Q}C = 16.3°$
			$0.8T_R + 0.28T_S = 0.9\times 9.8$ $(= 8.82)$	A1	Allow $\sin 53.1°$, etc	
			$T_R \cos\alpha + T_S \cos\beta = m\dfrac{v^2}{r}$	M1	Horizontal equation of motion	Three terms, and v^2 / r
			$0.6T_R + 0.96T_S = 0.9\times\dfrac{5^2}{2.4}$ $(= 9.375)$	A1		
				M1	Obtaining T_R or T_S	*Dependent on previous M1M1*
			Tension in string RQ is 9.737 N	A1		
			Tension in string SQ is 3.68 N	A1		
				[7]		

Question			Answer	Marks	Guidance	
3	(i)		Length of each string is 7.8 m	B1		
			$T=\frac{728}{6.4}(7.8-6.4)$	M1	Using Hooke's law	Must use extension
			Tension is 159.25 N	A1 **[3]**		
3	(ii)		$2T\cos\theta=mg$	M1	Resolving vertically	$\theta=\mathrm{X\hat{P}M}=67.4°$
			$2\times159.25\times\frac{5}{13}=m\times9.8$	A1	FT	
			$m=\frac{122.5}{9.8}=12.5\text{ kg}$	E1 **[3]**	*Working must lead to 12.5 to 3 sf*	
3	(iii)		New length of each string is 7.5 m	M1	Hooke's law with new extension	
			$T=\frac{728}{6.4}(7.5-6.4)\quad(=125.125)$	A1		
			$mg-2T\cos\theta=ma$	M1	Vertical equation of motion (3 terms)	
			$12.5\times9.8-2\times125.125\times0.28=12.5a$	A1	FT for incorrect T	
			Acceleration is 4.19 m s^{-2} downwards (3 sf)	A1 **[5]**	*Some indication of downwards required*	
3	(iv)		At maximum speed, acceleration is zero Acceleration is zero in equilibrium position	B1 **[1]**	Mention of zero acceleration	*Reference to* $v^2=\omega^2(A^2-x^2)$, *SHM, etc, will usually be B0*

Question			Answer	Marks	Guidance	
3	(v)		Change of PE is $12.5\times9.8\times3 \quad (=367.5)$	B1		
			Initial EE is $2\times\dfrac{728\times0.8^2}{2\times6.4} \quad (=72.8)$	B1	Allow one string (36.4)	
			Final EE is $2\times\dfrac{728\times1.4^2}{2\times6.4} \quad (=222.95)$	B1	Allow one string (111.475)	
				M1	Equation involving KE, PE and EE	
			$\frac{1}{2}(12.5)v^2-367.5+222.95=72.8$	A1	FT from any B0 above All signs must be correct	*All terms must be non-zero*
			Maximum speed is 5.90 m s^{-1} (3 sf)	A1	CAO	
				[6]		
4	(a)		$V=\int_0^h \pi(y^{\frac{1}{4}})^2\,dy$	M1	For $\int \dots (y^{\frac{1}{4}})^2\,dy$	
			$=\pi\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^h \quad (=\frac{2}{3}\pi h^{\frac{3}{2}})$	A1	For $\frac{2}{3}y^{\frac{3}{2}}$	
			$V\bar{y}=\int \pi x^2 y\,dy=\int_0^h \pi y^{\frac{1}{2}}y\,dy$	M1	For $\int x^2 y\,dy$	
			$=\pi\left[\frac{2}{5}y^{\frac{5}{2}}\right]_0^h \quad (=\frac{2}{5}\pi h^{\frac{5}{2}})$	A1	For $\frac{2}{5}y^{\frac{5}{2}}$	
			$\bar{y}=\dfrac{\frac{2}{5}\pi h^{\frac{5}{2}}}{\frac{2}{3}\pi h^{\frac{3}{2}}}=\frac{3}{5}h$	A1		
				[5]		

