

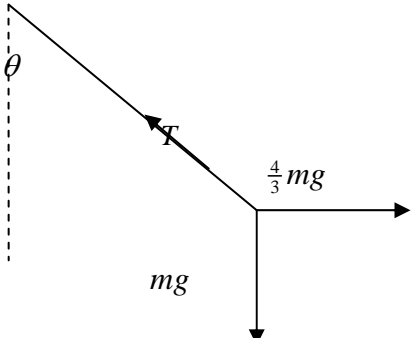
Mark Scheme (Results)

January 2009

GCE

GCE Mathematics (6679/01)

January 2009
6679 Mechanics M3
Mark Scheme

| Question Number | Scheme | Marks |
|-----------------|---|--|
| 1 | <p>N2L</p> $3a = -\left(9 + \frac{15}{(t+1)^2}\right)$ $3v = -9t + \frac{15}{t+1} (+A)$ $v = 0, t = 4 \Rightarrow 0 = -36 + 3 + A \Rightarrow A = 33$ $v = -3t + \frac{5}{t+1} + 11$ $t = 0 \Rightarrow v = 16$ | <p>B1</p> <p>M1 A1ft</p> <p>M1 A1</p> <p>M1 A1 (7) [7]</p> |
| 2 | <div style="text-align: center;">  </div> <p>(a)</p> <p>(←) $T \sin \theta = \frac{4}{3} mg$</p> <p>(↑) $T \cos \theta = mg$</p> $T^2 = \left(\frac{4}{3} mg\right)^2 + (mg)^2$ <p>Leading to $T = \frac{5}{3} mg$</p> <p>(b)</p> <p>HL $T = \frac{\lambda x}{a} \Rightarrow \frac{5}{3} mg = \frac{3mge}{a}$ ft their T</p> $e = \frac{5}{9} a$ $E = \frac{\lambda x^2}{2a} = \frac{3mg}{2a} \times \left(\frac{5}{9} a\right)^2 = \frac{25}{54} mga$ | <p>M1 A1</p> <p>A1</p> <p>M1</p> <p>A1 (5)</p> <p>M1 A1ft</p> <p>M1 A1 (4) [9]</p> |

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| 3 | $\omega = \frac{80 \times 2\pi}{60} \text{ rad s}^{-1} \left(= \frac{8\pi}{3} \approx 8.377... \right)$ <p style="text-align: center;">Accept $v = \frac{16\pi}{75} \approx 0.67 \text{ ms}^{-1}$ as equivalent</p> $(\uparrow) R = mg$ <p>For least value of μ $(\leftarrow) \mu mg = mr\omega^2$</p> $\mu = \frac{0.08}{9.8} \times \left(\frac{8\pi}{3} \right)^2 \approx 0.57 \quad \text{accept } 0.573$ | <p>B1</p> <p>B1</p> <p>M1 A1=A1</p> <p>M1 A1 (7)</p> <p>[7]</p> |
| 4 | <p>(a)</p> $a = 8$ $T = \frac{25}{2} = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{4\pi}{25} (\approx 0.502 \dots)$ $v^2 = \omega^2 (a^2 - x^2) \Rightarrow v^2 = \left(\frac{4\pi}{25} \right)^2 (8^2 - 3^2) \quad \text{ft their } a, \omega$ $v = \frac{4\pi}{25} \sqrt{55} \approx 3.7 \text{ (m h}^{-1}\text{)} \quad \text{awrt } 3.7$ <p>(b)</p> $x = a \cos \omega t \Rightarrow 3 = 8 \cos \left(\frac{4\pi}{25} t \right) \quad \text{ft their } a, \omega$ $t \approx 2.3602 \dots$ <p>time is 12 22</p> | <p>B1</p> <p>M1 A1</p> <p>M1 A1ft</p> <p>M1 A1 (7)</p> <p>M1 A1ft</p> <p>M1 A1 (4)</p> <p>[11]</p> |

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| 5 | <p>(a) Let x be the distance from the initial position of B to C GPE lost = EPE gained $mgx \sin 30^\circ = \frac{6mgx^2}{2a}$ Leading to $x = \frac{a}{6}$ $AC = \frac{7a}{6}$</p> <p>(b) The greatest speed is attained when the acceleration of B is zero, that is where the forces on B are equal. $(\curvearrowright) \quad T = mg \sin 30^\circ = \frac{6mge}{a}$ $e = \frac{a}{12}$ CE $\frac{1}{2}mv^2 + \frac{6mg}{2a} \left(\frac{a}{12}\right)^2 = mg \frac{a}{12} \sin 30^\circ$ Leading to $v = \sqrt{\left(\frac{ga}{24}\right)} = \frac{\sqrt{6ga}}{12}$</p> <p><i>Alternative approaches to (b) are considered on the next page.</i></p> | <p>M1 A1=A1</p> <p>M1</p> <p>A1 (5)</p> <p>M1</p> <p>A1</p> <p>M1 A1=A1</p> <p>M1 A1 (7)</p> <p>[12]</p> |

