

Edexcel Maths M3

Past Paper Pack

2007-2013

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1. A particle *P* moves along the *x*-axis. At time $t = 0$, *P* passes through the origin *O*, moving in the positive *x*-direction. At time t seconds, the velocity of *P* is $v \text{ m s}^{-1}$ and $OP = x$ metres. The acceleration of *P* is $\frac{1}{12}(30 - x) \text{ m s}^{-2}$, measured in the positive *x*-direction.

(a) Give a reason why the maximum speed of *P* occurs when $x = 30$. **(1)**

Given that the maximum speed of *P* is 10 m s^{-1} ,

(b) find an expression for v^2 in terms of x . **(5)**

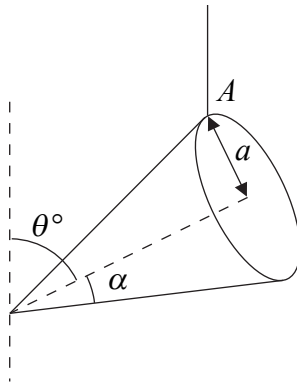
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Figure 1



A uniform solid right circular cone has base radius a and semi-vertical angle α , where $\tan \alpha = \frac{1}{3}$. The cone is freely suspended by a string attached at a point A on the rim of its base, and hangs in equilibrium with its axis of symmetry making an angle of θ° with the upward vertical, as shown in Figure 1.

Find, to one decimal place, the value of θ .



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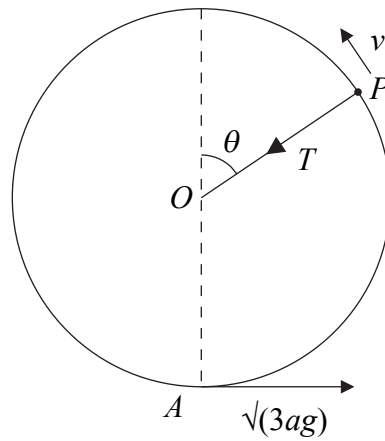
3. A particle *P* of mass *m* is attached to one end of a light elastic string, of natural length *a* and modulus of elasticity $3.6mg$. The other end of the string is fixed at a point *O* on a rough horizontal table. The particle is projected along the surface of the table from *O* with speed $\sqrt{2ag}$. At its furthest point from *O*, the particle is at the point *A*, where $OA = \frac{4}{3}a$.
- (a) Find, in terms of *m*, *g* and *a*, the elastic energy stored in the string when *P* is at *A*. **(3)**
- (b) Using the work-energy principle, or otherwise, find the coefficient of friction between *P* and the table. **(6)**



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Figure 2



A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a point O . The point A is vertically below O , and $OA = a$. The particle is projected horizontally from A with speed $\sqrt{3ag}$. When OP makes an angle θ with the upward vertical through O and the string is still taut, the tension in the string is T and the speed of P is v , as shown in Figure 2.

(a) Find, in terms of a, g and θ , an expression for v^2 . (3)

(b) Show that $T = (1 - 3 \cos \theta)mg$. (3)

The string becomes slack when P is at the point B .

(c) Find, in terms of a , the vertical height of B above A . (2)

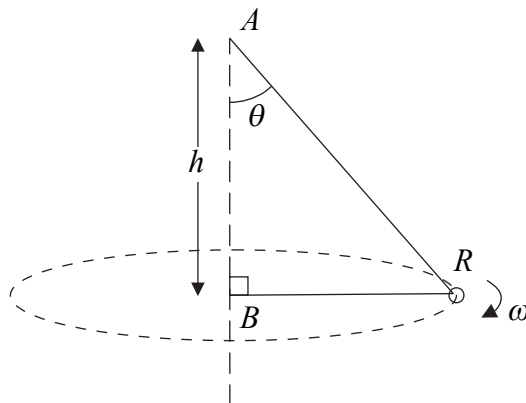
After the string becomes slack, the highest point reached by P is C .

(d) Find, in terms of a , the vertical height of C above B . (5)



5.

Figure 3



One end of a light inextensible string is attached to a fixed point A . The other end of the string is attached to a fixed point B , vertically below A , where $AB = h$. A small smooth ring R of mass m is threaded on the string. The ring R moves in a horizontal circle with centre B , as shown in Figure 3. The upper section of the string makes a constant angle θ with the downward vertical and R moves with constant angular speed ω . The ring is modelled as a particle.

(a) Show that $\omega^2 = \frac{g}{h} \left(\frac{1 + \sin \theta}{\sin \theta} \right)$. **(7)**

(b) Deduce that $\omega > \sqrt{\frac{2g}{h}}$. **(2)**

Given that $\omega = \sqrt{\frac{3g}{h}}$,

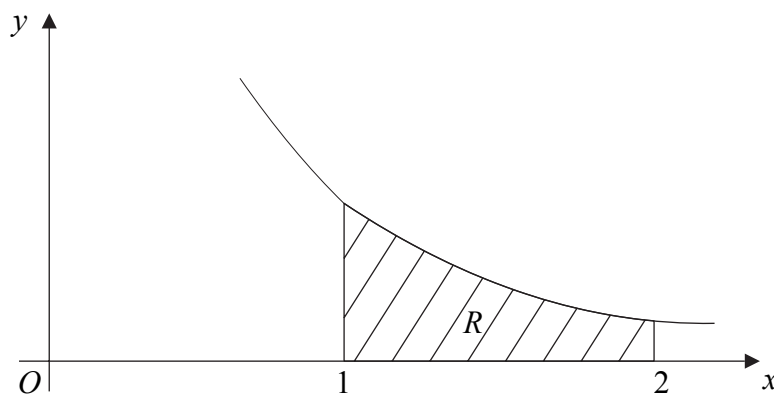
(c) find, in terms of m and g , the tension in the string. **(4)**



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Figure 4

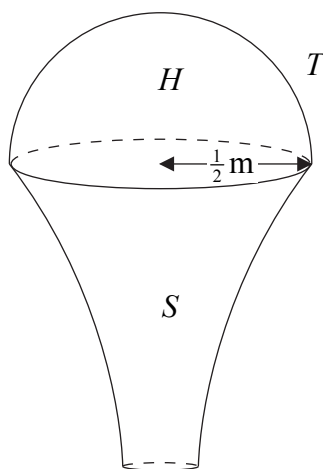


The shaded region R is bounded by the curve with equation $y = \frac{1}{2x^2}$, the x -axis and the lines $x = 1$ and $x = 2$, as shown in Figure 4. The unit of length on each axis is 1 m. A uniform solid S has the shape made by rotating R through 360° about the x -axis.

(a) Show that the centre of mass of S is $\frac{2}{7}$ m from its larger plane face.

(6)

Figure 5



A sporting trophy T is a uniform solid hemisphere H joined to the solid S . The hemisphere has radius $\frac{1}{2}$ m and its plane face coincides with the larger plane face of S , as shown in Figure 5. Both H and S are made of the same material.

(b) Find the distance of the centre of mass of T from its plane face.

(7)



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4. A light inextensible string of length l has one end attached to a fixed point A . The other end is attached to a particle P of mass m . The particle moves with constant speed v in a horizontal circle with the string taut. The centre of the circle is vertically below A and the radius of the circle is r .

Show that

$$gr^2 = v^2 \sqrt{(l^2 - r^2)}. \quad (9)$$



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5. A particle P moves on the x -axis with simple harmonic motion about the origin O as centre. When P is a distance 0.04 m from O , its speed is 0.2 m s^{-1} and the magnitude of its acceleration is 1 m s^{-2} .

(a) Find the period of the motion. (3)

The amplitude of the motion is a metres.

Find

(b) the value of a , (3)

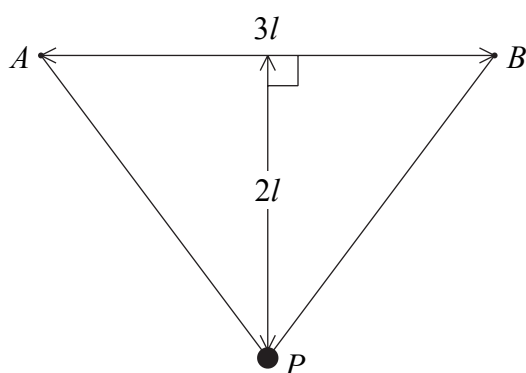
(c) the total time, within one complete oscillation, for which the distance OP is greater than $\frac{1}{2}a$ metres. (5)



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Figure 1



A light elastic string, of natural length $3l$ and modulus of elasticity λ , has its ends attached to two points A and B , where $AB = 3l$ and AB is horizontal. A particle P of mass m is attached to the mid-point of the string. Given that P rests in equilibrium at a distance $2l$ below AB , as shown in Figure 1,

- (a) show that $\lambda = \frac{15mg}{16}$. (9)

The particle is pulled vertically downwards from its equilibrium position until the total length of the elastic string is $7.8l$. The particle is released from rest.

- (b) Show that P comes to instantaneous rest on the line AB . (6)



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2. A particle P of mass 0.1 kg moves in a straight line on a smooth horizontal table. When P is a distance x metres from a fixed point O on the line, it experiences a force of magnitude $\frac{16}{5x^2}$ N away from O in the direction OP . Initially P is at a point 2 m from O and is moving towards O with speed 8 m s^{-1} .

Find the distance of P from O when P first comes to rest.

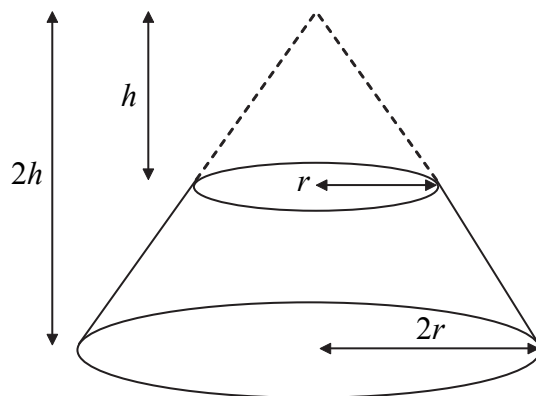
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Figure 1



A uniform solid S is formed by taking a uniform solid right circular cone, of base radius $2r$ and height $2h$, and removing the cone, with base radius r and height h , which has the same vertex as the original cone, as shown in Figure 1.