Question			Answer	Marks	Guidance	
4	(b)	(i)	$A=\int_0^4 (x+\sqrt{x})\,\mathrm{d}x$	M1	For $\int (x+\sqrt{x})\,\mathrm{d}x$	
			$=\left[\frac{1}{2}x^2+\frac{2}{3}x^{\frac{3}{2}}\right]_0^4 \quad (=\frac{40}{3})$	A1	For $\frac{1}{2}x^2+\frac{2}{3}x^{\frac{3}{2}}$	
			$A\bar{x}=\int xy\,\mathrm{d}x=\int_0^4 x(x+\sqrt{x})\,\mathrm{d}x$	M1	For $\int xy\,\mathrm{d}x$	
			$=\left[\frac{1}{3}x^3+\frac{2}{5}x^{\frac{5}{2}}\right]_0^4 \quad (=\frac{512}{15})$	A1	For $\frac{1}{3}x^3+\frac{2}{5}x^{\frac{5}{2}}$	
			$\bar{x}=\frac{\frac{512}{15}}{\frac{40}{3}}=\frac{64}{25}=2.56$	E1		
			$A\bar{y}=\int \frac{1}{2}y^2\,\mathrm{d}x=\int_0^4 \frac{1}{2}(x+\sqrt{x})^2\,\mathrm{d}x$	M1	For $\int \ldots y^2\,\mathrm{d}x$	
			$=\left[\frac{1}{6}x^3+\frac{2}{5}x^{\frac{5}{2}}+\frac{1}{4}x^2\right]_0^4 \quad (=\frac{412}{15})$	A1A1	For $\frac{1}{6}x^3+\frac{2}{5}x^{\frac{5}{2}}+\frac{1}{4}x^2$	Give A1 for two correct terms
			$\bar{y}=\frac{\frac{412}{15}}{\frac{40}{3}}=\frac{103}{50}=2.06$	A1		
				[9]		

Question			Answer	Marks	Guidance	
4	(b)	(ii)	Area of B is $24-\frac{40}{3}=\frac{32}{3}$ $\frac{32}{3}\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}+\frac{40}{3}\begin{pmatrix}2.56\\ 2.06\end{pmatrix}=24\begin{pmatrix}2\\ 3\end{pmatrix}$ $\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}1.3\\ 4.175\end{pmatrix}$	M1 M1 A1 A1	CM of composite body Correct strategy CAO FT requires $0<\bar{y}<6$	(One coordinate sufficient) *FT is* $6.75-1.25\bar{y}_A$ *No FT from wrong area*
		OR	$\int \frac{1}{4}\left(\sqrt{1+4y}-1\right)^2 y\,dy$ *or* $\int x(6-x-\sqrt{x})\,dx$ *or* $\int \frac{1}{32}\left(\sqrt{1+4y}-1\right)^4 dy$ *or* $\int \frac{1}{2}(6-x-\sqrt{x})(6+x+\sqrt{x})\,dx$ $\bar{x}=1.3,\quad \bar{y}=4.175$		M1 For any one of these M1 For one successful integration A1A1	
				[4]		

Question			Answer	Marks	Guidance	
1	(a)	(i)	$T\sin\theta = mr\omega^2$	M1	Equation involving $r\omega^2$ or $l\omega^2$	*All marks in (a) can be earned anywhere in (i) or (ii)*
			$r = 3.2\sin\theta$	B1		
			$T\sin\theta = (1.5)(3.2\sin\theta)(2.5)^2$	A1	$T = (1.5)(3.2)(2.5)^2$ *with no wrong working* earns M1B1A1	
			Tension is 30 N	A1		
				[4]		
1	(a)	(ii)	$T\cos\theta = mg$	M1	Resolving vertically	
			$30\cos\theta = 1.5\times 9.8$			
			Angle is 60.7° (3 sf)	A1	or 1.06 rad	
				[2]		
1	(b)	(i)	$[\, k\,] = (\mathrm{M\,L\,T^{-2}})\mathrm{L^{-1}} = \mathrm{M\,T^{-2}}$	E1	Can use $u = \sqrt{\dfrac{4kd^2}{3m}}$ or $k = \dfrac{\lambda}{l}$	
				[1]		
1	(b)	(ii)	$\left[\sqrt{\dfrac{4kd^2}{3m}}\right] = \left(\dfrac{\mathrm{M\,T^{-2}\,L^2}}{\mathrm{M}}\right)^{\frac{1}{2}} = \mathrm{L\,T^{-1}}$	M1	Obtaining dimensions of RHS	
			$[\, u\,] = \mathrm{L\,T^{-1}}$, so eqn is dimensionally consistent	E1	Condone circular argument	
				[2]		
1	(b)	(iii)	$\mathrm{T} = (\mathrm{M\,T^{-2}})^{\alpha}\,\mathrm{L}^{\beta}\,\mathrm{M}^{\gamma}$			
			$\alpha = -\frac{1}{2}$	B1		
			$\beta = 0$	B1		
			$\alpha + \gamma = 0$	M1	Considering powers of M	
			$\gamma = \frac{1}{2}$	A1	FT from wrong non-zero α	
				[4]		