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| 5 | <p><i>Alternative approach to (b) using calculus with energy.</i></p> <p>Let distance moved by B be x</p> <p>CE $\frac{1}{2}mv^2 + \frac{6mg}{2a}x^2 = mgx \sin 30^\circ$</p> $v^2 = gx - \frac{6g}{a}x^2$ <p>For maximum v $\frac{d}{dx}(v^2) = 2v \frac{dv}{dx} = g - \frac{12g}{a}x = 0$</p> $x = \frac{a}{12}$ $v^2 = g\left(\frac{a}{12}\right) - \frac{6g}{a}\left(\frac{a}{12}\right)^2 = \frac{ga}{24}$ $v = \sqrt{\left(\frac{ga}{24}\right)}$ | <p>M1 A1=A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (7)</p> |
| | <p><i>Alternative approach to (b) using calculus with Newton's second law.</i></p> <p>As before, the centre of the oscillation is when extension is $\frac{a}{12}$</p> <p>N2L $mg \sin 30^\circ - T = m\ddot{x}$</p> $\frac{1}{2}mg - \frac{6mg\left(\frac{a}{12} + x\right)}{a} = m\ddot{x}$ $\ddot{x} = -\frac{6g}{a}x \Rightarrow \omega^2 = \frac{6g}{a}$ $v_{\max} = \omega a = \sqrt{\left(\frac{6g}{a}\right)} \times \frac{a}{12} = \sqrt{\left(\frac{ga}{24}\right)}$ | <p>M1 A1</p> <p>M1 A1</p> <p>A1</p> <p>M1 A1 (7)</p> |

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| 6 (a) | $\int y^2 dx = \int (4-x^2)^2 dx = \int (16-8x^2+x^4) dx$ $= 16x - \frac{8x^3}{3} + \frac{x^5}{5}$ $\left[16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_0^2 = \frac{256}{15}$ $\int xy^2 dx = \int x(4-x^2)^2 dx = \int (16x-8x^3+x^5) dx$ $= 8x^2 - 2x^4 + \frac{x^6}{6}$ $\left[8x^2 - 2x^4 + \frac{x^6}{6} \right]_0^2 = \frac{32}{3}$ $\bar{x} = \frac{\int xy^2 dx}{\int y^2 dx} = \frac{32}{3} \times \frac{15}{216} = \frac{5}{8} *$ | M1 A1 M1 A1 M1 A1 M1A1 M1 A1 (10) |
| (b) | $A \times \bar{x} = (\pi r^2 l) \times \frac{l}{2}$ $\frac{256}{15} \pi \times \frac{5}{8} = \pi \times 16l \times \frac{l}{2}$ Leading to $l = \frac{2\sqrt{3}}{3}$ accept exact equivalents or awrt 1.15 | M1 A1 ft M1 A1 (4) |
| | | [14] |

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| 7 (a) | <p>Let speed at C be u</p> <p>CE $\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mga(1 - \cos\theta)$</p> $u^2 = \frac{9ga}{4} - 2ga\cos\theta$ $mg\cos\theta (+R) = \frac{mu^2}{a}$ $mg\cos\theta = \frac{9mg}{4} - 2mg\cos\theta \quad \text{eliminating } u$ <p>Leading to $\cos\theta = \frac{3}{4} *$</p> <p>(b) At C $u^2 = \frac{9ga}{4} - 2ga \times \frac{3}{4} = \frac{3}{4}ga$</p> <p>($\rightarrow$) $u_x = u\cos\theta = \sqrt{\left(\frac{3ga}{4}\right)} \times \frac{3}{4} = \sqrt{\left(\frac{27ga}{64}\right)} = 2.033\sqrt{a}$</p> <p>($\downarrow$) $u_y = u\sin\theta = \sqrt{\left(\frac{3ga}{4}\right)} \times \frac{\sqrt{7}}{4} = \sqrt{\left(\frac{21ga}{64}\right)} = 1.792\sqrt{a}$</p> $v_y^2 = u_y^2 + 2gh \Rightarrow v_y^2 = \frac{21}{64}ga + 2g \times \frac{7}{4}a = \frac{245}{64}ga$ $\tan\psi = \frac{v_y}{u_x} = \sqrt{\left(\frac{245}{27}\right)} \approx 3.012 \dots$ <p>$\psi \approx 72^\circ$ awrt 72°</p> <p>Or 1.3° (1.2502$^\circ$) awrt 1.3°</p> | <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1 (7)</p> <p>B1</p> <p>M1 A1ft</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (8)</p> <p>[15]</p> |
| | <p><i>Alternative for the last five marks</i></p> <p>Let speed at P be v.</p> <p>CE $\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mg \times 2a$ or equivalent</p> $v^2 = \frac{17mga}{4}$ $\cos\psi = \frac{u_x}{v} = \sqrt{\left(\frac{27}{64} \times \frac{4}{17}\right)} = \sqrt{\left(\frac{27}{272}\right)} \approx 0.315$ <p>$\psi \approx 72^\circ$ awrt 72°</p> <p><i>Note: The time of flight from C to P is $\frac{\sqrt{235} - \sqrt{21}}{8} \sqrt{\left(\frac{a}{g}\right)} \approx 1.38373 \sqrt{\left(\frac{a}{g}\right)}$</i></p> | <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> |