Mark Scheme (Results)

Summer 2017

Pearson Edexcel GCE
In Mechanics M3 (6679/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award **zero marks if the candidate's response is not worthy of credit according to** the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

- When examiners are in doubt regarding the application of the **mark scheme to a candidate's response, the team leader must be** consulted.

- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ✱ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. **All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to** indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   a. If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b. If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking
(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- dM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a **question i.e. (a) or (b) or (c),……then that working can only score marks for that part of** the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations

  M(A)   Taking moments about A.

  N2L   **Newton's Second Law (Equation of Motion)**

  NEL   **Newton's Experimental Law (Newton's Law of Impact)**

  HL   **Hooke's Law**

  SHM   Simple harmonic motion

  PCLM   Principle of conservation of linear momentum

  RHS, LHS   Right hand side, left hand side.

Question Number	Scheme	Marks
1	Area $= \int_0^2 \left(4 - x^2\right) dx = \left[4x - \dfrac{1}{3} x^3 \right]_0^2 , \quad = \dfrac{16}{3}$	M1,A1
	$\int xy\, dx = \int_0^2 \left(4x - x^3\right) dx$	M1
	$= \left[2x^2 - \dfrac{1}{4} x^4 \right]_0^2 , \quad = 4$	dM1, A1
	$\bar{x} = \dfrac{\int xy\, dx}{\int y\, dx} = 4 \div \dfrac{16}{3} , \quad = \dfrac{3}{4} \quad$ oe	M1,A1cso
		[7]

Use of volumes scores 0/7
Ignore any work for $\bar{y}$ whether before or after $\bar{x}$

M1   Using $\int y\, dx$ with lower limit 0, an attempt at the upper limit and an attempt at algebraic integration.

A1   Correct result for the area. May be implied by a correct final answer. No need to show substitution of either limit providing algebraic integration and attached limits are correct.

M1   Using $\int xy\, dx$ with the same limits as the first integral

dM1   Attempting the algebraic integration, including substitution of their limits. Depends on the second M mark.

A1   Correct result for this integral. May be implied by a correct final answer.

M1   Using $\bar{x} = \dfrac{\int xy\, dx}{\int y\, dx}$. If a constant for mass/unit area is seen it must be with both integrals or neither.

A1cso   Correct final answer.

1

Question Number	Scheme	Marks
2	$1.2mg\cos\theta = mg$ or $T\cos\theta = mg$	M1A1
(i)	$\cos\theta° = \dfrac{1}{1.2}$ $\theta° = \cos^{-1}\dfrac{1}{1.2}$, $\theta = $ 33.55... (accept 34, 33.6 or better)	A1
	$1.2mg\sin\theta = mr\omega^2$ or $T\cos\theta = mr\omega^2$	M1A1
	$1.2mg\sin\theta = m\times l\sin\theta\omega^2$	A1
(ii)	$1.2mg = 58.8lm \implies l = \dfrac{1.2\times 9.8}{58.8} = 0.2\,(\text{m})$	dM1A1 (8)

M1   Resolve vertically. Tension to be resolved, weight not resolved.
A1   Fully correct equation with substitution for *T* made.
(i)A1   Correct value of θ Min 2 sf   Use of radians scores A0
M1   Attempt NL2 horizontally. Tension must be resolved, acceleration can be in either form.

A1   LHS correct, RHS can be $mr\omega^2$ or $m\dfrac{v^2}{r}$ here. *T* substituted now or later

A1   RHS correct, acceleration as shown. $\sin\theta$ may be numerical $\dfrac{\sqrt{11}}{6}$ or 0.5527... (min 3 sf) or

        a numerical value for r ($\dfrac{\sqrt{11}}{30}$ or 0.110...) may be seen.

dM1   Use the above equation to obtain a numerical value for *l*. Depends on the second M mark

(ii)A1   Correct value of *l*. Accept 0.2, 0.20, 0.200. Exceptionally allow $\dfrac{1}{5}$ here.