(a) Show that the distance of the centre of mass of S from its larger plane face is $\frac{11}{28}h$. (5)

The solid S lies with its larger plane face on a rough table which is inclined at an angle θ° to the horizontal. The table is sufficiently rough to prevent S from slipping.
Given that $h = 2r$,

(b) find the greatest value of θ for which S does not topple. (3)



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5. A car of mass m moves in a circular path of radius 75 m round a bend in a road. The maximum speed at which it can move without slipping sideways on the road is 21 m s^{-1} . Given that this section of the road is horizontal,

(a) show that the coefficient of friction between the car and the road is 0.6. (3)

The car comes to another bend in the road. The car's path now forms an arc of a horizontal circle of radius 44 m. The road is banked at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the car and the road is again 0.6. The car moves at its maximum speed without slipping sideways.

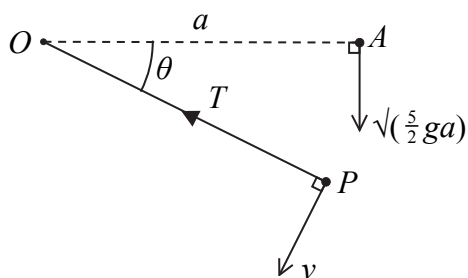
(b) Find, as a multiple of mg , the normal reaction between the car and road as the car moves round this bend. (4)

(c) Find the speed of the car as it goes round this bend. (5)



6.

Figure 2



A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . At time $t = 0$, P is projected vertically downwards with speed $\sqrt{\frac{5}{2}ga}$ from a point A which is at the same level as O and a distance a from O . When the string has turned through an angle θ and the string is still taut, the speed of P is v and the tension in the string is T , as shown in Figure 2.

(a) Show that $v^2 = \frac{ga}{2}(5 + 4\sin\theta)$. (3)

(b) Find T in terms of m , g and θ . (3)

The string becomes slack when $\theta = \alpha$.

(c) Find the value of α . (3)

The particle is projected again from A with the same velocity as before. When P is at the same level as O for the first time after leaving A , the string meets a small smooth peg B which has been fixed at a distance $\frac{1}{2}a$ from O . The particle now moves on an arc of a circle centre B . Given that the particle reaches the point C , which is $\frac{1}{2}a$ vertically above the point B , without the string going slack,

(d) find the tension in the string when P is at the point C . (6)

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Question 7 continued

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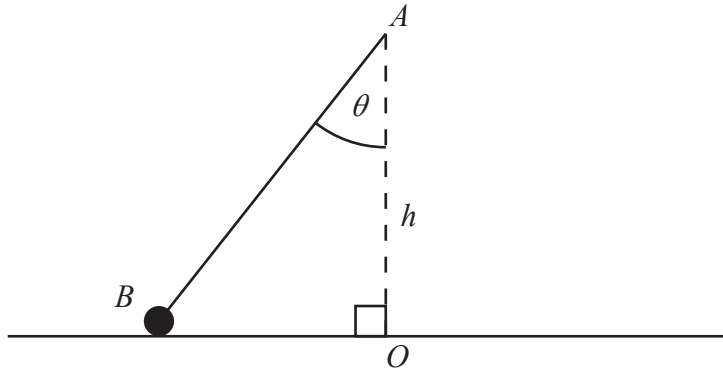


Figure 2

Figure 2 shows a particle B , of mass m , attached to one end of a light elastic string. The other end of the string is attached to a fixed point A , at a distance h vertically above a smooth horizontal table. The particle moves on the table in a horizontal circle with centre O , where O is vertically below A . The string makes a constant angle θ with the downward vertical and B moves with constant angular speed ω about OA .

- (a) Show that $\omega^2 \leq \frac{g}{h}$. (8)

The elastic string has natural length h and modulus of elasticity $2mg$.

Given that $\tan \theta = \frac{3}{4}$,

- (b) find ω in terms of g and h . (5)



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Question 3 continued

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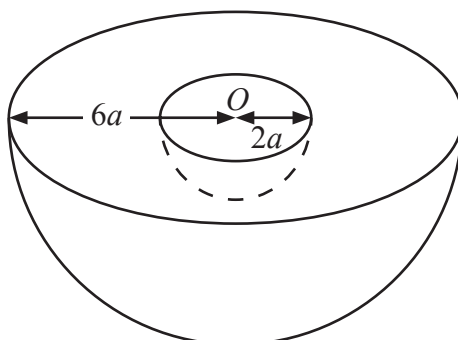


Figure 3

A uniform solid hemisphere, of radius $6a$ and centre O , has a solid hemisphere of radius $2a$, and centre O , removed to form a bowl B as shown in Figure 3.

(a) Show that the centre of mass of B is $\frac{30}{13}a$ from O . (5)

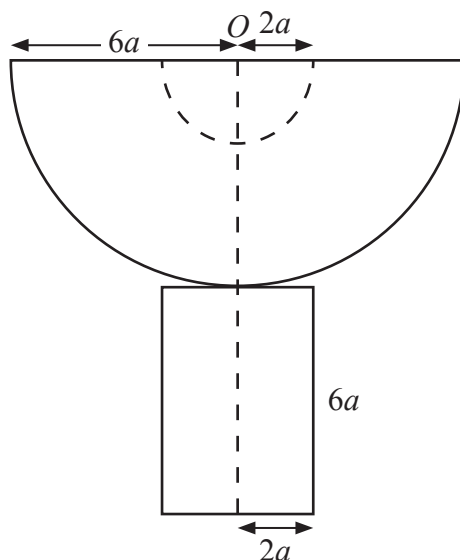


Figure 4

The bowl B is fixed to a plane face of a uniform solid cylinder made from the same material as B . The cylinder has radius $2a$ and height $6a$ and the combined solid S has an axis of symmetry which passes through O , as shown in Figure 4.

(b) Show that the centre of mass of S is $\frac{201}{61}a$ from O . (4)

The plane surface of the cylindrical base of S is placed on a rough plane inclined at 12° to the horizontal. The plane is sufficiently rough to prevent slipping.

(c) Determine whether or not S will topple. (4)



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Question 4 continued

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5. A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle is released from rest with the string taut and OP horizontal.

(a) Find the tension in the string when OP makes an angle of 60° with the downward vertical.

(6)

A particle Q of mass $3m$ is at rest at a distance a vertically below O . When P strikes Q the particles join together and the combined particle of mass $4m$ starts to move in a vertical circle with initial speed u .

(b) Show that $u = \sqrt{\left(\frac{ga}{8}\right)}$.

(3)

The combined particle comes to instantaneous rest at A .

(c) Find

(i) the angle that the string makes with the downward vertical when the combined particle is at A ,

(ii) the tension in the string when the combined particle is at A .

(6)



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Question 5 continued

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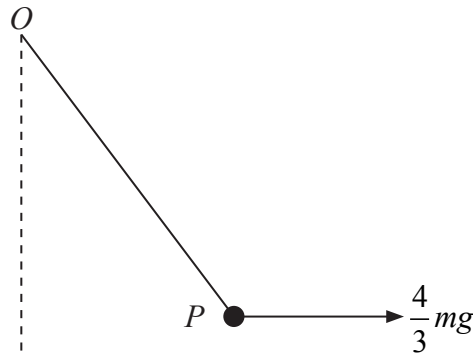


Figure 1

A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $3mg$. The other end of the string is attached to a fixed point O .

The particle P is held in equilibrium by a horizontal force of magnitude $\frac{4}{3}mg$ applied to P .

This force acts in the vertical plane containing the string, as shown in Figure 1. Find

(a) the tension in the string, (5)

(b) the elastic energy stored in the string. (4)



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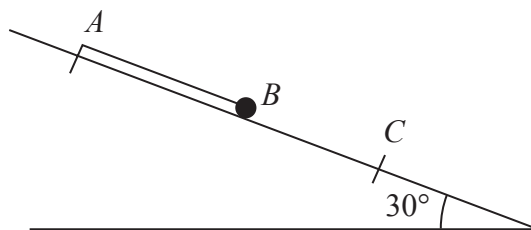


Figure 2

One end A of a light elastic string, of natural length a and modulus of elasticity $6mg$, is fixed at a point on a smooth plane inclined at 30° to the horizontal. A small ball B of mass m is attached to the other end of the string. Initially B is held at rest with the string lying along a line of greatest slope of the plane, with B below A and $AB = a$. The ball is released and comes to instantaneous rest at a point C on the plane, as shown in Figure 2. Find

- (a) the length AC , (5)
- (b) the greatest speed attained by B as it moves from its initial position to C . (7)



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Question 5 continued

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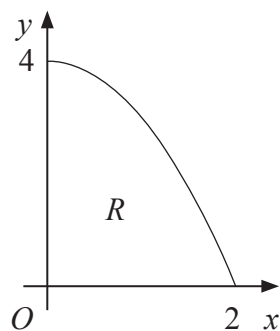


Figure 3

The region R is bounded by part of the curve with equation $y = 4 - x^2$, the positive x -axis and the positive y -axis, as shown in Figure 3. The unit of length on both axes is one metre. A uniform solid S is formed by rotating R through 360° about the x -axis.

- (a) Show that the centre of mass of S is $\frac{5}{8}$ m from O . (10)

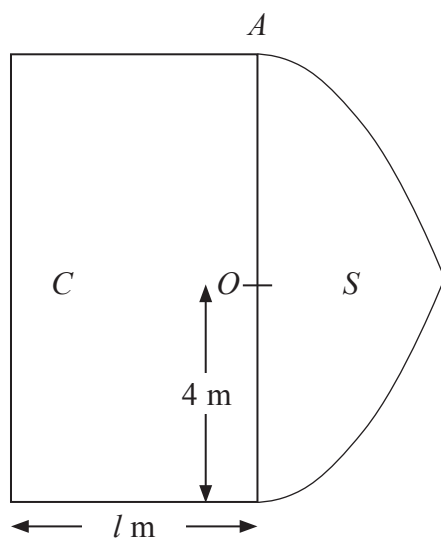


Figure 4

Figure 4 shows a cross section of a uniform solid P consisting of two components, a solid cylinder C and the solid S . The cylinder C has radius 4 m and length l metres. One end of C coincides with the plane circular face of S . The point A is on the circumference of the circular face common to C and S . When the solid P is freely suspended from A , the solid P hangs with its axis of symmetry horizontal.

