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2. [The centre of mass of a uniform hollow cone of height h is $\frac{1}{3}h$ above the base on the line from the centre of the base to the vertex.]

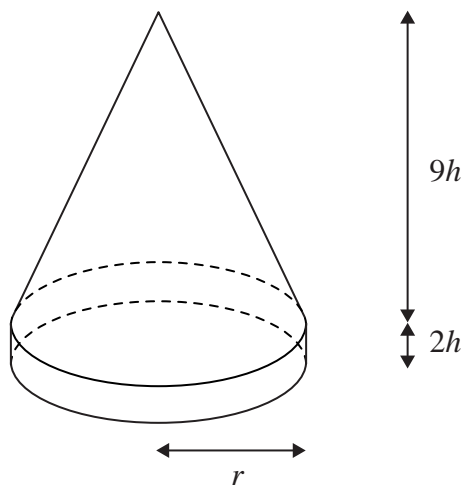


Figure 1

A marker for the route of a charity walk consists of a uniform hollow cone fixed on to a uniform solid cylindrical ring, as shown in Figure 1. The hollow cone has base radius r , height $9h$ and mass m . The solid cylindrical ring has outer radius r , height $2h$ and mass $3m$. The marker stands with its base on a horizontal surface.

- (a) Find, in terms of h , the distance of the centre of mass of the marker from the horizontal surface. **(5)**

When the marker stands on a plane inclined at $\arctan \frac{1}{12}$ to the horizontal it is on the point of toppling over. The coefficient of friction between the marker and the plane is large enough to be certain that the marker will not slip.

- (b) Find h in terms of r . **(3)**



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Question 2 continued

Lined writing area for the answer to Question 2.

Q2

(Total 8 marks)



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3.

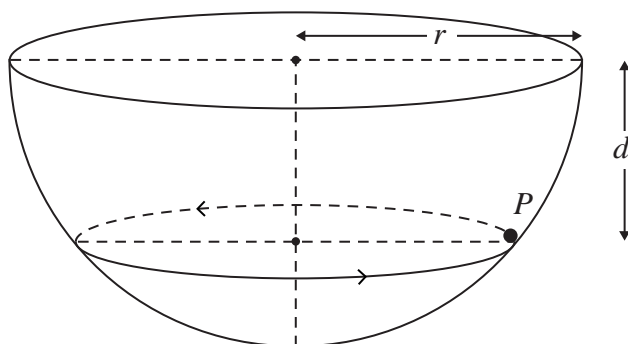


Figure 2

A particle P of mass m moves on the smooth inner surface of a hemispherical bowl of radius r . The bowl is fixed with its rim horizontal as shown in Figure 2. The particle moves with constant angular speed $\sqrt{\left(\frac{3g}{2r}\right)}$ in a horizontal circle at depth d below the centre of the bowl.

- (a) Find, in terms of m and g , the magnitude of the normal reaction of the bowl on P . **(4)**

- (b) Find d in terms of r . **(4)**

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Question 3 continued

Lined area for writing the answer to Question 3.



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5. One end of a light inextensible string of length l is attached to a fixed point A . The other end is attached to a particle P of mass m , which is held at a point B with the string taut and AP making an angle $\arccos \frac{1}{4}$ with the downward vertical. The particle is released from rest. When AP makes an angle θ with the downward vertical, the string is taut and the tension in the string is T .

(a) Show that

$$T = 3mg \cos \theta - \frac{mg}{2}$$

(6)

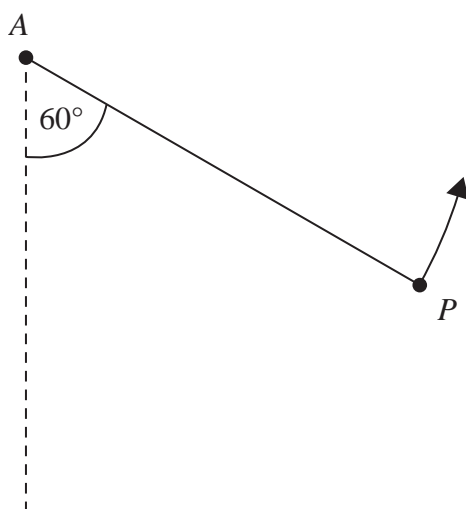


Figure 3

At an instant when AP makes an angle of 60° to the downward vertical, P is moving upwards, as shown in Figure 3. At this instant the string breaks. At the highest point reached in the subsequent motion, P is at a distance d below the horizontal through A .

(b) Find d in terms of l .

(5)

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- 6. A cyclist and her bicycle have a combined mass of 100 kg. She is working at a constant rate of 80 W and is moving in a straight line on a horizontal road. The resistance to motion is proportional to the square of her speed. Her initial speed is 4 m s⁻¹ and her maximum possible speed under these conditions is 20 m s⁻¹. When she is at a distance x m from a fixed point O on the road, she is moving with speed v m s⁻¹ away from O .

(a) Show that

$$v \frac{dv}{dx} = \frac{8000 - v^3}{10000v} \quad (5)$$

(b) Find the distance she travels as her speed increases from 4 m s⁻¹ to 8 m s⁻¹. (5)

(c) Use the trapezium rule, with 2 intervals, to estimate how long it takes for her speed to increase from 4 m s⁻¹ to 8 m s⁻¹. (4)



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7.

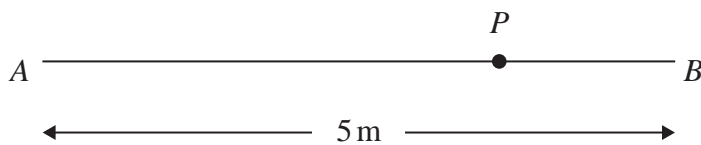


Figure 4

A and *B* are two points on a smooth horizontal floor, where $AB = 5 \text{ m}$.

A particle *P* has mass 0.5 kg . One end of a light elastic spring, of natural length 2 m and modulus of elasticity 16 N , is attached to *P* and the other end is attached to *A*. The ends of another light elastic spring, of natural length 1 m and modulus of elasticity 12 N , are attached to *P* and *B*, as shown in Figure 4.

- (a) Find the extensions in the two springs when the particle is at rest in equilibrium. (5)

Initially *P* is at rest in equilibrium. It is then set in motion and starts to move towards *B*. In the subsequent motion *P* does not reach *A* or *B*.

- (b) Show that *P* oscillates with simple harmonic motion about the equilibrium position. (4)

- (c) Given that the initial speed of *P* is $\sqrt{10} \text{ m s}^{-1}$, find the proportion of time in each complete oscillation for which *P* stays within 0.25 m of the equilibrium position. (7)