1

Question Number	Scheme	Marks
3(a)	$mv\dfrac{\mathrm{d}v}{\mathrm{d}x} = mg\sin 30 - \dfrac{1}{2}mx^2$	M1A1A1
	$\dfrac{1}{2}v^2 = xg\sin 30 - \dfrac{1}{6}x^3 \ (+c)$	dM1A1ft
	$x = 3 \quad \dfrac{1}{2}v^2 = 3g\sin 30 - \dfrac{9}{2}$	dM1
	$(v = 4.5166...)$	
	$v = 4.5 \text{ or } 4.52 \ \left(\mathrm{ms}^{-1}\right)$	A1cso (7)
(b)	$v = 0 \Rightarrow x^2 = 6g\sin 30 \ (x \neq 0)$	
	$x = 5.4 \text{ or } 5.42 \ (\mathrm{m})$	M1A1 (2)
		[9]

(a)M1 Attempt NL2 parallel to the plane. Acceleration must be $v\dfrac{\mathrm{d}v}{\mathrm{d}x}$ and weight must be resolved. (Variable force not resolved.) *m* may be cancelled. Integrating *a* to obtain $\dfrac{1}{2}v^2$ gains this mark by implication.

A1 A1 Deduct 1 mark for each error in the equation. Both signs incorrect on RHS is one error.

dM1 Attempt the integration (wrt *x*) of both sides of the equation. Depends on the first M mark.
A1ft Correct integration with or without the constant. Follow through their integrand.
dM1 Substitute $x = 3$ in their integrated equation. Depends on both previous M marks.
A1cso Correct value of *v*. Must be 2 or 3 sf. **CSO**: Evidence of a constant of integration must be seen. *C* included and then crossed out or disappearing is sufficient evidence.
Definite integration:
M1A1A1 as above
dM1A1ft For the integration - ignore any limits shown
dM1 Use of correct limits. No sub need be shown for 0.
A1 Correct value of *v*. Must be 2 or 3 sf. **CSO**: Evidence of a zero lower limit must be seen.
By work-energy:
F is variable, so if no integral seen score 0/7

$.\ \dfrac{1}{2}v^2(-0) = xg\sin 30 - \int \dfrac{1}{2}x^2\mathrm{d}x..$ M1A1A1

$\dfrac{1}{2}v^2(-0) = xg\sin 30 - \dfrac{1}{6}x^3$ M1A1

For the final A mark, evidence of initial KE being 0 must be seen.

(b)
M1 Substitute $v = 0$ in their equation for v^2 (from (a)) and obtain a numerical value of *x*
A1 Correct value of *x*. Must be 2 or 3 sf. Do not penalise missing constant here.

1

Question Number	Scheme	Marks
4(a)	Ratio of masses: πa^2 $2\pi a \times 4a$ $2\pi a^2$ $11\pi a^2$	M1A1
	Distances: $\quad\quad\quad$ 0 $\quad$ 2a $\quad\quad$ $4\frac{1}{2}a$ $\quad\quad$ $\bar{y}$	B1
	$(0+)2a \times 8 + 4\frac{1}{2}a \times 2 = 11\bar{y}$	M1A1ft
	$\bar{y} = \frac{25}{11}a \ \left(=2.272...a\right)$	A1 $\quad$ (6)
(b)	$\tan\theta° = \dfrac{a}{\dfrac{25}{11}a} \ \left(=\dfrac{11}{25}\right)$	M1A1ft
	$\theta = 23.749...$ $\quad$ Accept 24 or better	A1 $\quad$ (3)
		[9]

(a)

M1 Attempt the ratio of the masses of the separate parts. Formulae used must be correct; allow if cylinder has a top as well as a bottom - ignore top - or neither. Similarly if hemisphere has a base. Allow if the base of the cylinder and the curved surface are combined.

A1 Correct ratio seen eg 1:8:2:11 (or any equivalent) (Top of cylinder/base of hemisphere not ignored now; combined curved surface and base for cylinder not allowed.)

B1 Correct distances from *O* or any other point for the curved surface of the cylinder and the hemispherical shell.

M1 Form a moments equation about their chosen point. Must be dimensionally correct (ie no a^3 seen). Extra terms score M0.

A1ft Correct equation, follow through their mass ratio and distances. 1 or 2 signs may be negative if a point other than *O* has been used.

A1cao Correct distance from *O* exact or min 2sf. (Must be obtained from a correct equation.)

ALT Find c of m of cylinder (inc base) first and then combine with hemisphere. All marks available. Award second M when combining with the hemisphere.

NB If c of m of cylinder with base is given as 2*a* then only M1A0B0M0A0A0 available.