- (b) Find the value of l . (4)



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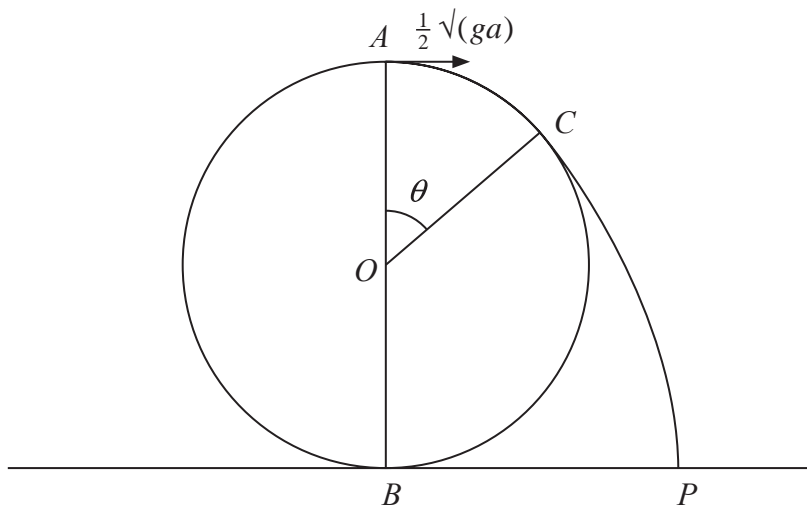


Figure 5

A particle is projected from the highest point A on the outer surface of a fixed smooth sphere of radius a and centre O . The lowest point B of the sphere is fixed to a horizontal plane. The particle is projected horizontally from A with speed $\frac{1}{2}\sqrt{ga}$. The particle leaves the surface of the sphere at the point C , where $\angle AOC = \theta$, and strikes the plane at the point P , as shown in Figure 5.

(a) Show that $\cos \theta = \frac{3}{4}$. (7)

(b) Find the angle that the velocity of the particle makes with the horizontal as it reaches P . (8)



1. A light elastic string has natural length 8 m and modulus of elasticity 80 N.

The ends of the string are attached to fixed points P and Q which are on the same horizontal level and 12 m apart. A particle is attached to the mid-point of the string and hangs in equilibrium at a point 4.5 m below PQ .

(a) Calculate the weight of the particle. (6)

(b) Calculate the elastic energy in the string when the particle is in this position. (3)



2. [The centre of mass of a uniform hollow cone of height h is $\frac{1}{3}h$ above the base on the line from the centre of the base to the vertex.]

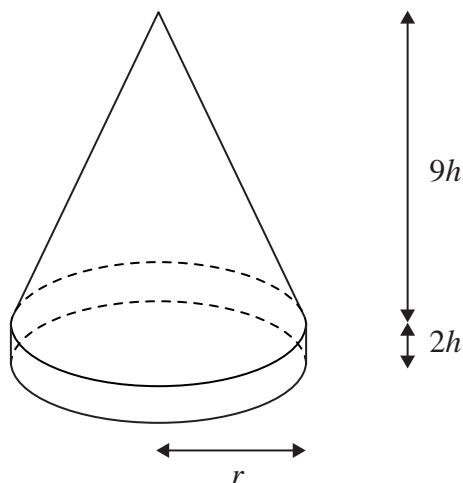


Figure 1

A marker for the route of a charity walk consists of a uniform hollow cone fixed on to a uniform solid cylindrical ring, as shown in Figure 1. The hollow cone has base radius r , height $9h$ and mass m . The solid cylindrical ring has outer radius r , height $2h$ and mass $3m$. The marker stands with its base on a horizontal surface.

- (a) Find, in terms of h , the distance of the centre of mass of the marker from the horizontal surface. (5)

When the marker stands on a plane inclined at $\arctan \frac{1}{12}$ to the horizontal it is on the point of toppling over. The coefficient of friction between the marker and the plane is large enough to be certain that the marker will not slip.

- (b) Find h in terms of r . (3)



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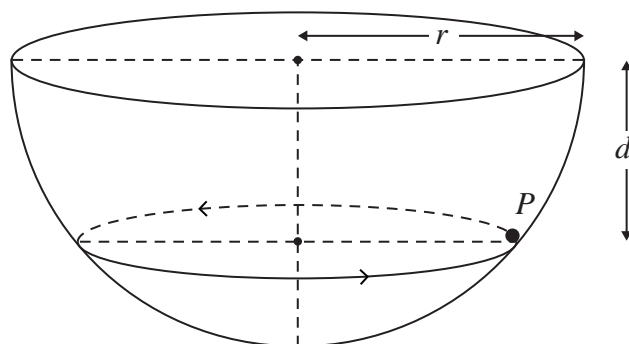


Figure 2

A particle P of mass m moves on the smooth inner surface of a hemispherical bowl of radius r . The bowl is fixed with its rim horizontal as shown in Figure 2. The particle moves with constant angular speed $\sqrt{\left(\frac{3g}{2r}\right)}$ in a horizontal circle at depth d below the centre of the bowl.

- (a) Find, in terms of m and g , the magnitude of the normal reaction of the bowl on P . (4)
- (b) Find d in terms of r . (4)



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4. The finite region bounded by the x -axis, the curve $y = \frac{1}{x^2}$, the line $x = \frac{1}{4}$ and the line $x = 1$, is rotated through one complete revolution about the x -axis to form a uniform solid of revolution.

(a) Show that the volume of the solid is 21π .

(4)

(b) Find the coordinates of the centre of mass of the solid.

(5)



5. One end of a light inextensible string of length l is attached to a fixed point A . The other end is attached to a particle P of mass m , which is held at a point B with the string taut and AP making an angle $\arccos \frac{1}{4}$ with the downward vertical. The particle is released from rest. When AP makes an angle θ with the downward vertical, the string is taut and the tension in the string is T .

- (a) Show that

$$T = 3mg \cos \theta - \frac{mg}{2}.$$

(6)

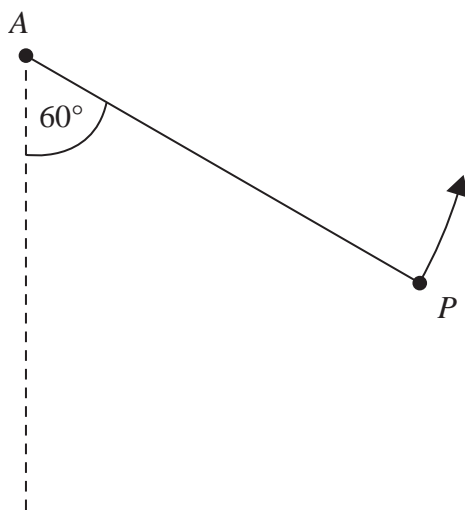


Figure 3

At an instant when AP makes an angle of 60° to the downward vertical, P is moving upwards, as shown in Figure 3. At this instant the string breaks. At the highest point reached in the subsequent motion, P is at a distance d below the horizontal through A .

- (b) Find d in terms of l .

(5)



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6. A cyclist and her bicycle have a combined mass of 100 kg. She is working at a constant rate of 80 W and is moving in a straight line on a horizontal road. The resistance to motion is proportional to the square of her speed. Her initial speed is 4 m s⁻¹ and her maximum possible speed under these conditions is 20 m s⁻¹. When she is at a distance x m from a fixed point O on the road, she is moving with speed v m s⁻¹ away from O .

(a) Show that

$$v \frac{dv}{dx} = \frac{8000 - v^3}{10000v} .$$

(5)

(b) Find the distance she travels as her speed increases from 4 m s⁻¹ to 8 m s⁻¹.

(5)

(c) Use the trapezium rule, with 2 intervals, to estimate how long it takes for her speed to increase from 4 m s⁻¹ to 8 m s⁻¹.

(4)



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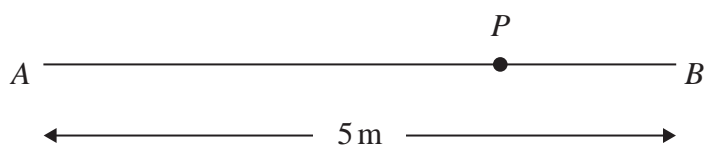


Figure 4

A and B are two points on a smooth horizontal floor, where $AB = 5$ m.

A particle P has mass 0.5 kg. One end of a light elastic spring, of natural length 2 m and modulus of elasticity 16 N, is attached to P and the other end is attached to A . The ends of another light elastic spring, of natural length 1 m and modulus of elasticity 12 N, are attached to P and B , as shown in Figure 4.

(a) Find the extensions in the two springs when the particle is at rest in equilibrium.

(5)

Initially P is at rest in equilibrium. It is then set in motion and starts to move towards B . In the subsequent motion P does not reach A or B .

(b) Show that P oscillates with simple harmonic motion about the equilibrium position.

(4)

(c) Given that the initial speed of P is $\sqrt{10}$ m s⁻¹, find the proportion of time in each complete oscillation for which P stays within 0.25 m of the equilibrium position.

(7)



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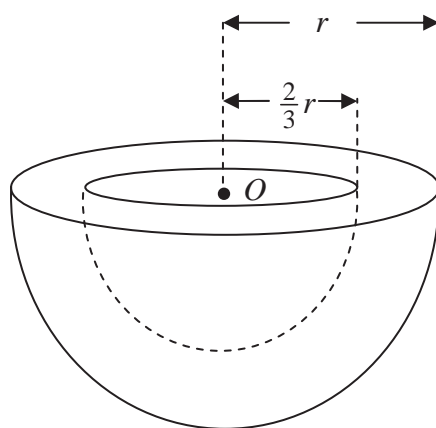


Figure 1

A bowl B consists of a uniform solid hemisphere, of radius r and centre O , from which is removed a solid hemisphere, of radius $\frac{2}{3}r$ and centre O , as shown in Figure 1.