Question			Answer	Marks	Guidance	
1	(b)	(iv)	$u=\sqrt{\dfrac{4\times 60\times 0.7^2}{3\times 5}}=2.8\text{ m s}^{-1}$	B1		
			Elastic energy is $\frac{1}{2}\times 60\times 0.7^2$ $(=14.7)$	M1A1	M1A0 if one error	
			$\frac{1}{2}(5)(2.8)^2-\frac{1}{2}(5)v^2=14.7$	M1	Equation involving initial KE, final KE and EE	
			Speed is 1.4 m s^{-1}	A1		*No FT in any part of Q1 except (iii)*
				[5]		
2	(i)			M1	Equation involving initial KE, final KE and PE	
			$\frac{1}{2}m(8.4)^2-\frac{1}{2}mv^2=mg(a-a\cos\theta)$	A1		$(m=0.25)$
			$v^2=70.56-19.6a(1-\cos\theta)$	A1		
				M1	Using acceleration $\dfrac{v^2}{a}$	
			$T-mg\cos\theta=m\dfrac{v^2}{a}$	A1		
			$T-2.45\cos\theta=0.25(\dfrac{70.56}{a}-19.6+19.6\cos\theta)$	M1	Equation relating T, a, θ	*Dependent on previous M1M1*
			$T-2.45\cos\theta=\dfrac{17.64}{a}-4.9+4.9\cos\theta$			
			$T=\dfrac{17.64}{a}+7.35\cos\theta-4.9$	E1		
				[7]		
2	(ii)		If $a=0.9$, $T=14.7+7.35\cos\theta$	M1	Expression for T when $a=0.9$	In terms of θ or when $\theta=\pi$
			$T>0$ for all θ, so P moves in a complete circle	E1	Any correct explanation	
				M1	Using $\theta=0$ or $\theta=\pi$	
			Maximum tension is $14.7+7.35=22.05$ N Minimum tension is $14.7-7.35=7.35$ N	A1	Both correct	
				[4]		

Question			Answer	Marks	Guidance	
2	(iii)		If P just completes the circle, $T=0$ when $\theta=\pi$	M1		
			$\frac{17.64}{a}-7.35-4.9=0$	A1		
			$a=1.44$	A1 **[3]**	For 1.44	Condone $a<1.44$ etc
2	(iv)		If $a=1.6$, $T=6.125+7.35\cos\theta$	M1	Using expression for T when $a=1.6$	
			String becomes slack when $T=0$	M1		
			$\cos\theta=-\frac{6.125}{7.35}=-\frac{5}{6}$ [$\theta=2.56$ rad or $146°$]			
			$v^2=70.56-19.6\times1.6(1+\frac{5}{6})$	M1	Obtaining an equation for v Or $-mg(-\frac{5}{6})=m\frac{v^2}{1.6}$	*Dependent on previous M1M1*
			Speed is $3.61\,\text{m}\,\text{s}^{-1}$ (3 sf)	A1 **[4]**		*No FT in any part of Q2*
3	(i)		$\frac{686(2.2-l)}{l}=18\times9.8$	M1	Using Hooke's law	
			Natural length is 1.75 m	A1 **[2]**		
3	(ii)		Tension in AP is $\frac{686}{1.75}(0.45+x)$	M1		
			$=176.4+392x$	E1		
			Thrust in BP is $\frac{145}{2.5}x$ $(=58x)$	B1 **[3]**	Allow $-58x$	Condone thrust / tension confusion

Question			Answer	Marks	Guidance	
3	(iii)			M1	Equation of motion	2 forces from (ii), *mg* and *ma*
			$18\times 9.8-(176.4+392x)-58x=18\dfrac{d^2x}{dt^2}$	A1	Correct LHS equated to $\pm 18a$	*No FT*
			$176.4-176.4-450x=18\dfrac{d^2x}{dt^2}$			
			$\dfrac{d^2x}{dt^2}=-25x$	E1	Fully correct derivation	
				[3]		
3	(iv)		Period is $\dfrac{2\pi}{5}=1.26\text{ s}$ (3 sf)	B1	Allow $\dfrac{2\pi}{5}$	
			$A\omega=3.4$	M1		
			Amplitude ($A=\dfrac{3.4}{5}$) is 0.68 m	A1		
				[3]		
3	(v)		$v=3.4\cos 5t$	M1	Using $\cos\omega t$ or $\sin\omega t$	$\cos\frac{2}{5}\pi t$ is M0
			When $t=2.4$, $v=2.87$			
			Magnitude of velocity is 2.87 m s^{-1} (3 sf)	A1		
			Since $v>0$ the direction is downwards	A1	*Dependent on M1A1*	'Downwards' is sufficient
				[3]		
		OR	When $t=2.4$, $x=-0.3649$			Earns B1M1 from (vi)
			$v^2=25(0.68^2-0.3649^2)$		M1 Using $v^2=\omega^2(A^2-x^2)$	
			Magnitude of velocity is 2.87 m s^{-1} (3 sf)		A1	*No FT*
			Between 1¾ and 2 periods; hence downwards		A1 *Dependent on M1A1*	Must be justified
3	(vi)		$x=0.68\sin 5t$	B1	FT (from wrong amplitude)	
			When $t=2.4$, $x=-0.3649$	M1		B1M1 can be earned in (v)
			2.4 s is $\dfrac{2.4}{1.26}=1.91$ periods (between 1¾ and 2)			
			Distance is $8\times 0.68-0.3649$	M1	$8A+x_{t=2.4}$ with $x_{t=2.4}<0$	Strictly, only for this
			Distance is 5.08 m (3 sf)	A1	FT is 7.463*A*	
				[4]		