(b)

M1 Use $\tan\theta = \dfrac{a}{\bar{y}}$ or $\dfrac{\bar{y}}{a}$ with their answer from (a)

A1ft Correct expression for $\tan\theta$ follow through their $\bar{y}$

A1cao Correct value for θ Min 2 sf. (NB Equivalent in radians scores A0)

1

Question Number	Scheme	Marks
5 (a)	$\dfrac{1}{2}mv^2 - \dfrac{1}{2}m \times 7ag = mga\sin\theta$	M1A1A1
	$v^2 = 7ag + 2ag\sin\theta = ag(7 + 2\sin\theta)$ $*$	A1 (4)
(b)	At top $v^2 = 5ag$	M1A1
	$R + mg = m\dfrac{v^2}{a}$ or $m\dfrac{v^2}{a} > mg$	M1A1
	$R = 4mg$ or substitute for v^2	dM1
	$R > 0$ $\therefore$ complete circles	A1 cso (6)
(c)	Max v at lowest point	
	$\sin\theta = 1 \Rightarrow v^2 = 9ag$	M1
	$v = 3\sqrt{ag}$	A1 (2)
		[12]

(a)

M1   Energy equation from the point of projection to a general point. Must have 3 terms and the PE term must include a trig function.

A1   Correct difference of KE terms.

A1   Correct PE term and all signs correct.

A1cso   Obtain correct **given** expression for v^2 with no errors in the solution.

(b)

M1   Use the result given in (a) with $\theta = 270°$ to obtain v^2 at the top. Substitution for θ may occur later.

A1   Correct expression for v^2. May be implied by correct work later.

M1   Attempt NL2 at the **top.** This mark cannot be awarded if a general position is used but can be awarded later when the motion at the highest point is considered.

A1   Correct NL2 at the top with $R + mg$

dM1   Eliminate v^2 between the 2 equations. Depends on the 2 previous M marks in (b).

A1cso   Correct result for R (at the top) seen **and** the conclusion stated. (Do not need to see $R > 0$). If working with the resultant, resultant $> mg$ must be seen.

Full marks can be awarded if it is stated that $v^2 > 0$ and $R > 0$ at the top - mark the work relevant to R.

ALT   Last 4 marks:

If $m\dfrac{v^2}{a} > mg$ is seen, give M1A1. M1 substitute for v^2; $5mg > mg$ $\therefore$ complete circles

(c)

M1   Using $\sin\theta = 1$ in the result given in (a) to obtain v^2 at the lowest point. Any other **complete** method may be used, eg an energy equation provided it leads to the speed at the lowest point.

A1 $v = 3\sqrt{ag}$ or $\sqrt{9ag}$ (Watch square root covers all necessary letters.)

1

Question Number	Scheme	Marks
6(a)	$8 = \dfrac{\lambda \times 0.20}{0.40}$ $\lambda = 16$ $*$	M1A1 A1cso (3)
(b)	Length of string = 1 m or 100cm $T = \dfrac{\lambda \times 0.6}{0.4}, \ = 24$ (or use half string) $2T\cos\theta = F$ $F = 2 \times 24 \times \dfrac{4}{5} = 38.4, \ \dfrac{192}{5}$ or $38\dfrac{2}{5}$	 M1,A1 M1 A1 (4)
(c)	Initial EPE $= \dfrac{16 \times 0.6^2}{2 \times 0.4} \left(= \dfrac{36}{5} \right)$ Final EPE $= \dfrac{16 \times 0.2^2}{2 \times 0.4} \left(= \dfrac{4}{5} \right)$ $\dfrac{16 \times 0.6^2}{2 \times 0.4} - \dfrac{16 \times 0.2^2}{2 \times 0.4} = \dfrac{1}{2} 0.3 v^2$ $0.3v^2 = 40 \left(0.6^2 - 0.2^2 \right)$ $v = 6.531...$ Accept $6.5 \left(\text{m s}^{-1} \right)$ or better or exact value $8\sqrt{\dfrac{2}{3}} \left(\text{m s}^{-1} \right)$	B1 (either) M1A1A1 dM1A1cso (6)
		[13]

(a)
M1 Attempt Hooke's Law using the whole string or a half string.
A1 Correct equation.
A1cso Correct **given** value of λ obtained with no errors seen.
(b)
M1 Use Hooke's Law with the new longer length for the string or half string. λ must be 16, but length need not be correct but use of 0.2 for extension of full string or 0.1 for extension of half string scores M0.
A1 Obtain $T = 24$
M1 Resolve parallel to F or in another direction which gives an equation connecting T and F.
A1 Obtain the correct value of F
(c)
B1 Correct initial or final EPE with one string ($l = 0.4$) or two half strings ($l = 0.2$)
M1 Attempt an energy equation with the difference of 2 EPE terms and a KE term. The EPE terms must be of the form $k\dfrac{\lambda x^2}{l}$.
A1A1 Deduct one mark per error.   (A1A1, A1A0 or A0A0)

dM1 Solve for v. Depends on the previous M mark.
A1cso Correct value of v, min 2 sf or exact value.
Energy terms wrong way round in the equation will lose this mark even if modulus sign inc here.
NB If the energy terms are subtracted the wrong way round, max score is B1M1A1A0M1A0