- (a) Show that the distance of the centre of mass of B from O is $\frac{65}{152}r$. (5)

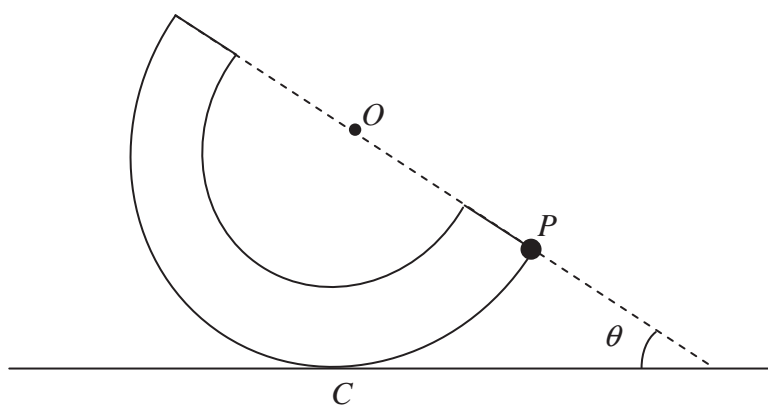


Figure 2

The bowl B has mass M . A particle of mass kM is attached to a point P on the outer rim of B . The system is placed with a point C on its outer curved surface in contact with a horizontal plane. The system is in equilibrium with P , O and C in the same vertical plane. The line OP makes an angle θ with the horizontal as shown in Figure 2. Given that

$$\tan \theta = \frac{4}{5},$$

- (b) find the exact value of k . (5)



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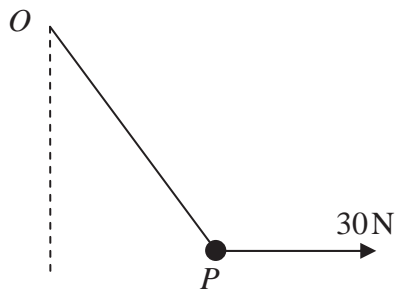


Figure 3

A particle P of weight 40 N is attached to one end of a light elastic string of natural length 0.5 m . The other end of the string is attached to a fixed point O . A horizontal force of magnitude 30 N is applied to P , as shown in Figure 3. The particle P is in equilibrium and the elastic energy stored in the string is 10 J .

Calculate the length OP .

(10)



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Question 4 continued

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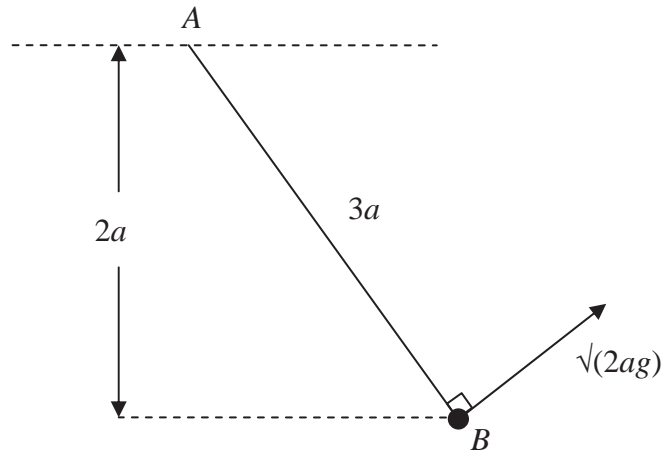


Figure 4

One end A of a light inextensible string of length $3a$ is attached to a fixed point. A particle of mass m is attached to the other end B of the string. The particle is held in equilibrium at a distance $2a$ below the horizontal through A , with the string taut. The particle is then projected with speed $\sqrt{2ag}$, in the direction perpendicular to AB , in the vertical plane containing A and B , as shown in Figure 4. In the subsequent motion the string remains taut. When AB is at an angle θ below the horizontal, the speed of the particle is v and the tension in the string is T .

(a) Show that $v^2 = 2ag(3 \sin \theta - 1)$. (5)

(b) Find the range of values of T . (6)



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Question 5 continued

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7. A light elastic string has natural length a and modulus of elasticity $\frac{3}{2}mg$. A particle P of mass m is attached to one end of the string. The other end of the string is attached to a fixed point A . The particle is released from rest at A and falls vertically. When P has fallen a distance $a + x$, where $x > 0$, the speed of P is v .

(a) Show that $v^2 = 2g(a + x) - \frac{3gx^2}{2a}$. **(4)**

(b) Find the greatest speed attained by P as it falls. **(4)**

After release, P next comes to instantaneous rest at a point D .

(c) Find the magnitude of the acceleration of P at D . **(6)**



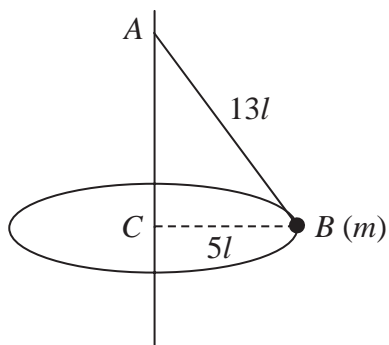
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Question 7 continued

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**Figure 1**

A garden game is played with a small ball B of mass m attached to one end of a light inextensible string of length $13l$. The other end of the string is fixed to a point A on a vertical pole as shown in Figure 1. The ball is hit and moves with constant speed in a horizontal circle of radius $5l$ and centre C , where C is vertically below A . Modelling the ball as a particle, find

(a) the tension in the string, (3)

(b) the speed of the ball. (4)



2. A particle P of mass m is above the surface of the Earth at distance x from the centre of the Earth. The Earth exerts a gravitational force on P . The magnitude of this force is inversely proportional to x^2 .

At the surface of the Earth the acceleration due to gravity is g . The Earth is modelled as a sphere of radius R .

- (a) Prove that the magnitude of the gravitational force on P is $\frac{mgR^2}{x^2}$. (3)

A particle is fired vertically upwards from the surface of the Earth with initial speed $3U$. At a height R above the surface of the Earth the speed of the particle is U .

- (b) Find U in terms of g and R . (7)



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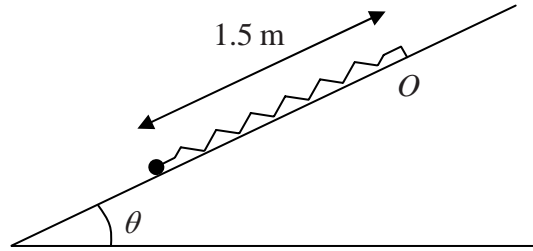


Figure 2

A particle of mass 0.5 kg is attached to one end of a light elastic spring of natural length 0.9 m and modulus of elasticity λ newtons. The other end of the spring is attached to a fixed point O on a rough plane which is inclined at an angle θ to the horizontal, where $\sin \theta = \frac{3}{5}$. The coefficient of friction between the particle and the plane is 0.15. The particle is held on the plane at a point which is 1.5 m down the line of greatest slope from O , as shown in Figure 2. The particle is released from rest and first comes to rest again after moving 0.7 m up the plane.

Find the value of λ .

(9)



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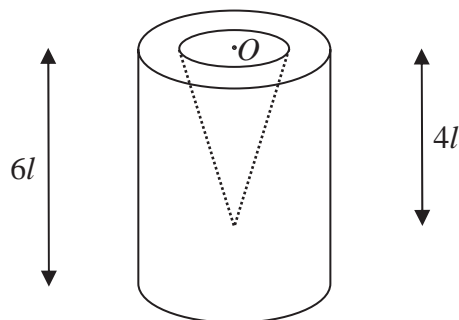


Figure 3

A container is formed by removing a right circular solid cone of height $4l$ from a uniform solid right circular cylinder of height $6l$. The centre O of the plane face of the cone coincides with the centre of a plane face of the cylinder and the axis of the cone coincides with the axis of the cylinder, as shown in Figure 3. The cylinder has radius $2l$ and the base of the cone has radius l .

(a) Find the distance of the centre of mass of the container from O .

(6)

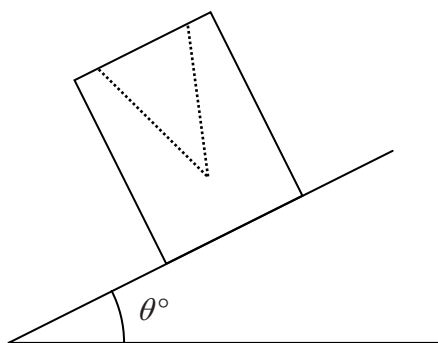


Figure 4

The container is placed on a plane which is inclined at an angle θ° to the horizontal. The open face is uppermost, as shown in Figure 4. The plane is sufficiently rough to prevent the container from sliding. The container is on the point of toppling.

(b) Find the value of θ .

(4)



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Question 4 continued

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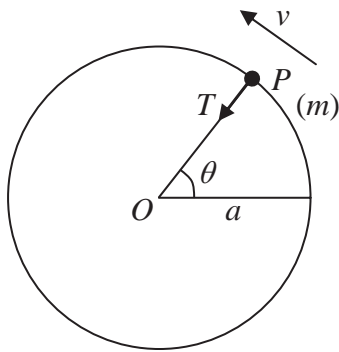


Figure 5

A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is fixed at the point O . The particle is initially held with OP horizontal and the string taut. It is then projected vertically upwards with speed u , where $u^2 = 5ag$. When OP has turned through an angle θ the speed of P is v and the tension in the string is T , as shown in Figure 5.

(a) Find, in terms of a , g and θ , an expression for v^2 . (3)

(b) Find, in terms of m , g and θ , an expression for T . (4)

(c) Prove that P moves in a complete circle. (3)

(d) Find the maximum speed of P . (2)



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7. A light elastic string, of natural length $3a$ and modulus of elasticity $6mg$, has one end attached to a fixed point A . A particle P of mass $2m$ is attached to the other end of the string and hangs in equilibrium at the point O , vertically below A .

(a) Find the distance AO .

(3)

The particle is now raised to point C vertically below A , where $AC > 3a$, and is released from rest.

(b) Show that P moves with simple harmonic motion of period $2\pi\sqrt{\left(\frac{a}{g}\right)}$.

(5)

It is given that $OC = \frac{1}{4}a$.

(c) Find the greatest speed of P during the motion.

(3)

The point D is vertically above O and $OD = \frac{1}{8}a$. The string is cut as P passes through D , moving upwards.

(d) Find the greatest height of P above O in the subsequent motion.

(4)



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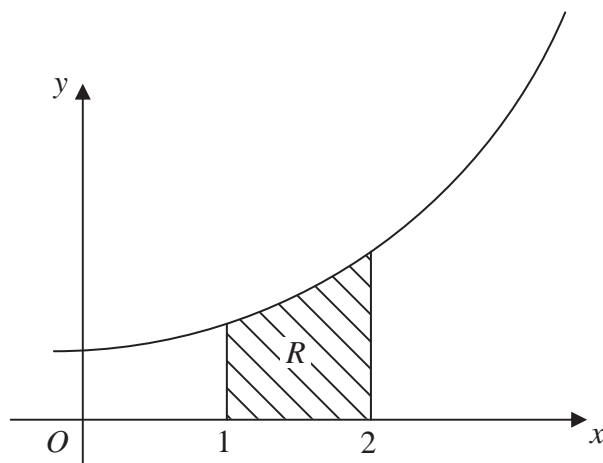


Figure 2

The region R is bounded by the curve with equation $y = e^x$, the line $x = 1$, the line $x = 2$ and the x -axis as shown in Figure 2. A uniform solid S is formed by rotating R through 2π about the x -axis.