Question			Answer	Marks	Guidance	
4	(a)	(i)	$V = \int_0^4 \pi x^2 (4-x)\,\mathrm{d}x$	M1	For $\int \left(x\sqrt{4-x}\right)^2 \mathrm{d}x$	π may be omitted throughout
			$= \pi\left[\frac{4}{3}x^3 - \frac{1}{4}x^4\right]_0^4 \quad (= \frac{64\pi}{3})$	A1	For $\frac{4}{3}x^3 - \frac{1}{4}x^4$	
			$V\bar{x} = \int \pi x y^2\,\mathrm{d}x = \int_0^4 \pi x^3 (4-x)\,\mathrm{d}x$	M1	For $\int xy^2\,\mathrm{d}x$	
			$= \pi\left[x^4 - \frac{1}{5}x^5\right]_0^4 \quad (= 51.2\pi)$	A1	For $x^4 - \frac{1}{5}x^5$	
			$\bar{x} = \frac{51.2\pi}{\frac{64}{3}\pi}$	M1	*Dependent on previous M1M1*	
			$= 2.4$	A1 **[6]**		
4	(a)	(ii)		M1	Taking moments	
			$W(2.4\sin\theta) = W(4\cos\theta)$	A1	FT Correct equation for required angle	$W(2.4\cos\phi) = W(4\sin\phi)$ is A0 unless $\theta = 90° - \phi$ also appears
			$\tan\theta = \frac{4}{2.4} = \frac{5}{3}$			
			$\theta = 59.0°$ (3 sf)	A1	FT is $\tan^{-1}\frac{4}{\bar{x}}$	FT requires $\bar{x} < 4$
				[3]		

Question			Answer	Marks	Guidance	
4	(b)		$x = 2 + y^{\frac{1}{3}}$	B1		
			$A = \int_0^8 (2 + y^{\frac{1}{3}})\,dy \quad = \left[\ 2y + \frac{3}{4}y^{\frac{4}{3}}\ \right]_0^8 \quad (= 28)$	B1	FT	Or $32 - \left[\ \frac{1}{4}(x-2)^4\ \right]_2^4$
			$A\overline{x} = \int \frac{1}{2}x^2\,dy = \int_0^8 \frac{1}{2}\left(4 + 4y^{\frac{1}{3}} + y^{\frac{2}{3}}\right)dy$	M1	For $\int x^2\,dy$	Or $32 \times 2 - \int_2^4 xy\,dx$
			$= \left[\ 2y + \frac{3}{2}y^{\frac{4}{3}} + \frac{3}{10}y^{\frac{5}{3}}\ \right]_0^8 \quad (= 49.6)$	B2	FT for $2y + \frac{3}{2}y^{\frac{4}{3}} + \frac{3}{10}y^{\frac{5}{3}}$ Give B1 for one minor slip in integration, or if ½ omitted	Or $\frac{1}{5}(x-2)^5 + \frac{1}{2}(x-2)^4$ Or $\frac{1}{4}x(x-2)^4 - \frac{1}{20}(x-2)^5$ Or $\frac{1}{5}x^5 - \frac{3}{2}x^4 + 4x^3 - 4x^2$
			$\overline{x} = \frac{49.6}{28} = \frac{62}{35} = 1.77 \quad$ (3 sf)	A1	CAO	*Must be* $\overline{x}$
			$A\overline{y} = \int xy\,dy = \int_0^8 \left(2y + y^{\frac{4}{3}}\right)dy$	M1	For $\int xy\,dy$	Or $32 \times 4 - \int_2^4 (\frac{1}{2})y^2\,dx$
			$= \left[\ y^2 + \frac{3}{7}y^{\frac{7}{3}}\ \right]_0^8 \quad (= \frac{832}{7})$	A1	FT for $y^2 + \frac{3}{7}y^{\frac{7}{3}}$	Or B2 for $\frac{1}{14}(x-2)^7$ Give B1 for one minor slip in integration, or if ½ omitted
			$\overline{y} = \frac{\frac{832}{7}}{28} = \frac{208}{49} = 4.24 \quad$ (3 sf)	A1	CAO	*Must be* $\overline{y}$
				[9]		
		OR	Region under curve has CM $(3.6,\ \frac{16}{7})$		B2B2	For integrals, as above
			$28\overline{x} + 4 \times 3.6 = 32 \times 2$		B1 (for 28) M1	
			$\overline{x} = 1.77$		A1	
			$28\overline{y} + 4 \times \frac{16}{7} = 32 \times 4$		M1	
			$\overline{y} = 4.24$		A1	