1

Question Number	Scheme	Marks
7 **(a)**	$T = \dfrac{45e}{1.8} = \dfrac{20(1-e)}{1.2}$ or $\dfrac{45(1-e')}{1.8} = \dfrac{20e'}{1.2}$ $e = 0.4$ or $e' = 0.6$ $AO = 2.2$ m $\qquad\qquad *$	M1A1 A1 A1cso (4)
ALT:	$AO = y$ $\dfrac{45(y-1.8)}{1.8} = \dfrac{20(2.8-y)}{1.2} \qquad$ M1A1A1 $54y - 97.2 = 100.8 - 36y$ $y = 2.2$ m $\qquad\qquad * \qquad$ A1cso	
(b)	$0.6\ddot{x} = \dfrac{20(0.6-x)}{1.2} - \dfrac{45(0.4+x)}{1.8}$ $0.6\ddot{x} = -\dfrac{125}{3}x$ $\ddot{x} = -\dfrac{625}{9}x \quad \therefore \text{SHM}$	M1A1A1 dM1A1cso (5)
(c)	$\omega^2 = \dfrac{625}{9} \qquad$ oe Time from C to D: $-0.4 = 0.5\cos\dfrac{25}{3}t$ $t = \dfrac{3}{25}\cos^{-1}(-0.8)$ Speed at D: $v^2 = \dfrac{625}{9}\left(0.5^2 - 0.4^2\right)$ or use $v = -a\omega\sin\omega t$ $v = \dfrac{25}{3}\times 0.3 = 2.5$ Time from D to A $= \dfrac{1.8}{2.5}$ Total time: $= \dfrac{1.8}{2.5} + \dfrac{3}{25}\cos^{-1}(-0.8) = 1.01977...$ (s) (accept 1.0 or better)	B1 M1A1ft A1 M1 A1 M1 A1 cao (8) [17]

1

(a)

M1 Use Hooke's law to obtain the tension in each string and equate the 2 tensions. Either extension can be used as the unknown. ALT: Use both extensions and use $e + e' = 1$ later

A1 Correct equation.

A1 Correct result for their choice of unknown.

A1cso Correct completion to the **given** answer with no errors seen.

(b)

M1 Form an equation of motion with the difference of 2 tensions (from applying Hooke's law) which must have different extensions which both include a variable. Acceleration can be a or $\ddot{x}$

If the variable is measured from *A* or *B* a substitution is required to obtain the necessary SHM equation. **Do not** award this mark until this substitution is attempted.

A1A1 Deduct 1 for each error. Difference of tensions the wrong way round counts as one error. Acceleration can be a or $\ddot{x}$ but if a is used it must be in the same direction as $\ddot{x}$. The numbers in the extensions must be as shown (as SHM cannot be assumed before it is shown and so the numbers must come from the situation and not just chosen so that the constant terms cancel out).

dM1 Solve the equation to $\pm\ddot{x} = ...$ Acceleration a scores M0 now.

A1cso Correct equation (must be $\ddot{x} = ...$ now) **and** the conclusion.

(c)

B1 Correct value for ω^2 or ω seen explicitly or used

M1 Obtain the time from *C* to *D*. $x = a\cos\omega t$ or $x = a\sin\omega t$ can be used as long as the method is complete. Amplitude to be 0.5, $x = \pm 0.4$ with their ω which must have come from an equation of the form $\ddot{x} = \pm\omega^2 x$

Sometimes done in two parts: Time *C* to *O* $\left(\dfrac{3\pi}{50}\ \text{or}\ 0.1884...\right)$ and *O* to *D* $\left(0.111...\right)$

added now or later.

A1ft Correct equation. Follow through their ω

A1 Correct time, as shown or 0.2997...seen now or later

M1 Find the speed of *P* as it reaches *D* with their ω.

A1 Correct speed at *D*

M1 Use their speed at *D* to find the time from *D* to *A*

A1cao Add the 2 (or 3) times to obtain the required time.

1