(a) Show that the volume of S is $\frac{1}{2}\pi(e^4 - e^2)$. (4)

(b) Find, to 3 significant figures, the x -coordinate of the centre of mass of S . (6)



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Question 4 continued

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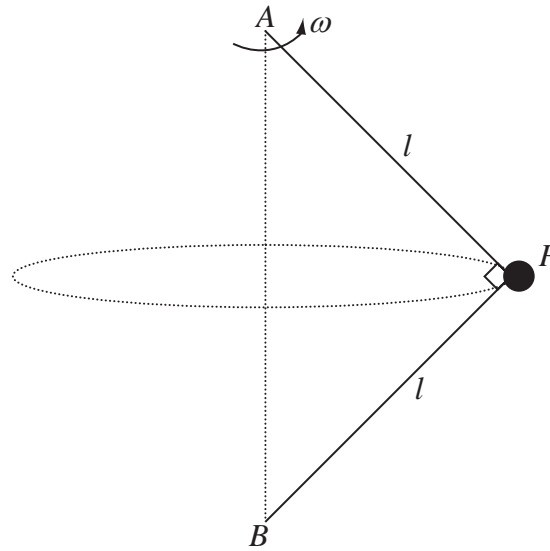


Figure 3

A small ball P of mass m is attached to the ends of two light inextensible strings of length l . The other ends of the strings are attached to fixed points A and B , where A is vertically above B . Both strings are taut and AP is perpendicular to BP as shown in Figure 3. The system rotates about the line AB with constant angular speed ω . The ball moves in a horizontal circle.

(a) Find, in terms of m , g , l and ω , the tension in AP and the tension in BP . (8)

(b) Show that $\omega^2 > \frac{g\sqrt{2}}{l}$. (2)



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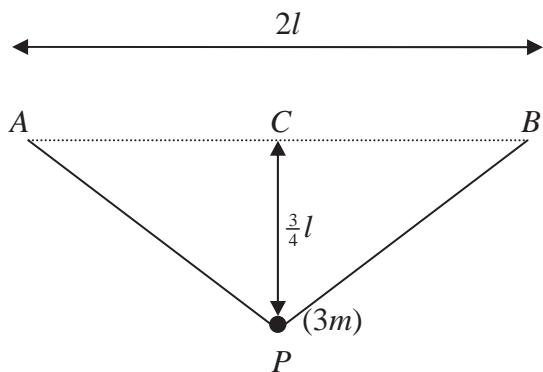


Figure 4

A small ball of mass $3m$ is attached to the ends of two light elastic strings AP and BP , each of natural length l and modulus of elasticity kmg . The ends A and B of the strings are attached to fixed points on the same horizontal level, with $AB = 2l$. The mid-point of AB is C . The ball hangs in equilibrium at a distance $\frac{3}{4}l$ vertically below C as shown in Figure 4.

(a) Show that $k = 10$ (7)

The ball is now pulled vertically downwards until it is at a distance $\frac{12}{5}l$ below C . The ball is released from rest.

(b) Find the speed of the ball as it reaches C . (6)

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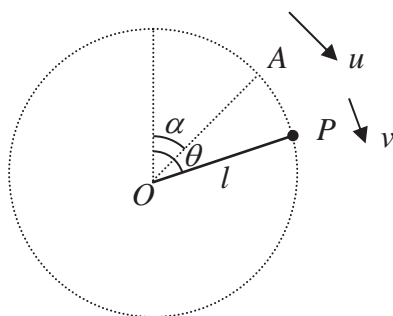


Figure 5

A particle P of mass m is attached to one end of a light rod of length l . The other end of the rod is attached to a fixed point O . The rod can turn freely in a vertical plane about O . The particle is projected with speed u from a point A , where OA makes an angle α with the upward vertical through O and $0 < \alpha < \frac{\pi}{2}$. When OP makes an angle θ with the upward vertical through O the speed of P is v as shown in Figure 5.

(a) Show that $v^2 = u^2 + 2gl(\cos \alpha - \cos \theta)$. **(4)**

It is given that $\cos \alpha = \frac{3}{5}$ and that P moves in a complete vertical circle.

(b) Show that $u > 2\sqrt{\left(\frac{gl}{5}\right)}$. **(4)**

As the rod rotates the least tension in the rod is T and the greatest tension is $5T$.

(c) Show that $u^2 = \frac{33}{10}gl$. **(9)**



Centre No.						Paper Reference						Surname	Initial(s)	
Candidate No.						6	6	7	9	/	0	1	Signature	

Paper Reference(s)

6679/01

Edexcel GCE

Mechanics M3

Advanced/Advanced Subsidiary

Thursday 16 June 2011 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

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Team Leader's use only

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Question Number	Leave Blank
1	
2	
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Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper.

Answer ALL the questions.

You must write your answer to each question in the space following the question.

Whenever a numerical value of *g* is required, take $g = 9.8 \text{ m s}^{-2}$.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).

There are 7 questions in this question paper. The total mark for this paper is 75.

There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You should show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.



Turn over

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1. A particle P of mass 0.5 kg moves on the positive x -axis under the action of a single force directed towards the origin O . At time t seconds the distance of P from O is x metres, the magnitude of the force is $0.375x^2$ N and the speed of P is v m s $^{-1}$.

When $t = 0$, $OP = 8$ m and P is moving towards O with speed 2 m s $^{-1}$.

- (a) Show that $v^2 = 260 - \frac{1}{2}x^3$. (4)

- (b) Find the distance of P from O at the instant when $v = 5$. (2)



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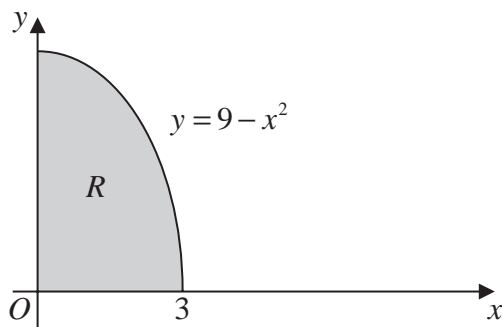


Figure 1

The shaded region R is bounded by the curve with equation $y = 9 - x^2$, the positive x -axis and the positive y -axis, as shown in Figure 1. A uniform solid S is formed by rotating R through 360° about the x -axis.

Find the x -coordinate of the centre of mass of S .

(9)



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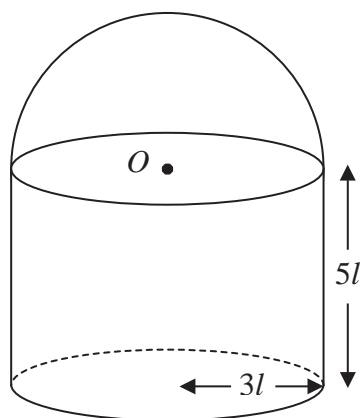


Figure 2

A solid consists of a uniform solid right cylinder of height $5l$ and radius $3l$ joined to a uniform solid hemisphere of radius $3l$. The plane face of the hemisphere coincides with a circular end of the cylinder and has centre O , as shown in Figure 2.

The density of the hemisphere is **twice** the density of the cylinder.

(a) Find the distance of the centre of mass of the solid from O .

(5)

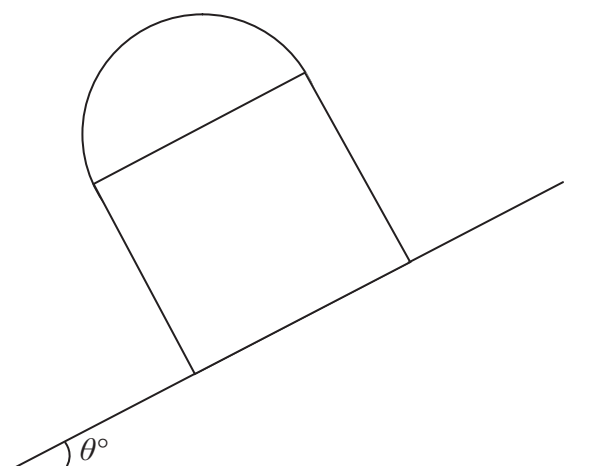


Figure 3

The solid is now placed with its circular face on a plane inclined at an angle θ° to the horizontal, as shown in Figure 3. The plane is sufficiently rough to prevent the solid slipping. The solid is on the point of toppling.

(b) Find the value of θ .

(4)



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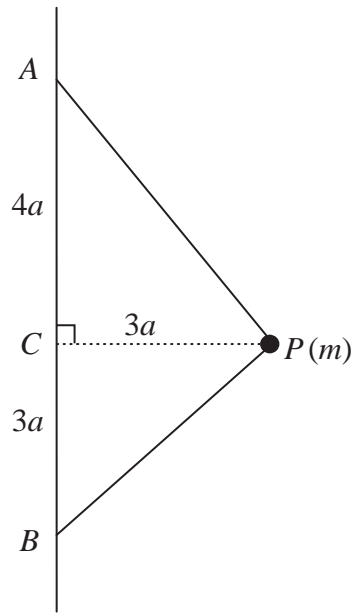


Figure 4

A light inextensible string has its ends attached to two fixed points A and B . The point A is vertically above B and $AB = 7a$. A particle P of mass m is fixed to the string and moves in a horizontal circle of radius $3a$ with angular speed ω . The centre of the circle is C where C lies on AB and $AC = 4a$, as shown in Figure 4. Both parts of the string are taut.

(a) Show that the tension in AP is $\frac{5}{7}m(3a\omega^2 + g)$. (8)

(b) Find the tension in BP . (2)

(c) Deduce that $\omega \geq \frac{1}{2}\sqrt{\left(\frac{g}{a}\right)}$. (2)



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5. A particle P of mass m is attached to one end of a light elastic string of natural length l and modulus of elasticity $3mg$. The other end of the string is attached to a fixed point O on a rough horizontal table. The particle lies at rest at the point A on the table, where $OA = \frac{7}{6}l$. The coefficient of friction between P and the table is μ .

(a) Show that $\mu \geq \frac{1}{2}$. (4)

The particle is now moved along the table to the point B , where $OB = \frac{3}{2}l$, and released from rest. Given that $\mu = \frac{1}{2}$, find

(b) the speed of P at the instant when the string becomes slack, (5)

(c) the total distance moved by P before it comes to rest again. (3)



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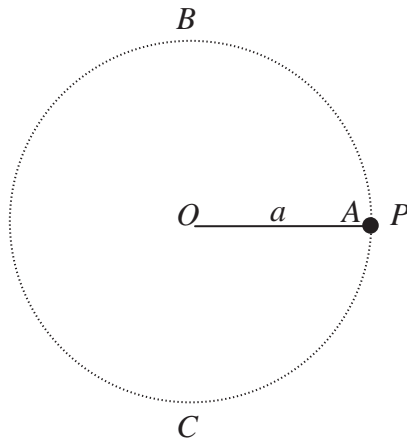


Figure 5

A particle P is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle is held at the point A , where $OA = a$ and OA is horizontal. The point B is vertically above O and the point C is vertically below O , with $OB = OC = a$, as shown in Figure 5. The particle is projected vertically upwards with speed $3\sqrt{ag}$.

(a) Show that P will pass through B . (6)

(b) Find the speed of P as it reaches C . (2)

As P passes through C it receives an impulse. Immediately after this, the speed of P is $\frac{5}{12}\sqrt{11ag}$ and the direction of motion of P is unchanged.

(c) Find the angle between the string and the downward vertical when P comes to instantaneous rest. (4)



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7. A particle P of mass 0.5 kg is attached to the mid-point of a light elastic string of natural length 1.4 m and modulus of elasticity 2 N. The ends of the string are attached to the points A and B on a smooth horizontal table, where $AB = 2$ m. The mid-point of AB is O and the point C is on the table between O and B where $OC = 0.2$ m. At time $t = 0$ the particle is released from rest at C . At time t seconds the length of the string AP is $(1+x)$ m.

(a) Show that the tension in BP is $\frac{2}{7}(3-10x)$ N. (2)

(b) Find, in terms of x , the tension in AP . (1)

(c) Show that P performs simple harmonic motion with period $2\pi\sqrt{\left(\frac{7}{80}\right)}$ s. (6)

(d) Find the greatest speed of P during the motion. (2)

The point D lies between O and A , where $OD = 0.1$ m.

(e) Find the time taken by P to move directly from C to D . (4)



Centre No.						Paper Reference						Surname	Initial(s)	
Candidate No.						6	6	7	9	/	0	1	Signature	

Paper Reference(s)

6679/01

Edexcel GCE

Mechanics M3

Advanced/Advanced Subsidiary

Friday 27 January 2012 – Morning

Time: 1 hour 30 minutes

Examiner’s use only

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Team Leader’s use only

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Question Number	Leave Blank
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Materials required for examination Mathematical Formulae (Pink) Items included with question papers Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

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5. Above the Earth's surface, the magnitude of the gravitational force on a particle due to the Earth is inversely proportional to the square of the distance of the particle from the centre of the Earth. The Earth is modelled as a sphere of radius R and the acceleration due to gravity at the Earth's surface is g . A particle P of mass m is at a height x above the surface of the Earth.

- (a) Show that the magnitude of the gravitational force acting on P is

$$\frac{mgR^2}{(R+x)^2} \quad (3)$$

A rocket is fired vertically upwards from the surface of the Earth. When the rocket is at height $2R$ above the surface of the Earth its speed is $\sqrt{\left(\frac{gR}{2}\right)}$. You may assume that air resistance can be ignored and that the engine of the rocket is switched off before the rocket reaches height R .

Modelling the rocket as a particle,

- (b) find the speed of the rocket when it was at height R above the surface of the Earth. (9)



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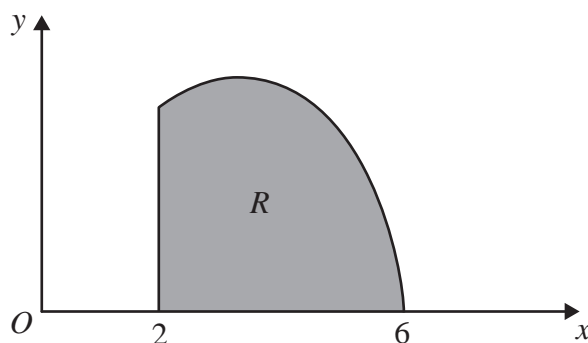


Figure 1

The shaded region R is bounded by the curve with equation $y = \frac{1}{2}x(6-x)$, the x -axis and the line $x=2$, as shown in Figure 1. The unit of length on both axes is 1 cm. A uniform solid P is formed by rotating R through 360° about the x -axis.

- (a) Show that the centre of mass of P is, to 3 significant figures, 1.42 cm from its plane face. **(9)**

The uniform solid P is placed with its plane face on an inclined plane which makes an angle θ with the horizontal. Given that the plane is sufficiently rough to prevent P from sliding and that P is on the point of toppling when $\theta = \alpha$,

- (b) find the angle α . **(4)**

Given instead that P is on the point of sliding down the plane when $\theta = \beta$ and that the coefficient of friction between P and the plane is 0.3,

- (c) find the angle β . **(3)**



Centre No.							Paper Reference						Surname	Initial(s)
Candidate No.						6	6	7	9	/	0	1	Signature	

Paper Reference(s)
6679/01

Edexcel GCE
Mechanics M3
Advanced/Advanced Subsidiary
Thursday 14 June 2012 – Morning
Time: 1 hour 30 minutes

Examiner's use only		
Team Leader's use only		

Question Number	Leave Blank
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<u>Materials required for examination</u> Mathematical Formulae (Pink)	<u>Items included with question papers</u> Nil
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1. A particle P is moving along the positive x -axis. At time $t = 0$, P is at the origin O . At time t seconds, P is x metres from O and has velocity $v = 2e^{-x}$ m s⁻¹ in the direction of x increasing.

(a) Find the acceleration of P in terms of x . **(3)**

(b) Find x in terms of t . **(6)**



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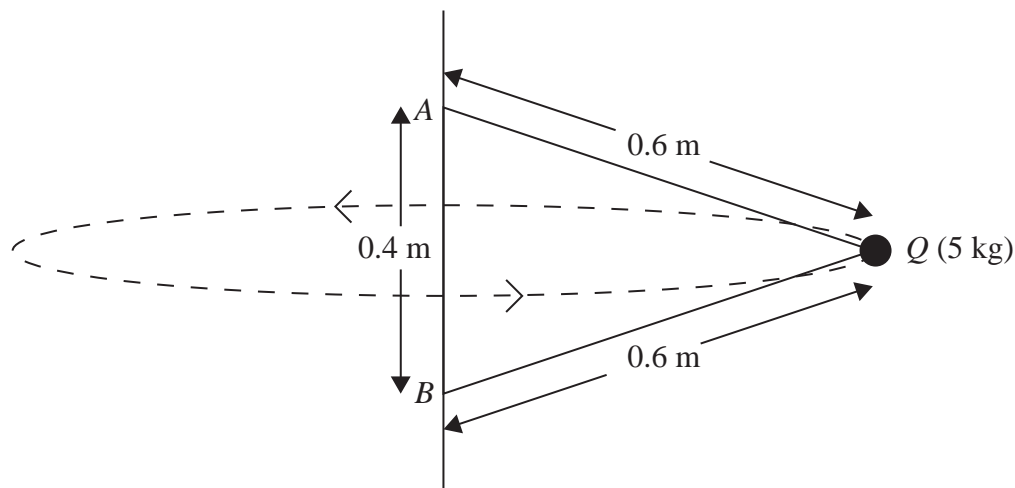


Figure 1

A particle Q of mass 5 kg is attached by two light inextensible strings to two fixed points A and B on a vertical pole. Each string has length 0.6 m and A is 0.4 m vertically above B , as shown in Figure 1.

Both strings are taut and Q is moving in a horizontal circle with constant angular speed 10 rad s^{-1} .

Find the tension in

(i) AQ ,

(ii) BQ .

(10)



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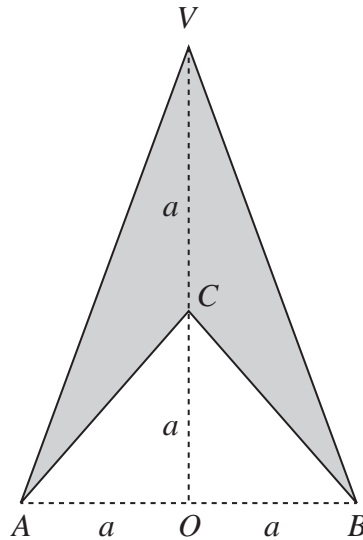


Figure 2

Figure 2 shows the cross-section $AVBC$ of the solid S formed when a uniform right circular cone of base radius a and height a , is removed from a uniform right circular cone of base radius a and height $2a$. Both cones have the same axis VCO , where O is the centre of the base of each cone.

- (a) Show that the distance of the centre of mass of S from the vertex V is $\frac{5}{4}a$. (5)

The mass of S is M . A particle of mass kM is attached to S at B . The system is suspended by a string attached to the vertex V , and hangs freely in equilibrium. Given that VA is at an angle 45° to the vertical through V ,

- (b) find the value of k . (5)



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5. A fixed smooth sphere has centre O and radius a . A particle P is placed on the surface of the sphere at the point A , where OA makes an angle α with the upward vertical through O . The particle is released from rest at A . When OP makes an angle θ to the upward vertical through O , P is on the surface of the sphere and the speed of P is v .

Given that $\cos \alpha = \frac{3}{5}$

(a) show that

$$v^2 = \frac{2ga}{5}(3 - 5 \cos \theta) \tag{4}$$

(b) find the speed of P at the instant when it loses contact with the sphere. (8)



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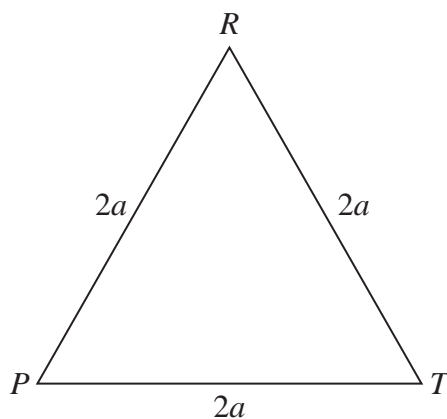


Figure 3

Figure 3 shows a uniform equilateral triangular lamina PRT with sides of length $2a$.

- (a) Using calculus, prove that the centre of mass of PRT is at a distance $\frac{2\sqrt{3}}{3}a$ from R . (6)

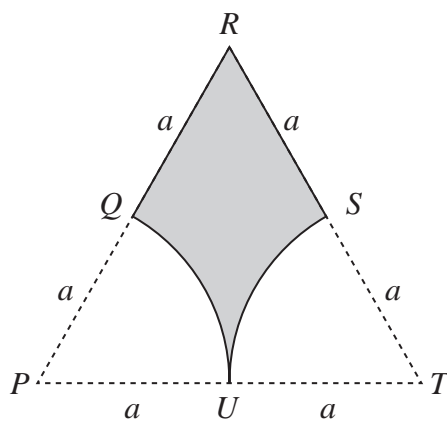


Figure 4

The circular sector PQU , of radius a and centre P , and the circular sector TUS , of radius a and centre T , are removed from PRT to form the uniform lamina $QRSU$ shown in Figure 4.

- (b) Show that the distance of the centre of mass of $QRSU$ from U is $\frac{2a}{3\sqrt{3}-\pi}$. (6)



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7. A particle B of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.75 m and modulus of elasticity 24.5 N. The other end of the string is attached to a fixed point A . The particle is hanging in equilibrium at the point E , vertically below A .

(a) Show that $AE = 0.9$ m.

(3)

The particle is held at A and released from rest. The particle first comes to instantaneous rest at the point C .

(b) Find the distance AC .

(5)

(c) Show that while the string is taut, B is moving with simple harmonic motion with centre E .

(4)

(d) Calculate the maximum speed of B .

(2)



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Question 7 continued

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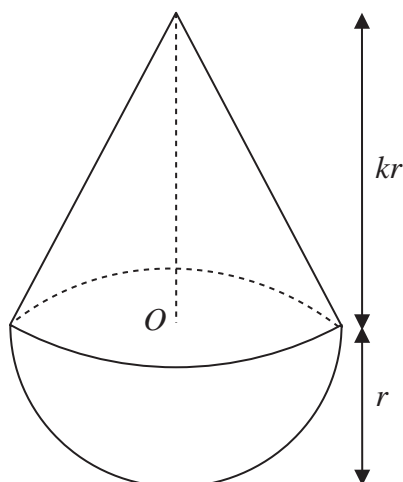


Figure 1

A uniform solid consists of a right circular cone of radius r and height kr , where $k > \sqrt{3}$, fixed to a hemisphere of radius r . The centre of the plane face of the hemisphere is O and this plane face coincides with the base of the cone, as shown in Figure 1.

(a) Show that the distance of the centre of mass of the solid from O is

$$\frac{(k^2 - 3)r}{4(k + 2)} \tag{5}$$

The point A lies on the circumference of the base of the cone. The solid is suspended by a string attached at A and hangs freely in equilibrium. The angle between AO and the vertical is θ , where $\tan \theta = \frac{11}{14}$

(b) Find the value of k . (4)



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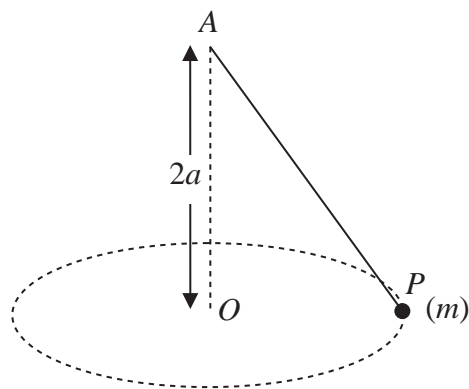


Figure 2

A particle P of mass m is attached to one end of a light elastic string, of natural length $2a$ and modulus of elasticity $6mg$. The other end of the string is attached to a fixed point A . The particle moves with constant speed v in a horizontal circle with centre O , where O is vertically below A and $OA = 2a$, as shown in Figure 2.

(a) Show that the extension in the string is $\frac{2}{5}a$. (6)

(b) Find v^2 in terms of a and g . (5)



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5. A particle P is moving in a straight line with simple harmonic motion on a smooth horizontal floor. The particle comes to instantaneous rest at points A and B where AB is 0.5 m. The mid-point of AB is O . The mid-point of OA is C . The mid-point of OB is D . The particle takes 0.2 s to travel directly from C to D . At time $t = 0$, P is moving through O towards A .

(a) Show that the period of the motion is $\frac{6}{5}$ s. (5)

(b) Find the distance of P from B when $t = 2$ s. (3)

(c) Find the maximum magnitude of the acceleration of P . (2)

(d) Find the maximum speed of P . (2)



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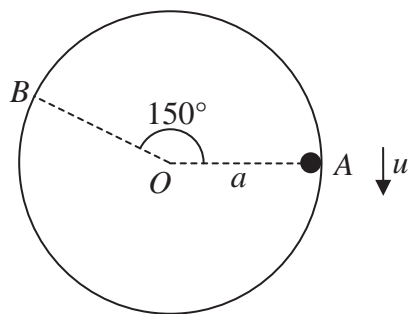


Figure 3

A smooth hollow cylinder of internal radius a is fixed with its axis horizontal. A particle P moves on the inner surface of the cylinder in a vertical circle with radius a and centre O , where O lies on the axis of the cylinder. The particle is projected vertically downwards with speed u from point A on the circle, where OA is horizontal. The particle first loses contact with the cylinder at the point B , where $\angle AOB = 150^\circ$, as shown in Figure 3. Given that air resistance can be ignored,

(a) show that the speed of P at B is $\sqrt{\left(\frac{ag}{2}\right)}$, (3)

(b) find u in terms of a and g . (4)

After losing contact with the cylinder, P crosses the diameter through A at the point D . At D the velocity of P makes an angle θ° with the horizontal.

(c) Find the value of θ . (7)



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Question 3 continued

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4. A particle P is moving along the positive x -axis. At time t seconds, $t \geq 0$, P is x metres from the origin O and is moving away from O with velocity v m s⁻¹, where $v = \frac{4}{(x + 2)}$. When $t = 0$, P is at O . Find

(a) the distance of P from O when $t = 2$ (5)

(b) the magnitude and direction of the acceleration of P when $t = 2$ (5)



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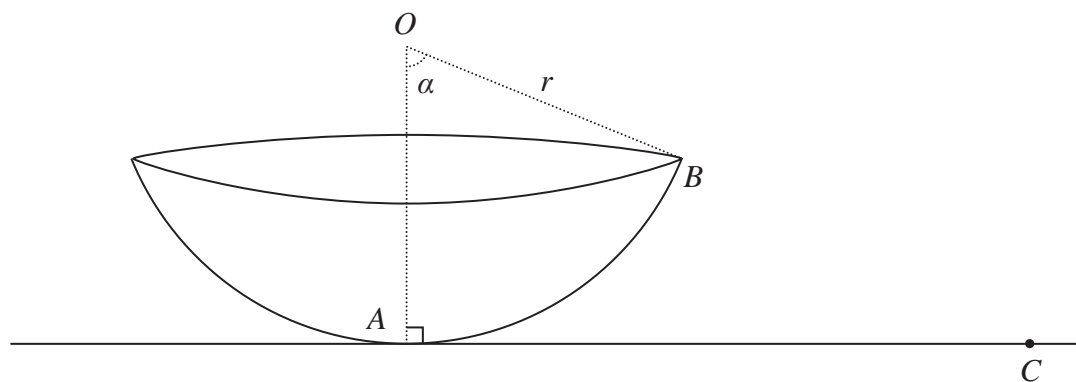


Figure 2

Part of a hollow spherical shell, centre O and radius r , forms a bowl with a plane circular rim. The bowl is fixed to a horizontal surface at A with the rim uppermost and horizontal.

The point A is the lowest point of the bowl. The point B , where $\angle AOB = \alpha$ and $\tan \alpha = \frac{3}{4}$, is on the rim of the bowl, as shown in Figure 2. A small smooth marble M is placed inside the bowl at A , and given an initial horizontal speed \sqrt{gr} . The motion of M takes place in the vertical plane OAB .

- (a) Show that the speed of M as it reaches B is $\sqrt{\left(\frac{3}{5}gr\right)}$. (4)

After leaving the surface of the bowl at B , M moves freely under gravity and first strikes the horizontal surface at the point C . Given that $r = 0.4$ m,

- (b) find the distance AC . (8)



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Question 5 continued

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6. (a) A uniform lamina is in the shape of a quadrant of a circle of radius a . Show, by integration, that the centre of mass of the lamina is at a distance of $\frac{4a}{3\pi}$ from each of its straight edges. (7)

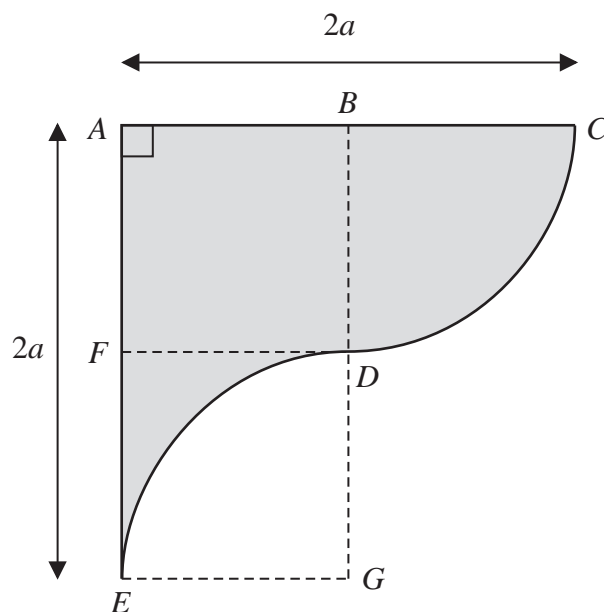


Figure 3

A second uniform lamina $ABCDEFA$ is shown shaded in Figure 3. The straight sides AC and AE are perpendicular and $AC = AE = 2a$. In the figure, the midpoint of AC is B , the midpoint of AE is F , and $ABDF$ and $DGEF$ are squares of side a . BCD is a quadrant of a circle with centre B . DGE is a quadrant of a circle with centre G .

- (b) Find the distance of the centre of mass of the lamina from the side AE . (5)

The lamina is smoothly hinged to a horizontal axis which passes through E and is perpendicular to the plane of the lamina. The lamina has weight W newtons. The lamina is held in equilibrium in a vertical plane, with A vertically above E , by a horizontal force of magnitude X newtons applied at C .

- (c) Find X in terms of W . (3)



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7. Two points A and B are 4 m apart on a smooth horizontal surface. A light elastic string, of natural length 0.8 m and modulus of elasticity 15 N, has one end attached to the point A . A light elastic string, of natural length 0.8 m and modulus of elasticity 10 N, has one end attached to the point B . A particle P of mass 0.2 kg is attached to the free end of each string. The particle rests in equilibrium on the surface at the point C on the straight line between A and B .

(a) Show that the length of AC is 1.76 m.

(4)

The particle P is now held at the point D on the line AB such that $AD = 2.16$ m. The particle is then released from rest and in the subsequent motion both strings remain taut.

(b) Show that P moves with simple harmonic motion.

(4)

(c) Find the speed of P as it passes through the point C .

(2)

(d) Find the time from the instant when P is released from D until the instant when P is first moving with speed 2 m s^{-1} .

(4)



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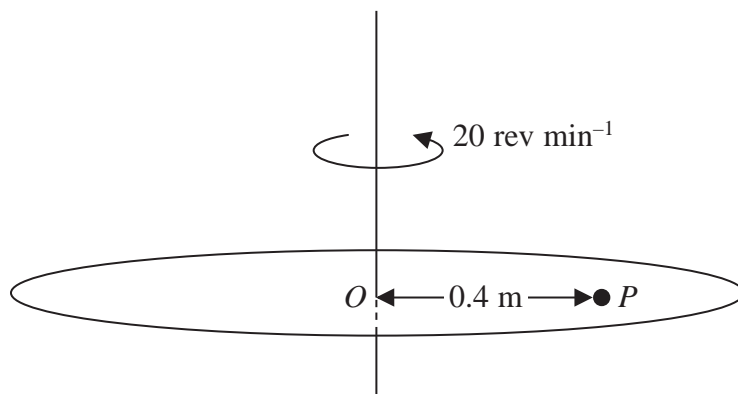


Figure 1

A rough disc is rotating in a horizontal plane with constant angular speed 20 revolutions per minute about a fixed vertical axis through its centre O . A particle P rests on the disc at a distance 0.4 m from O , as shown in Figure 1. The coefficient of friction between P and the disc is μ . The particle P is on the point of slipping.

Find the value of μ .

(6)



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2. A particle P of mass 0.5 kg is moving along the positive x -axis in the positive x -direction. The only force on P is a force of magnitude $\left(2t + \frac{1}{2}\right)$ N acting in the direction of x increasing, where t seconds is the time after P leaves the origin O . When $t=0$, P is at rest at O .

(a) Find an expression, in terms of t , for the velocity of P at time t seconds. **(3)**

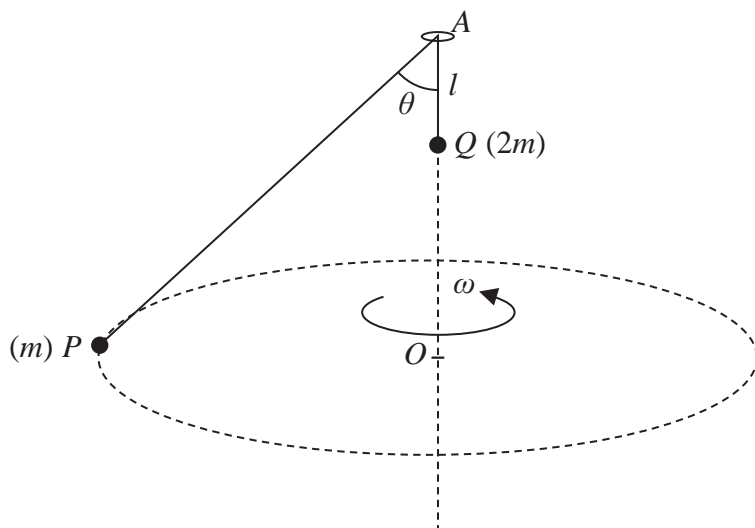
The particle passes through the point A with speed 6 m s⁻¹.

(b) Find the distance OA . **(6)**



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**Figure 2**

Two particles P and Q , of mass m and $2m$ respectively, are attached to the ends of a light inextensible string of length $6l$. The string passes through a small smooth fixed ring at the point A . The particle Q is hanging freely at a distance l vertically below A . The particle P is moving in a horizontal circle with constant angular speed ω . The centre O of the circle is vertically below A . The particle Q does not move and AP makes a constant angle θ with the downward vertical, as shown in Figure 2.

Show that

(i) $\theta = 60^\circ$

(ii) $\omega = \sqrt{\left(\frac{2g}{5l}\right)}$

(8)



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Question 3 continued

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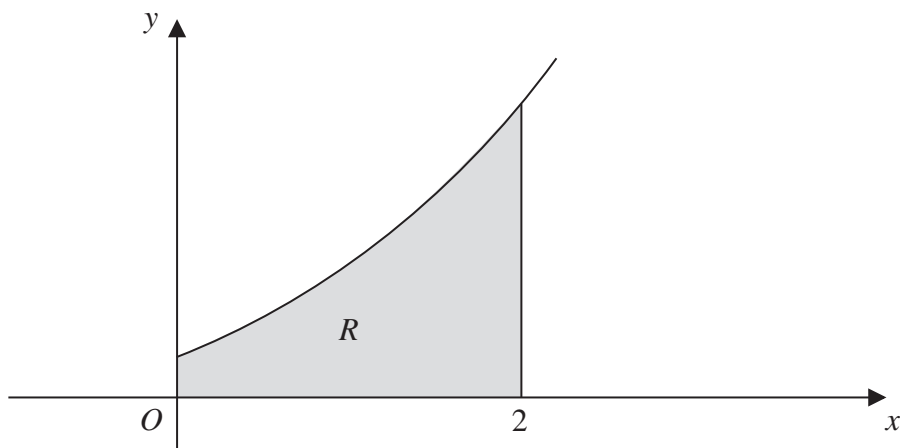


Figure 3

The shaded region R is bounded by the curve with equation $y = (x + 1)^2$, the x -axis, the y -axis and the line with equation $x = 2$, as shown in Figure 3. The region R is rotated through 2π radians about the x -axis to form a uniform solid S .

(a) Use algebraic integration to find the x coordinate of the centre of mass of S .

(8)

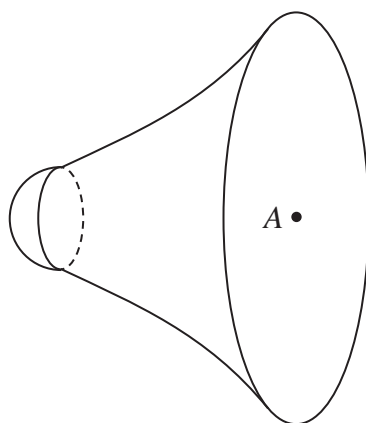


Figure 4

A uniform solid hemisphere is fixed to S to form a solid T . The hemisphere has the same radius as the smaller plane face of S and its plane face coincides with the smaller plane face of S , as shown in Figure 4. The mass per unit volume of the hemisphere is 10 times the mass per unit volume of S . The centre of the circular plane face of T is A . All lengths are measured in centimetres.

(b) Find the distance of the centre of mass of T from A .

(5)

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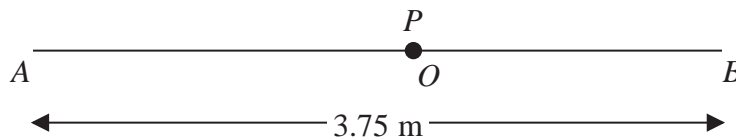


Figure 5

The points A and B are 3.75 m apart on a smooth horizontal floor. A particle P has mass 0.8 kg. One end of a light elastic spring, of natural length 1.5 m and modulus of elasticity 24 N, is attached to P and the other end is attached to A . The ends of another light elastic spring, of natural length 0.75 m and modulus of elasticity 18 N, are attached to P and B . The particle P rests in equilibrium at the point O , where AOB is a straight line, as shown in Figure 5.

(a) Show that $AO = 2.4$ m. (4)

The point C lies on the straight line AOB between O and B . The particle P is held at C and released from rest.

(b) Show that P moves with simple harmonic motion. (5)

The maximum speed of P is $\sqrt{2}$ m s⁻¹.

(c) Find the time taken by P to travel 0.3 m from C . (5)



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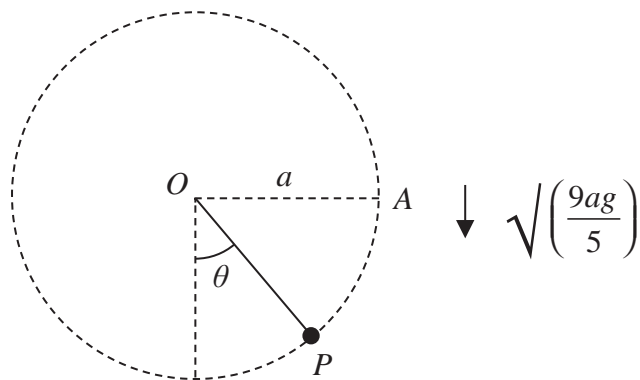


Figure 6

A particle P of mass $5m$ is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle is held at the point A , where $OA = a$ and OA is horizontal, as shown in Figure 6. The particle is projected vertically downwards with speed $\sqrt{\left(\frac{9ag}{5}\right)}$. When the string makes an angle θ with the downward vertical through O and the string is still taut, the tension in the string is T .

(a) Show that $T = 3mg(5 \cos \theta + 3)$. (6)

At the instant when the particle reaches the point B the string becomes slack.

(b) Find the speed of P at B . (3)

At time $t = 0$, P is at B .

At time t , before the string becomes taut once more, the coordinates of P are (x, y) referred to horizontal and vertical axes with origin O . The x -axis is directed along OA produced and the y -axis is vertically upward.

(c) Find

- x in terms of t , a and g ,
- y in terms of t , a and g .

(7)



